The Quantification of Strike Pitch and Pitch Shifts in Church Bells

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Abstract

The primary objective of the work reported in this thesis was to quantify how the pitch or strike note of a bell is determined by the frequencies of its partials. Pitches of bells are generally virtual pitch or missing fundamental effects, generated in the ear rather than present as a frequency in the radiated sound. The exact pitch is shifted from that expected for the missing fundamental by changes in the frequency of various partials. This can cause bells whose partials are in theory tuned precisely, to sound out of tune by considerable fractions of a semitone.

The pitch shifts were quantified at frequencies across the audible spectrum by a set of experiments carried out on 30 subjects. Subsidiary experiments established which partials create a bell's pitch or pitches at different frequencies, and showed that partial amplitude does not significantly affect bell pitch. A simple model of pitch shift was devised from the test results which gave good agreement with the stretch tuning in a number of peals of bells. Stretch tuning has not previously been satisfactorily explained. The pitch shifts were also compared against Terhardt's algorithm for virtual pitch, which did not predict the shifts seen in practice.

To prepare for these experiments, a comprehensive investigation was done of the partial frequencies of over 2,000 bells with a wide range of dates, weights and founders. An unexpected and straightforward relationship was found between the frequencies of the upper partials which generate virtual pitches, which seems to apply to all bronze and steel bells of Western shape. The relative frequencies of these partials are in turn determined by the thickness of the bell's wall near the rim. This relationship between the partials has not been previously reported, and explains previous failed attempts by bellfounders to tune these partials independently. The modified version of Chladni's law proposed by Perrin and Rossing for these partial frequencies was found not to give as good a fit to their frequencies as the relationship discovered in this research.

The work presented in this thesis is important for at least two reasons:

- It provides new practical guidance for the design and tuning of bells
- The shifts in virtual pitches observed as a result of upper partial changes support current research into pitch generation mechanisms in the human ear.

Acknowledgements

I am grateful to my supervisor Dr Shahram Taherzadeh and his colleague Dr David Sharp, both of the Open University Acoustics Research Group, for their constant insight, encouragement and support over the past five and a half years.

Robert Perrin, Ernst Terhardt and the late André Lehr have proved good teachers and correspondents, willing to impart knowledge and make suggestions for areas of fruitful investigation. Ernst Terhardt kindly allowed me to translate his 1984 paper into English.

I am also grateful to David Kelly and Stephen Ivin who took part in many discussions about bell acoustics in the early stages of my investigations, and acted as sponsors when I applied to The Open University to embark on the formal research reported in this thesis.

My thanks are due to the staff of The Whitechapel Bellfoundry and Taylors, Eayre and Smith Ltd, and in particular to Nigel Taylor, Andrew Higson and David Marshall for many interesting discussions on bell tuning and acoustics. The subscribers to the Bell Historians Yahoo Group have provided historical insights, details of peals of bells, and descriptions of historical tuning practices which are not documented elsewhere. I wish to thank the many bellringers and church authorities who allowed me to record and analyse their bells.

Over 50 people participated in one or more experiments and I am grateful to them for their efforts and test results.

Thanks are due to my son Christopher who was a trial subject for all the experiments done as part of this research and made many helpful suggestions. He also helped take many of the bell recordings in my collection.

Finally, this work would not have been possible without the loving and patient support of my wife Margaret, who believed in the project from the start, even though she understood how much of my time it would take. Her encouragement and co-operation have been essential and invaluable.

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- Figure 3-6 from (Elphick 1988) page 56
- Figure 3-7 from a photograph supplied by David Bryant
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1 INTRODUCTION

The acoustics, and in particular the musical pitches, of bells are of considerable interest to both musicians and acousticians. Typical wind or stringed instruments have harmonic partials whose frequencies are related by the ratios of small integers. Bells, like gongs, drums, cymbals and other percussion instruments, have inharmonic partials. At the same time, unlike many percussion instruments, bells have a defined note or pitch, and a set of bells can be tuned as a melodic instrument. It is the unique position of bells on this borderline between inharmonic and melodic instruments that makes them of such interest. Investigation of the mechanisms that give rise to the perceived pitch of bells provides insight into the general problem of pitch perception.

An example of the inharmonic partials of handbells is given by Helmholtz (1877 p. 73) in his ground-breaking work on musical acoustics: *'... inharmonic tones always disturb the harmony most unpleasantly ... A very striking example of this was furnished by a company of bell-ringers, said to be Scottish, that lately travelled around Germany, and performed all kinds of musical pieces, some of which had an artistic character. The accuracy and skill of the performance was undeniable, but the musical effect was detestable, on account of the heap of false secondary tones which accompanied the music, although care was taken to damp each bell as soon as the proper duration of its note had expired, by placing it on a table covered with cloth.'*

To assist with the reading of this thesis by those unfamiliar with the acoustics of bells, a glossary of technical terms is provided as Appendix 3.

A bell has many different modes of vibration, each of a defined frequency. The relative frequency of the vibrations is governed by the shape or profile of the bell. Different cultures have developed different bell shapes, but this study is confined to bells of conventional western shape. This particular shape was arrived at via heuristic methods in the middle ages. One of the characteristics of bells with this profile is that they tend to have a single dominant pitch or note.

An example of the spectrum of the sound of a bell (the old number 9 bell at St Mary le Tower, Ipswich) is given as Figure 1-1. It shows the significant partial frequencies and the names they are given. The vertical axis is loudness in decibels (dB), calculated as $20 \cdot \log_{10}\left({}^{a}\!/_{a_0} \right)$ where a is the amplitude of the partial and a_0 the amplitude of the loudest partial, in this case the superquint. The analysis was over the first 2s following the clapper strike. This short sample length is used because the higher frequency partials die away quite rapidly.

