ANALYSIS OF BELL VIBRATIONS

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Abstract. *The initial vibration of a bell during and shortly after a stroke is analyzed. For this purpose a small bell with diameter and height of about 30 cm is investigated in detail. Different finite element models, including elements with quadratic shape functions, are created. The vibration frequencies are computed to compare them with measured acoustic data of the bell's sound. The motion of the bell is simulated with modal superposition and non-linear time integration. `GiD` [1] is used for geometry modeling, mesh generation and result visualization. Important for the communication in the interdisciplinary project is a good depiction of the mode shapes and animation of the bell motion.*

1 INTRODUCTION

Although the shape of present bells was developed in the 15th century, their acoustic behavior is not completely understood. Bells are three dimensional shell like objects which show a variety of possible vibration modes. Bell founders design the shape of bells based on experience and simple formula, which ensures that some of the resulting vibration frequencies form a minor accord. The ideal ratio of eigenfrequencies is 1:2:2.4:3:4:5:... However, real bells never perfectly fit into this shape and need to be tuned [2].

Simple models, which are based on modal analysis, describe the overall motion as superposition of different modes corresponding to the natural frequencies. Detailed analyses of acoustic recordings correspond well with the theoretical results for the continuing vibrations after the stroke. However the sound of a bell during and shortly after a stroke shows some phenomena which are beyond the scope of these models. Therefore, a finite element model of a bell is created to investigate the direct non-linear effects of a stroke on the bell's vibrations.

A small bell with diameter and height of about 30 cm was cast especially for this research project at the "Eifeler Glockengießerei" (see figure 3).

2 GEOMETRY MODELING

2.1 Symmetric Model

The bell founder provided a rough copy of the iron template which was used to determine the shape of the bell (figure 1). From this, a simple model was created by measuring points on the template and rotating them around the bell's axis. The crown was added afterwards. A tetrahedra mesh was obtained by using `GiD`'s tetrahedra mesher [1] with elements as large as possible to reduce the computational cost.

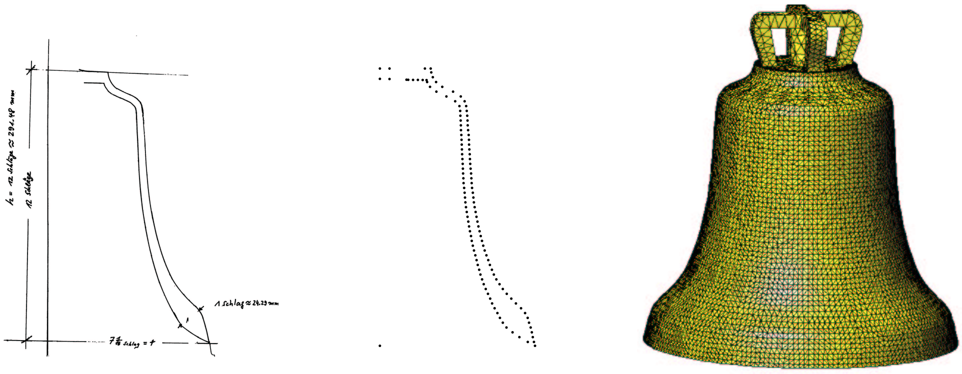

Figure 1: Template, points, and tetrahedra mesh

Additionally a hexahedra model as shown in figure 2 was created. For this purpose the cross section was divided into quadrilaterals. This section was rotated by angles of 5 degrees to obtain hexahedra volumes, which represent one finite element each. Again the crown was added afterwards.

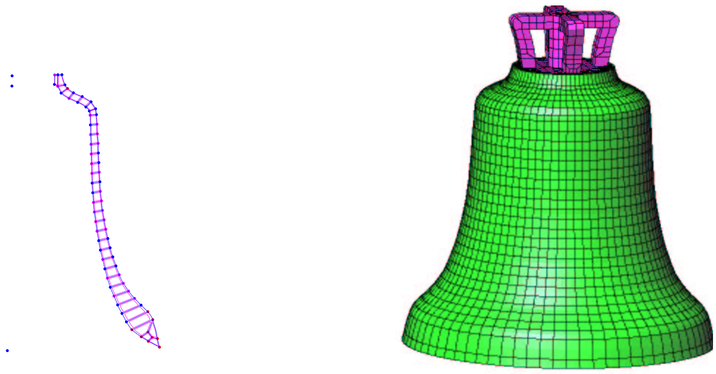

Figure 2: Bell section and hexahedra mesh

2.2 Model from Measured Data

To obtain a more precise geometry of the bell, the bell was measured at the Institute of Photogrammetry and GeoInformation. Stripes with marked points were attached to the surface of the bell as shown in figure 3. The locations of these points in three dimensional space were computed from 33 images by a bundle adjustment method using the software package PhotoModeler 5.0 [4]. The photogrammetric images were taken with a calibrated digital camera Nikon Coolpix 950 [5].