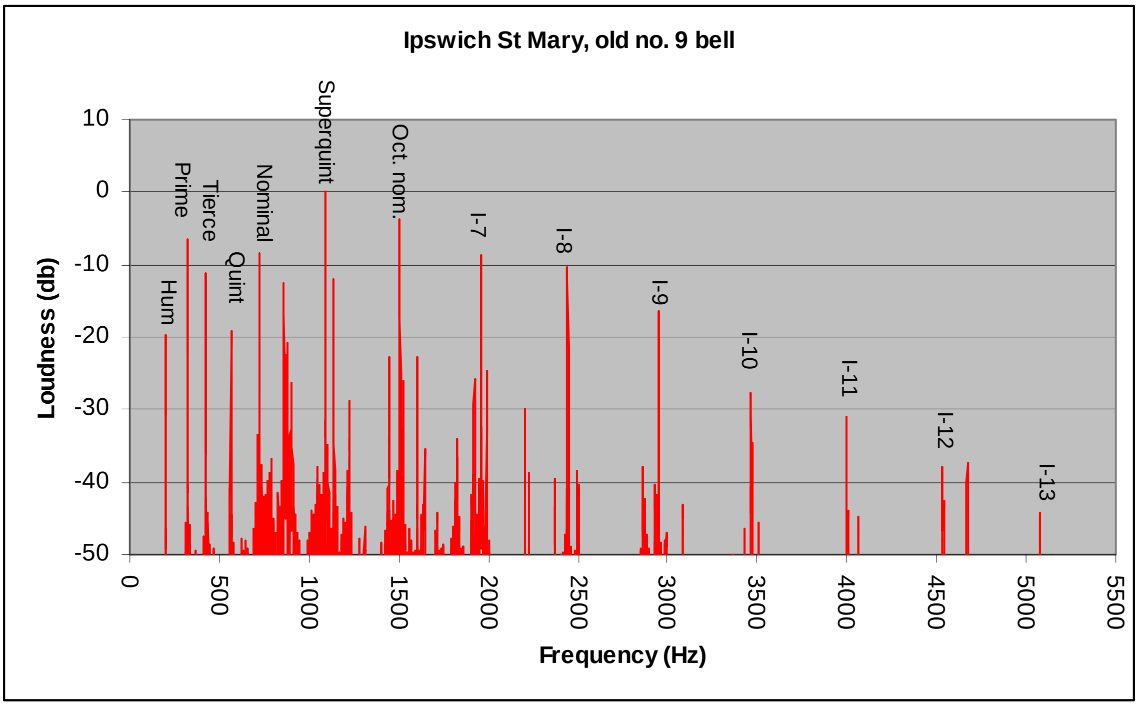


Figure 1-1 Spectrum in dB of Ipswich St Mary le Tower old 9th bell

Table 1-1 below shows the frequencies and musical notes of the main partials highlighted above. These frequencies were measured from a spectrum taken over 10s of sound. For an explanation of the constraints on frequency measurements in this thesis see section 1.3 and appendix 9. The information in the notes column comprises a note name, an indication of the octave, and a deviation in cents (hundredths of a semitone) from the equal-tempered note with A=440Hz.

Partial	Freq (Hz)	Note	Ratio to lowest frequency	Ratio to nominal
Hum	201.9	G#(3) - 48	1.00	0.28
Prime	323.7	E(4) - 30	1.60	0.45
Tierce	424.2	G#(4) + 37	2.10	0.59
Quint	562.8	C#(5) + 26	2.79	0.78
Nominal	723.4	F#(5) - 38	3.58	1.00
Superquint	1091.4	C#(6) - 26	5.41	1.51
Octave Nominal	1507.3	F#(6) + 32	7.47	2.08
I-7	1959.3	B(6) - 13	9.70	2.71
I-8	2443.2	D#(7) - 31	12.10	3.38
I-9	2948.3	F#(7) - 6	14.60	4.08
I-10	3467.6	A(7) - 25	17.17	4.79
I-11	4000.8	B(7) + 22	19.82	5.53
I-12	4536.5	C#(8) + 39	22.47	6.27
I-13	5077.1	D#(8) + 34	25.15	7.02

Table 1-1 Partial frequencies of Ipswich old 9th

From this table it is clear that no partials are related to the lowest frequency (the hum) by small integers, so that based on the hum the sound is inharmonic. The only partials close to harmonic are the nominal, superquint and octave nominal with ratios approximately $2/2 : 3/2 : 4/2$. The strike pitch of this bell (about 362Hz) does not appear as a frequency in the bell's sound, a phenomenon known as the strike note paradox. Neither the lowest partial (the hum) nor the loudest partial (the superquint) determine the pitch that is heard.

There is a distinction to be drawn between frequency and pitch. Frequency (e.g. of one of the partials above) is measured directly by instruments, either by observing the physical vibration of the bell, or by identifying frequencies in the emitted sound. These two are equivalent, apart from amplitude, and a particular mode of vibration in the bell, if it has an audible effect, gives rise to that frequency of vibration in the surrounding air.

Pitch, which is defined as that characteristic of a sound which allows it to be ordered in a musical scale, is not directly measurable – there may be no frequency of vibration in the bell or the surrounding air which corresponds with the observed pitch. Pitches only exist because we hear them, and the only way to assign pitch is to use our ears to compare one sound with another. Perceived pitch may not be single valued (we may hear more than one pitch in a sound) and may differ from one listener to another.

There are two alternative models of the pitch of bells: the partial frequency view – that the pitch is an octave below the nominal, and the holistic view – that pitch is some overall effect of the bell, not linked to one partial. The assumption used by most if not all current bellfounders is that the pitch is determined by the nominal, even though the weight of scientific evidence supports the holistic model. Confirmation that Taylor's bellfoundry traditionally record pitch as the nominal frequency is given in (Ayres 1983). Fletcher & Rossing (1998 p. 685) say *'Whether a founder tunes the nominal or the strike note makes little difference, however, because the nominal is one of the main partials that determines the tuning of the strike note'*.

Experiments show that the pitches of bells are generally virtual pitches, i.e. they arise from a missing fundamental effect, most commonly based on the nominal, superquint and octave nominal partials. If these partials were exactly harmonic the pitch would be exactly an octave below the nominal. However, as they are inharmonic the pitch is shifted from the half nominal. This shift can be a considerable fraction of a semitone, and so is musically significant; bells whose nominals are tuned exactly can have pitches which are audibly out of tune. This lack of acknowledgement of the difference between pitch and nominal frequency and of the associated pitch shifts is remarkable given the experimental evidence, but is due in part to the difficulty in measuring pitches.

1.1 Core proposition of this thesis

The core proposition of this thesis, to be proved through a range of experiments and investigation of the partial frequencies of more than 2,000 bells, has two parts. The first is that a set of approximately equally-spaced partials with frequencies roughly $2f$, $3f$, $4f$, etc. will give rise to a sensation of pitch at frequency f much stronger than the pitch sensation arising from any individual partial frequency. The second part of the proposition is that if this set of partials is shrunk together, the pitch flattens; if they are stretched apart, the pitch sharpens. Such a set of equally-spaced partials is found in all bells of western shape. In Figure 1-1 above, the nominal, superquint, octave nominal and partials I-7 upwards form such a series.

Both parts of the core proposition are documented in the literature. The new work in this thesis explores the actual pitch experienced by test subjects, provides further confirmation of the first part of the proposition, and shows for the first time the effect of the dominance region for virtual pitch on the pitches experienced by the subjects. A set of virtual pitch experiments quantifies the shifts in virtual pitch across the audible spectrum in a way that for the first time is directly applicable to the design and manufacture of bells.

1.2 Overview of chapter contents

The remainder of this introduction gives a brief overview of each chapter of the thesis.

1.2.1 Chapter 2 Historical Perspective and Literature Survey

This chapter gives a brief description of, and references to, significant published information on the acoustics of bells. It also gives an overview of pitch models and pitch perception, but does not go into detail as pitch detection and hearing mechanisms are not directly investigated in the current research. There is an emphasis on historical context; the history of investigations into bell acoustics has close parallels with the development of musical acoustics in general. The rest of chapter 2 covers the following subjects, illustrated by reference to the literature:

- Investigations into the partials of bells
- Theories of pitch perception
- Pitch shifts in complex tones and the dominance region for virtual pitch
- Previous studies into the pitch of bells
- Measurement techniques for partial frequencies and for pitch.

1.2.2 Chapter 3 Introduction to Bell Acoustics

This chapter gives a brief introduction to bell acoustics for those unfamiliar with the subject. The topics covered are as follows:

- An example of the spectrum and associated partial frequencies of a bell
- Examples of the historical development of the shapes of bells
- Explanation of the naming and numbering of bells in peals and carillons
- An overview of the processes by which bells are cast and tuned, together with a discussion of doublets and of true harmonic tuning
- An analysis of partial amplitude from recordings of a number of bells, highlighting the acoustically important partials
- An explanation of the virtual pitch effects which dominate pitch perception in bells, and the effect in heavier bells known as ‘secondary strike’
- Examples of virtual pitch shifts experienced in practice, with evidence of the need for stretch tuning to make small bells sound ‘in tune’

At the end of the chapter, examples are given from existing bellfounding practice (including investigation of the weights and partial frequencies of 1,759 bells) to show that heavy, thick bells have closer upper partials than thin, light ones. Small bells in a peal or carillon are usually cast proportionally heavier than larger ones for mechanical reasons and to increase the volume of their sound. It is proposed that this practice causes them to sound flat due to virtual pitch effects. This hypothesis provides the background to and motivation for the experiments described later in the thesis.

1.2.3 Chapter 4 Investigation into the Pitches of Bells

This chapter describes two experiments into the pitch or pitches perceived in bells. The first experiment investigates the phenomenon in large bells known as 'secondary strike' whereby some listeners perceive the bell to have a second strike pitch, often a musical fourth above the nominal pitch. The second experiment extends the technique developed for the first experiment to investigate what determines the pitches of bells across a range of frequencies and demonstrates the effect of the dominance region for virtual pitch.

At the end of the chapter, a brief discussion of timbre of bell sounds leads into a description of the effect of the dominance region for virtual pitch on the timbre of bells, especially those with non-true-harmonic tuning.

1.2.4 Chapter 5 Partial Classification Schemes and Frequencies

This chapter documents investigations into the following areas:

- The partial classification schemes proposed by André Lehr and Robert Perrin were compared, and reconciled with a recording of the actual bell investigated by Perrin
- A simple model for the relative frequencies of the upper partials of bells was derived from analysis of the partial frequencies of 2,066 bells. This model is a new discovery, and considerably simplifies the virtual pitch experiments reported in this thesis
- Before and after tuning figures for the Berlin Freedom bell illustrate this model and the effect of bell tuning on upper partials
- The 2,066 bells were compared with a model of upper partial frequencies previously proposed by Perrin and Rossing based on Chladni's law. The modified Chladni model was found not to explain the relationship between partial frequencies seen in practice, though it did explain why expressing partial frequencies as cents (i.e. as logarithmic ratios) makes the relationships linear.

This work was originally undertaken just with the aim of simplifying the virtual pitch tests. As it progressed, substantial new insights emerged not previously reported in the literature. The reconciliation between Lehr's and Perrin's classification schemes based on analysis of the sound of the actual bell used by Perrin has not been done before. The investigation into upper partial frequencies and the model developed for their relationship is also new work, and provides a robust explanation of some effects in bell design not previously understood.

1.2.5 Chapter 6 Practical Issues in Pitch Measurement

This chapter describes a number of different candidate designs for the virtual pitch tests reported on in this thesis, and gives justification for the choice of approach used. The extensive trials used to validate the approach and choose the test parameters are described. Literature references are given to a number of studies in pitch determination which further justify the choice of experimental method.

The chapter goes on to describe the detailed design of the virtual pitch tests, and present the results of various trials of the tests to prove their validity. Issues concerning equipment used to conduct the tests and musical skills of the test subjects are covered.

1.2.6 Chapter 7 Statistical Design of Virtual Pitch Tests

This chapter explains and justifies the statistical design of the four-factor analysis of variance employed in the virtual pitch tests. A large number of exploratory experiments were carried out to validate the experimental procedure, establish the range of various effects so that tests were appropriately calibrated, and to provide sample data to validate the analysis techniques. The results of some of these tests are used in this chapter to illustrate and validate the test design.

1.2.7 Chapter 8 Further verification of virtual pitch tests

The results of two further experiments conducted by the author are described in this chapter. The first uses an alternate experiment design to verify and confirm the pitch shifts measured in the virtual pitch experiments. The second explores the effect on virtual pitch of changes in partial amplitude of some key partials, confirming that the effect is not significant.

1.2.8 Chapter 9 Virtual Pitch Test Results

This chapter documents the results of the virtual pitch tests. These tests quantify the changes or shifts in virtual pitch in bell-like sounds as a result of changes in frequency of various partials. 9 sets of tests explore pitch shifts at various points in the audible spectrum.

The tests, using 30 test subjects, were successful and showed to a high level of statistical significance that all test subjects experienced virtual pitch shifts. The upper partials had a statistically and musically significant effect at all frequencies, the prime and tierce a smaller effect. From these results, a simple numerical model of pitch shift was devised.

The chapter concludes with an investigation of the effect of musical experience of the test subjects, and a comparison of the precision of the chosen test methods with other candidate designs.

1.2.9 Chapter 10 Validation of Pitch Shift Model

Two pieces of work are documented in this chapter. The first compares the observed pitch shifts with the results of Terhardt's model of virtual pitch. The shifts observed are not predicted by the Terhardt model, showing that the effect measured in the virtual pitch tests was not catered for in his model design.

The second piece of work involved the analysis of the tuning of a number of peals of bells. Some of these are tuned with stretch tuning, i.e. with sharp treble bells. The model of pitch shift derived in the previous chapter gave a good quantitative prediction of the degree of stretch employed by the bell tuners, and showed that stretch tuning was employed to compensate for pitch shifts. This gives elegant and convincing confirmation of the existence of the virtual pitch effect and the validity of the derived model.

1.2.10 Chapter 11 Concluding Remarks

This chapter covers the following subjects:

- Practical applicability of the experiment results
- Restrictions of the derived model; in particular the need to investigate the effect of the hum partial on virtual pitch and the relative contribution of virtual and spectral effects to the pitch of small bells
- The opportunities for further investigation into factors affecting the timbre of bells

- Loose ends and areas for further investigation
- Relevance of the research results to general research into pitch perception.