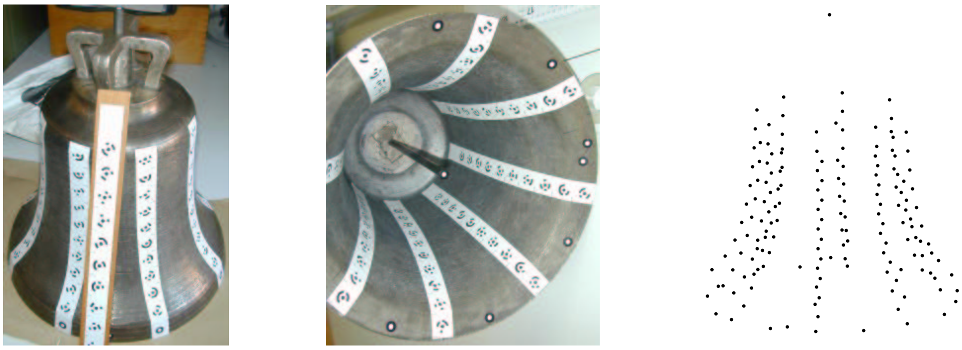

Figure 3: Bell with marked points, points in `GiD`

With this data, about 20 points of eight sections of the bell, the shape of the sections can be corrected. This yields an improved approximation, which allows a comparison of the data with some sound recordings.

3 VIBRATION ANALYSIS

3.1 Comparison of Vibration Frequencies

The finite element models were used to compute mode shapes with the subspace algorithm. This is necessary to compare the natural frequencies of the bell with those of the real bell, which are known from a Fourier analysis of the bell's sound after a stroke.

Table 1: Vibration frequencies [Hz]

Degree	Acoustic data	Tetrahedra model	Hexahedra model	
		with crown	with crown	without crown
Hum	588.54	820.26, 820.72	629.11, 629.41	628.81
Prime	1190.58, 1196.97	1552.72, 1553.23	1192.52, 1192.90	1193.68
Tierce	1444.47	1846.81, 1847.81	1531.51	1531.53
Quint	1757.10	2309.45, 2310.39	1824.23	1824.34

Table 1 shows the results for a tetrahedra model with linear shape functions (56 000 DOF) and a hexahedra model with quadratic shape functions (104 000 DOF). In both cases, the bell is not fixed at any point and can vibrate freely in space. Plots of the

a. Hum b. Prime c. Tierce

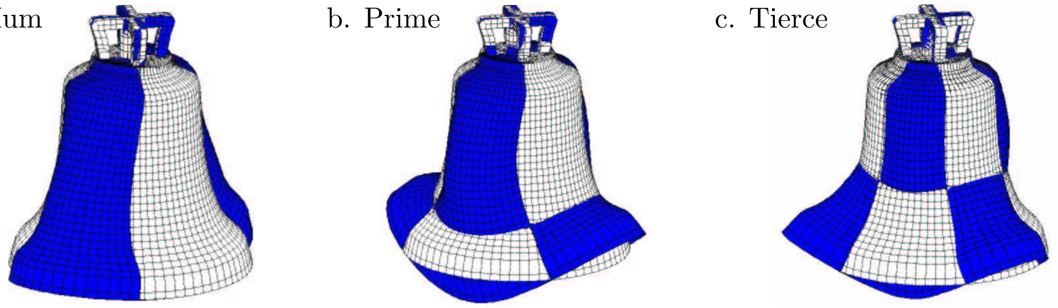

Figure 4: Vibration modes (white = undeformed mesh)

vibration modes as shown in figure 4 are used to match degrees and frequencies, as demonstrated in [3].

The results obtained with the linear tetrahedra model do not correspond well with the acoustic data due to the bad representation of bending deformation with these elements. However, using tetrahedra with quadratic shape functions leads to large systems (>300 000 DOF), which could not be solved with our equipment. The use of quadratic elements in the hexahedra models significantly improves the computed eigenfrequencies of the bell, while the crown only influences higher modes, but not the basic frequencies. The results of the final model with the measured data are not available yet, but are expected to be even closer to the measured frequencies. The present difference to the experimental data is caused by the inaccurate geometric model and only estimated material properties. After all, a rough fit is sufficient as only the general vibration phenomena are of interest.

3.2 Time Domain Analysis

In the course of a further investigation, the movement of the stroke of a clapper on the bell shall be computed with a time integration method. With this procedure the non-stationary motion is analyzed during the stroke and shortly afterwards. Within this analysis, eventually also non-linear deformations have to be considered.

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