1.2.11 Appendices

There are a number of appendices, as follows:

- Details of all specific references from this thesis to the literature and other similar material
- A general bibliography of published materials consulted during the research. Subjects covered include general acoustics and musical pitch; bell acoustics; general and topographical information on bells; mathematical techniques including FFT, matrix calculations and statistics; temperaments and tuning; and a list of publicly available CDs of bell sounds and music
- A glossary of technical terms specific to bell acoustics, and a glossary of basic terms in statistics
- Translations of portions of Isaac Beeckman's 1633 journal and letters giving the earliest known written account of multiple partials in bells
- The text of two articles on bell acoustics published by the author during the course of this research
- The author's translation from German to English of Terhardt's 1984 paper on the strike pitch of bells
- Calibration of the Wavanal program used for tuning analysis
- Details of all the test and reference sounds used in the virtual pitch tests.

1.3 Frequency measurements in this thesis

All the partial frequency measurements in this thesis were taken from recordings of bells using the author's 'Wavanal' programme. For a description of this programme and its calibration see appendix 9. The main factor affecting the precision with which frequencies can be measured is recording length. It is a feature of spectral analysis in Wavanal that for a recording of length t seconds, the spectral energy appears in bins $\frac{1}{t}$ Hz wide. This in turn implies that frequencies in the spectrum can be measured to ± $\frac{1}{2t}$ Hz. For example, frequencies in the spectrum of a sound sample of length 2 seconds are spaced 0.5Hz apart.

Frequency measurements in this thesis were generally taken from 2s samples, giving bin widths of 0.5Hz and measurements precise to ±0.25Hz. When partial frequencies measured in this way are quoted they are given with no decimal place or a final 0.5 as appropriate (e.g. 200Hz or 200.5Hz).

More precise measurements used sound samples 10s or more long, implying a bin width of 0.1Hz or less. Where this is the case, partial frequencies are given with one digit to the right of the decimal point, even if this digit is zero (e.g. 200.0Hz or 200.1Hz). Where partial frequencies are quoted in this way it can be assumed that sound samples of at least 10s in length were used.

Where very long recordings are available, more precise measurements are possible, in principle giving further significant decimal places. However, temperature effects (described towards the end of section 3.1.1) and issues with the equipment used to take recordings (described in appendix 9) mean that quoting partial frequencies with more than one decimal digit is not realistic (with the one exception of the frequency calibration measurements in appendix 9).

2 HISTORICAL PERSPECTIVE & LITERATURE SURVEY

2.1 Introduction

This chapter gives a brief description of, and references to, significant published information on the acoustics of bells, with a bias towards investigations into bell pitch. It also gives a general overview of pitch models and pitch perception, but at a high level, as pitch detection and hearing mechanisms are not investigated in the current research. There is an emphasis on historical context, as the history of investigations into bell acoustics has close parallels with the development of musical acoustics in general. The rest of this chapter covers the following subjects, illustrated by reference to the literature:

- Investigations into the partials of bells
- Theories of pitch perception
- Pitch shifts in complex tones and the dominance region for virtual pitch
- Previous studies into the pitch of bells
- Measurement techniques for partial frequencies and for pitch.

2.2 Studies into the partial structure of bells

It has probably always been recognised that bells vibrate at a range of different frequencies (different partials can easily be heard unaided by any instrument by striking the bell at different points in its height). However, the contribution of the various partials of a bell to its timbre, and the ability to tune the partials independently and so control the timbre, is a relatively recent discovery, first made in the 1630s, but for various reasons lost, and then rediscovered in the late 19th century

The first published work showing that musical sounds could consist of more than one partial frequency or overtone, and that these overtones contributed to the quality or timbre of a musical sound, was that by Nicole Oresme in 14th century France. This knowledge was rediscovered by a number of researchers in the 17th century. Isaac Beeckman, a Dutch scientist and philosopher, recounts in his diary (Beeckman 1633a) a meeting with Jacob van Eyck, who described to him the partial tones of a bell. Van Eyck was the musical brains behind the Hemony brothers, famous Low Country bell founders who were the first founders to methodically tune the partials of bells. Beeckman was also in correspondence with Marin Mersenne, who was the first to publish (in his Traité de l'Harmonie Universelle of 1636) the fact that vibrating bodies could do so with more than one frequency. Details of this fascinating correspondence, which I have had translated from the original Dutch and Latin, are included as Appendix 4.

Helmholtz (1877 p. 72) makes passing reference to the partial tones of bells and it is clear that it was common knowledge that bells had multiple partials. He quotes the notes for the partials of a bell 'Gloriosa' cast in 1477 in Erfurt which this author has verified against a recent recording of the bell. He also explains, probably as a result of his own work on glass bowls and metal plates, that bell partials have various numbers of nodal meridians spaced equally around the bell. He also notes the effect of doublets, saying *'If a bell is not perfectly symmetrical about its axis, if, for example, the wall is a little thicker at one point of its circumference than at another, it will give, on being struck, two notes of very nearly the same pitch, which will beat together'*.

The first scientific investigation of bell partials and bell pitch was done by Lord Rayleigh who investigated the partials of a number of bells and explained the phenomenon of doublets, though he did not use this term (Rayleigh 1890). Rayleigh used doublets to determine the number of nodal meridians (nodes running from the lip to the shoulder of the bell) for the six lowest frequencies of vibration.

Simpson (1895, 1896) refers to an article in Encyclopaedia Britannica of 1815 explaining that *'the sound of a bell is principally composed of a fundamental, or octave, and of a third'*. Simpson was the first writer to suggest the effect on bell timbre of the tuning of the various partials, and to document rules for tuning them. It is probable that his knowledge was obtained not just through his own research, but also through discussions with bellfounders, in particular Taylors of Loughborough, who at the time that Simpson was writing established the practical rules for tuning of partials, and re-introduced 'true harmonic' tuning (in which the hum, prime and nominal of a bell are tuned in octaves) which since then has become the predominant tuning style because of the resulting good quality of the bell sound. See (Hibbert 2002) included as appendix 6 for an account of this. Taylor's breakthrough owes much to the shape of their cast bells as well as subsequent tuning, a shape which may well be derived from their research into continental and Hemony bells.

Through the 20th century a number of researchers carried out increasingly detailed and authoritative investigations into the modes of vibration of bells. Jones (1928) did much of his work using tuning forks, Helmholtz resonators and hand-calculated Fourier transforms, but an increasing use of electronic methods as explained by Tyzzer (1930), Curtiss & Gianni (1933), Slaymaker & Meeker (1954), and Grützmacher, Kallenbach & Nellessen (1965) allowed progressively more detailed identification of frequencies, nodal circles and nodal meridians. Van Heuven (1949) reports on a large number of bells he was able to investigate because they had been removed from their towers by the occupying German forces during the 1939-1945 war – though his results are limited by the measurement equipment at his disposal.

The two definitive studies of bell partials were carried out by Lehr (1965, 1986) and Perrin, Charnley & De Pont (1983). André Lehr provides the practical bell-founder's view (he was latterly Managing Director of the Eijsbout bellfoundry in Asten). He groups the many partials according to the number and position of their nodal circles in a manner of practical benefit to the bell-tuner. A useful summary of Lehr's work from a practical perspective is given in (Ayres 1983). Perrin, Charnley and De Pont carried an exhaustive practical and theoretical investigation of a bell from Taylor's bellfoundry, identifying 134 separate modes of vibration. They built a finite element analysis model of the bell which predicted with reasonable accuracy the modes as observed in practice, and helped identify some they had missed. They proposed a classification scheme based on nodal circles and meridians, though in later papers (Perrin & Charnley 1987a, Perrin & Charnley 1987b) they explain that their identification of nodal circles was sometimes in error.

The Lehr and Perrin studies are of such practical importance that a detailed comparison of them, and of the Perrin results against a recent recording of the bell they investigated, forms an important part of chapter 5 of this thesis.

At least two other investigations into bell partials have recently been reported in the literature. (McLachlan & Cabrera 2002) and (McLachlan 2002) describe work to design radical new bell profiles with special arrangements of partials using finite element design. A number of bells have been manufactured to the new designs. The bells meet the musical objectives set but sound quite different from traditional bells of Western profile. Özakça & Göğüş (2004) report on work, also using finite element design, to optimise the shape of bells of Western profile to produce an exact musical relationship for the lowest seven partials. As far as I am aware no bells have yet been manufactured to their designs.

2.3 Theories of pitch

Pitch may be defined as the characteristic of sounds that allows them to be ordered in a musical scale. This definition has two aspects:

- Pitch determination involves comparison; the pitch of a sound is determined as being higher or lower than other sounds
- This comparison necessarily involves the human ear; there is no way to measure pitch with an instrument alone.

The importance of pitch to the study of bells is that it is the pitch of a bell that determines whether the listener will judge it to be in tune with others in a set, vital to the quality of bells as musical instruments.

There is general agreement that the perception of pitch arises from physiological mechanisms in the ear, audio nerve and / or brain. However, there are two competing theories of pitch perception. Both of these theories are supported by experimental evidence, and neither provides a full explanation of all the observed effects, especially with complex tones, i.e. those consisting of multiple partial frequencies. The two theories are:

- The ‘place’ theory, which proposes that the pitch perception arises because particular parts of the ear respond to specific partial frequencies
- The ‘pattern’ theory, proposing that the ear perceives pitch according to the overall shape of the sound waveform or repeating patterns in it.

Individual people can perceive pitch in a way supported by either model under appropriate circumstances.

It is not proposed to provide further explanation of these two theories in this thesis, nor details of the physiology of the ear, because this is not necessary to an understanding of the current research. Good comparison of the models with many references to other work appear in (de Cheveigné 2004) and (Roberts 2005).

One practical aspect of pitch perception is however of great importance to the current research. Sounds whose partials comprise a harmonic series (a fundamental frequency together with overtones at 2 times the fundamental, 3 times, 4 times etc.) provide a strong perception of pitch at the fundamental frequency. This perception of pitch, remarkably, persists if the fundamental frequency is removed from the sound. This effect is variously known as the missing fundamental, residue pitch, or virtual pitch. For clarity, the term virtual pitch is used for this effect throughout this thesis. The effect is very powerful and generally dominates pitch perception even when individual partials can be clearly heard. For example, Moore (2004 p. 207) says *'the perception of a residue pitch should not be regarded as unusual. Rather, residue pitches are what we normally hear when we listen to complex tones'*.

Virtual pitch has been shown to make a critical contribution to the perceived pitch of bells. The use of the description ‘missing fundamental’ for this effect was probably coined by Fletcher. His work was first applied to bells by Jones (1928). Schouten, who coined the phrase residue pitch for this phenomenon, applied his investigations to the pitch of bells in

(Schouten & ‘t Hart 1965). Ernst Terhardt, who was the first to use the phrase ‘virtual pitch’ for this effect, proposed an algorithm for the determination of pitch in the case that partials are inharmonic (Terhardt, Stoll & Seewann 1982a, 1982b), and applied his theory to bells (Terhardt & Seewann 1984) in a paper which was translated into English by the author as part of this research. The English translation is included as Appendix 8. Both of these papers are further described below.

Two different modes of listening have been identified, known as spectral and virtual pitch. These two modes can be experienced by different listeners, and by the same listeners in different circumstances. There is a parallel between these two listening modes and the place and pattern theories of pitch perception. In the case of spectral pitch perception, the pitch or pitches of a sound are determined by the frequencies of individual partials even if multiple partials are present. In the case of virtual pitch perception, multiple partials contribute holistically to the perceived pitch. Both listening mechanisms are encountered when investigating the pitch of bells. In particular, bell tuners, because they spend their working life hearing out individual partials, tend to experience spectral pitch in circumstances where other listeners experience virtual pitch. This is the author’s experience, as well as that of others (Perrin, private communication, 2006), and (Andrew Higson, private communication, 2007).

In the case of virtual pitch, because all the partials contribute in a complex way to the overall pitch perception, the principle of linear superposition emphatically does not apply.

2.4 Pitch shifts and the dominance region for virtual pitch

Plomp (1967a) gives a good overview of the early history of investigations into the pitch of complex tones, i.e. sounds with multiple partial frequencies. (de Cheveigné 2004) and (Roberts 2005) update this historical perspective with the results of later research.

Where the partials in a sound are inharmonic, shifts in the frequency of any of the partials can shift the perceived virtual pitch. There is robust experimental evidence for this effect. As explained by Plomp (1967a), the effect was first noted by Hermann in 1912, though the cause at the time was not understood. Plomp cites work by de Boer in 1956 showing that shifts in the upper partials of a complex tone changed the perceived pitch. His own experiments (Plomp 1967a) confirm the effect. Patterson & Wightman (1976) report on further experiments which not only confirm the effect, but demonstrate that the size of the effect depends on the absolute position of the complex tone and virtual pitch in the audible spectrum, being larger at lower frequencies. The existence of the virtual pitch shift in piano tones with inharmonic upper partials is demonstrated by experiment in (Järveläinen 2003), and both the shift and the enhanced effect at lower frequencies, are further demonstrated in experiments on piano tones described by Anderson & Strong (2005).

The experiments described in (Plomp 1967a) showed no significant difference in pitch shifts between the two cases with the amplitude of all partials was equal, and with it falling off as $1/n$ where n was the partial number. The experiments in (Järveläinen 2001) were carried with partials falling off as $1/f$ where f was the partial frequency. Most other work on virtual pitch shifts in the literature uses partials with constant amplitude. The tentative conclusion from the literature is that partial amplitude has a minor effect if any on virtual pitch shifts. Other work, for example as described in (Lin & Hartmann 1996) investigates the effect of missing components of the harmonic series, as a side effect of other experiments. The consensus in the literature is that, with complex tones with half a dozen or more higher partials, a virtual pitch can be experienced with up to four of the lowest partials of the harmonic series missing. Both of these effects are investigated and confirmed by the experiment described in section 8.3 of this thesis.

Despite the strong evidence for the shift in virtual pitch when upper partials change, it has not been universally recognised. Moore (2004 p. 209) quotes Ernst Terhardt as saying *'a residue pitch will always be a subharmonic of a dominant partial'*. Terhardt in his description of the virtual pitch model (Terhardt, Stoll & Seewann 1982b) uses the term 'pitch shift' to refer to a different effect - that the pitch of inharmonic partials can be affected by other frequencies in a complex tone, and that partial pitch depends on intensity. Experimental confirmation of shifts in the pitch of inharmonic partials is given in (Lin & Hartmann 1996). This effect may be related to shifts in virtual pitch, but Terhardt's virtual pitch algorithm does not predict the virtual pitch shifts experienced in practice, as demonstrated in chapter 10 of this thesis.

Another important concept is that of a dominance region for virtual pitch. Virtual pitch is commonly experienced in a restricted frequency band. Moore (2004 p. 209) continues *'Terhardt suggested that these dominant partials lie in the region between 500 and 1500Hz'*. The experiments reported on by Plomp (1967a) suggest that virtual pitch effects apply for fundamental frequencies below 1400Hz. For fundamentals above this frequency, spectral pitch for a single partial predominates. Pfundner (1962), after successfully explaining the strike pitches of four bells by looking for near harmonic partials, goes to speculate that Fletcher-Munson or equal loudness effects explain why secondary strike pitches become apparent in large bells, and why small bells have spectral pitches based on their low partials.

In the description of the virtual pitch algorithm in (Terhardt, Stoll & Seewann 1982b) there is a discussion of the dominance region (called there spectral dominance), citing experimental evidence from a number of researchers. Plack & Oxenham (2005 section 3.2) give an updated review of research results. Although there is no definite conclusion about the lower and upper limits of the dominance region for virtual pitch, there is no doubt as to its existence, and that the region is centred near 600 to 800Hz. Various researchers report that pitch becomes very difficult to judge at frequencies above 5000Hz.

There is a possible explanation of the dominance region for virtual pitch effect in terms of the sensitivity of the ear to tones of different frequencies. The effect for pure tones (i.e. comprising a single frequency) is well understood and quantified in published work. Examples are the so-called Fletcher-Munson and Robinson-Dadson equal-loudness curves, and the A-weighting used in acoustic instruments to compensate for the different sensitivity of the ear at different frequencies. However, to the author's knowledge no attempt has been made to extend this work to explain the virtual pitch dominance region.

The virtual pitch shift effect and the virtual pitch dominance region are both confirmed and quantified for bells by the experiments reported in this thesis.

2.5 Pitch of bells

As explained in the introduction to this chapter, there are two theories of the strike pitch of bells, related to the distinction between spectral and virtual pitch:

- The partial frequency view – that the pitch is an octave below the nominal
- The holistic view – that pitch is some overall effect of the bell, not linked to one partial.

Rayleigh (1890) was the first to scientifically investigate this matter. He noted with great surprise that the notes of the bells he investigated did not match any of the frequencies of vibration he observed. In particular, a bell supplied to him by Mears and Stainbank for his investigations came with a tuning fork tuned by the founders to the strike pitch. The frequency of the fork was half the nominal frequency of the bell but did not correspond to any of its partials. Rayleigh proposed, without offering any explanation, that the pitch of a bell was half its nominal frequency. This has been adopted as standard practice by all bellfounders, who tune the nominal partial to ensure bells are in tune with each other, as noted in (Ayres 1983) and (Fletcher & Rossing 1998 page 685), both quoted earlier.

Interestingly, the Mears and Stainbank foundry at Whitechapel continued to work with 'tap tones' about an octave below the nominal until the 1920s but then changed their practice to tuning of the nominal. For an example of the practice see the report by Whitechapel on the bells of Coventry Cathedral in (Young 1926). The following description of the approach they used (from an informal communication from Richard Offen, onetime employee of Mears and Stainbank) shows they were tuning strike pitches holistically, not by tuning the nominal (Offen 2006): '*... at the time Stockport were put in [1897], Whitechapel were making a set of forks for each ring they produced. If I remember the process correctly, as related to me by Bill Hughes, forks were made to correspond with the strike note of each bell as cast. The tuning forks were then taken to a quiet corner of the foundry and adjusted to give a set of forks that sounded in tune to the tuner. These forks were then used to adjust the strike note of each bell, presumably by ear and not by placing the fork on the bell to get a sympathetic resonance. Bill Hughes always used to quote the note of a bell giving a frequency as half that of the nominal.*'

In work since Rayleigh's initial investigation, confusion over strike pitches has arisen in at least three different areas. First, in many bells, especially those of better quality tuned according to true-harmonic principles, the prime partial is an octave below the nominal. Even though the prime does not make a major contribution to strike pitch, some authors have assumed the contrary. The prime partial is often called the fundamental because of this, a practice the author avoids to remove the confusion.

Secondly, although there is no physical frequency at the strike pitch (unless the prime happens to lie there), various authors have supposedly demonstrated beats between the bell's pitch and a tuning fork or sine tone. These beats actually arise because of a phenomenon called secondary beats, whereby two tones an octave or other harmonic interval apart can form beats under appropriate circumstances. This effect was investigated and confirmed by Plomp (1967b) but it still causes confusion. For an example see the unpublished paper (Swallowe & Perrin 2001). The effect can confuse bellfounders too, as explained to me by Nigel Taylor, head tuner at the Whitechapel bellfoundry, and can easily be demonstrated with simulated bell sounds.

Thirdly, some investigators who accept a holistic (missing fundamental) theory for the strike pitch attempt to explain it as a difference tone between the generating partials. This explanation fails to predict the direction of change of the strike pitch when various partials move. When the partial frequencies generating the strike pitch change, the difference tone explanation of pitch predicts that the strike pitch changes in the opposite direction to that observed in practice, as explained by Schouten & ‘t Hart (1965).

Several authors have advanced the hypothesis that the strike pitch of bells is of very short duration. I believe this hypothesis was attractive as a way of explaining the inability to detect vibrational energy at the strike pitch. However, in their research on the strike pitches of bells, Ernst Terhardt and his colleagues showed (Terhardt and Seewann 1984) that the perceived pitch of bells was the same when the sound sample comprised:

- 3s of sound starting with the clapper impact
- 100mS of sound starting with the clapper impact
- 100mS of sound starting 600mS after the clapper impact.

This shows that the strike pitch effect is not restricted to the time immediately after the clapper impact.

As already explained, Rayleigh (1890) proposed that a bell’s pitch was an octave below its nominal, still the usual approach adopted in practice today. Simpson (1895, 1896) does not consider bell pitch at all, other than to note that English founders tend to pitch a bell by its nominal, and continental founders by the prime (an example perhaps of the first confusion above).

Jones in his two papers (Jones 1928, Jones 1930) mentions Rayleigh’s hypothesis, reports on investigations of strike pitch with a number of test subjects, and proposes a missing fundamental explanation for the strike pitch formed from the nominal, superquint and missing fundamental - the first example in the literature of the holistic view of bell pitch. He discusses the theory that the strike pitch is a difference tone but dismisses it. He also cites work by Griesbacher erroneously reporting beats against the strike pitch (an example of the second confusion). Finally he cites work by Biehle suggesting that strike pitches sometimes deviate from the half nominal. Jones’ proposal of a holistic explanation for perceived pitch has proved to be correct, though he did not have a physical or physiological explanation for his observations.

In the bells studied by Tyzzer (1930) the prime partial was very close to the half nominal and strike pitch and Tyzzer does not comment on the origin of bell pitch. The bells investigated by Slaymaker & Meeker (1954) also had this quality and the authors refer to the prime partial throughout as the strike note.

Meyer & Klaes (1933) report experiments on a bell deliberately cast with a non-octave prime to avoid this problem. They showed, using filters to selectively remove partials from a bell sound, that both the nominal and superquint needed to be present for a strike pitch to be heard. Bagot (1982) explains how he repeated this experiment on a bell recording with the same result – though the experiment in section 8.3 shows that with a number of higher partials present, the strike note survives removal of the superquint and octave nominal. Meyer and Klaes (1933) also report on another study of 500 bells in which adjustable tuning forks were used to ascertain the strike pitch. They reported discrepancies between strike pitch and half-nominal of up to half a semitone, with 60% of the bells tested showing a deviation of more than a quarter of a semitone.

Arts (1938, 1939) showed by examination of a number of bells that the strike pitch was not explicable as a difference tone between nominal and superquint. He favoured Rayleigh's hypothesis, but also reported on the phenomenon known as secondary strike, where in bigger bells a second, higher strike pitch is heard, an octave below one of the partials higher than the nominal. He also reported secondary beats between a tuning fork at the strike pitch and higher partials.

Van Heuven (1949) devotes a whole chapter of his thesis to strike pitches. He correctly dismisses the explanation as a difference tone, but proposes that the strike pitch is an octave below the nominal due to interaction between clapper and bell, i.e. the way the clapper bounces at the moment of impact. A further theory of the strike pitch, similar to Van Heuven's hypothesis, was proposed by George Elphick, an eminent bell historian (Elphick 1988 pp103-106). He subscribed to the theory that the strike pitch is an octave below the nominal, and that the difference of an octave is due to the initial asymmetric distortion of the bell when the clapper hits it. This would give rise to a strike pitch which rapidly disappeared after the clapper impact which, as has been explained, is not the case in practice. The missing fundamental effect is easily observed in artificially created waveforms, which disproves both theories.

Pfundner (1962) reports on investigations into four bells with unexpected strike pitches and shows that in all cases they can be explained by looking for higher partials with frequencies related by small integers to the strike pitch.

Lehr in an early paper (Lehr 1951) assumes the strike pitch is an exact octave below the nominal – the bells he studied in this paper were largely true-harmonic. In a later paper (Lehr 1965) in which he proposes his classification scheme for bell partials the only reference to strike pitches is to *'the problem of the strike note about which Prof. Schouten reports more extensively'*.

Schouten had previously done research into residue tones, i.e. the missing fundamental effect, and went on to apply this approach to bells (Schouten & 't Hart 1965). The work proved using electronic tones that the nominal and superquint generate a missing fundamental, an effect enhanced by the addition of the octave nominal. He confirmed that the strike pitch is due to a residue effect rather than difference tones by considering the effect of mistuned partials; the strike pitch moves the wrong way if a difference tone is assumed.

Following their publication of an algorithm for virtual pitch estimation in complex tones (Terhardt, Stoll & Seewann 1982a, 1982b), Terhardt and Seewann went on to carry out a major study of bell strike pitches (Terhardt & Seewann 1984). Their primary objective was to use bells as confirmation of their theory of virtual pitch, but the work is also important as a study of strike pitches. A number of test subjects were asked to determine the strike pitches of 137 bells using two different methods, and their results were compared with those predicted by the virtual pitch algorithm. The results proved that strike pitches were a virtual pitch effect. Although both the test subjects and the algorithm provided more than one candidate pitch, in 79% of the bells tested the highest ranked pitch identified by the test subjects agreed to the nearest semitone with the highest ranked pitch predicted by the algorithm. However, the exact frequency of the strike pitch was not predicted well by the algorithm. Terhardt later published a software implementation of the algorithm, which has been tested by the author. Terhardt's algorithm does not correctly predict the shift in perceived pitch caused by changes in upper partial frequency, despite these pitch shifts being clearly audible in practice. A description by the author of the effects and their cause from the early stages of this research is given in (Hibbert 2003).

Two attempts have been made so far to quantify the shift with partial frequency in bells. The first, reported on very briefly by Greenhough (1976), gives an analysis of the partials of two bells, and then reports on experiments on two subjects determining the pitch of synthesised sounds, with the relative frequencies of the partials derived from the bells, but with equal amplitude. The experiments showed that the nominal, superquint and octave nominal determined the pitch of the strike note.

The second attempt to quantify the shift in strike pitch with partial frequency in bells was the work by Eggen & Houtsma (1986). These researchers took a recording of a Hemony bell, and by digital manipulation of the waveform were able to shift the frequency of individual partials. They quantified the effect of the lowest nine partials on strike pitch, varying each individually, and discovered that the nominal, superquint and octave nominal had the greatest effect. They also looked at the effect of multiple partials changing together.

The work of Greenhough and of Eggen & Houtsma is of great importance (not least, it provides confirmation of virtual pitch as the explanation for the strike pitch). However, both sets of experiments have two drawbacks that mean the results cannot be applied in practice:

- The tests were carried out at a single point in the audible spectrum, though Patterson and Wightman had already shown that the effects vary at different frequencies
- The changes in partial frequency, singly or in combination, are not those encountered in practice in bell tuning.

The main objective of the work described in this thesis, building on the work of Terhardt and Seewann, Greenhough, and Eggen and Houtsma, is to establish the quantitative relationship between partial frequencies and strike pitch, in such a way that the results can be applied in practice in the tuning of bells, across the full range of frequencies encountered.

2.6 Measurement techniques for partial frequencies and pitch

The development of understanding of bell partials and strike pitches has been limited by the measurement techniques available. Partial frequency measurement and strike pitch measurement are two quite different problems.

2.6.1 Partial frequency measurement

Lehr (1965) compares the various methods of frequency measurement. Accurate determination of partial frequencies was not possible until the invention of the tuning fork; prior to this, bell pitches were compared with notes played on other musical instruments. Rayleigh (1890) and Jones (1930) also used Helmholtz resonators, and Rayleigh counted beats of partials against notes on a harmonium.

Van Eyck and the Hemonys used vibrating metal bars, similar in principle to tuning forks but more difficult to use in practice. Measuring the frequencies of all the significant partials in a bell using forks requires a large range of fork frequencies or adjustable forks that are well calibrated, experience in counting beats, and plenty of time. Work in the early part of the century on higher partials had to be restricted to big, low pitched bells because forks with the required high frequencies were not available. In any case, counting of beats at high frequencies is impractical. Work done using tuning forks is described by Jones

(1928, 1930). The use of forks for bell tuning beginning in the late 19th century allowed accurate tuning of bell partials for the first time since the techniques of the Hemonys were lost at the end of the 17th century. A description of the Taylor bellfoundry's rediscovery of true-harmonic tuning, and of the changes in their tuning practices brought about by the use of tuning forks, is given by the author in (Hibbert 2002).

The advent of electronic methods brought about a complete transformation in techniques. Most of the detailed work on determination of patterns and frequencies of vibration was done using excitation of the bell by an electronic oscillator linked to a transducer attached at a suitable point to the bell. Detection used either a stethoscope (to allow the location of nodes and antinodes to be discovered), or electronic transducers. Work done using these techniques is described by Tyzzer (1930), Curtiss & Giannini (1933), Van Heuven (1949), Grützmacher, Kallenbach & Nellessen (1965), and Perrin, Charnley & DePont (1983). These methods of determination were still very time intensive; I am told by Robert Perrin that the investigations done by Tom Charnley took months or years because the transducer had to be re-fixed to the bell with Plaster of Paris for each new point of excitation.

Electronic analysis of the radiated sound is very much faster. Slaymaker & Meeker (1954) used a Stroboconn for measurement of partial frequencies in the sound. These instruments pick up the sound via a microphone, use the sound vibrations to turn a neon strobe light on and off, and measure the frequency with spinning marked discs whose rotational speed is adjusted until the marks are stationary in the strobe light. The Whitechapel bellfoundry still uses these instruments for bell tuning. Pfundner (1962) reports on work done using an electronic frequency counter.

Even quicker are spectrum analysis packages used to analyse digitised sound on a computer. Packages with many different capabilities are now available for personal computers. All the analysis work done for this thesis uses a program (Wavanal) written by the author specifically designed for bell analysis. Accurate determination of all audible partial frequencies can now be done in seconds, compared with the hours it would have taken with forks. The relative amplitudes of the partials in the radiated sound are also easily observed.

2.6.2 Strike pitch measurement

No such simple methods as are used for measurement of frequency exist for the determination of pitch. As explained above, pitch determination necessarily requires the active involvement of a human subject, and involves comparison of test and reference sounds. In much of the work reported in the literature, strike pitches of bells were compared against sine tones, either from forks or generated electronically. Comparison of complex tones against sine tones has two disadvantages:

- The subject is comparing sounds with very different timbres, which introduces extra difficulty in experiments
- If the test and reference sounds are played simultaneously, there is a risk of the subject listening for beats, forcing spectral rather than virtual pitch determination.

Three different ways of pitch matching have been traditionally used:

- Adjustment of a test tone which is continuously variable using a potentiometer or other such device
- Matching against a set of test tones or tuning forks spaced some interval apart
- Binary tests between pairs of tones, i.e. a higher / lower judgement, where the experimenter is looking for a better success rate that the 50% expected from chance.

In the literature on the strike pitch of bells, Meyer and Klaes (1933) report results from comparison against tuning forks. Greenhough (1976) states only that a pitch matching technique was used. This is inferred to be a comparison against continuously variable sine tones, as were the tests reported on by Eggen and Houtsma (1986). Two experimental approaches were described by Terhardt & Seewann (1984), one involving matching against a continuously variable sine tone, and the other involving the test subject singing or humming the perceived pitch of the bell. In the latter, bell sound and the subject's performance are both recorded for later analysis.

This technique of vocal reproduction, although not suitable for test automation, is the preferred method of the author for quick estimation of pitches, and was employed in two of the experiments in this thesis.

To avoid the disadvantages of sine tone matching, and to allow test automation, the author has devised a new method of pitch comparison where the two sounds to be compared are complex tones (i.e. with multiple partials) with similar timbre. To allow test automation with simple software, the author's pitch tests involve comparison against a set of complex tones spaced a defined interval apart. The method used will be described in detail and justified in chapter 6.

Since this method was devised and trials of the tests began a few years ago, two reports have been published of similar tests, both co-incidentally investigating pitch shifts in piano strings due to inharmonic partials. Järveläinen (2003) and Anderson & Strong (2005) both matched artificially generated piano-like sounds with inharmonic partials against similar sounds with harmonic partials. Järveläinen (2003) says *'Sine tones were rejected for the practical reason that the aim is to correct the differences between harmonic and inharmonic tones, and because the pitch comparison was easier between sounds of similar timbre.'* Both sets of results confirm that 'stretching' the upper partials of sounds sharpens the perceived pitch, which is in line with the results of the current research.

3 INTRODUCTION TO BELL ACOUSTICS

3.1 Introduction

This chapter gives a brief introduction to bell acoustics. The topics covered are as follows:

- An example of the spectrum and associated partial frequencies of a bell
- Examples of the historical development of the shapes of bells
- An overview of the processes by which bells are cast and tuned, together with a discussion of doublets and of true harmonic tuning
- An analysis of partial amplitude from recordings of a number of bells, highlighting the acoustically important partials
- An explanation of the virtual pitch effects which dominate pitch perception in bells, and the effect in heavier bells known as 'secondary strike'
- Examples of virtual pitch shifts experienced in practice
- An explanation of the effect on upper partial spacing of the relative thickness or weight of bells.

3.1.1 Acoustics of a typical bell

Bells have distinct but inharmonious partials. In this thesis 'partial' is used to mean a frequency of vibration present in the sound of a bell. These partials in turn arise from modes of vibration of the bell. In most cases these modes of vibration occur in degenerate pairs as described in (Perrin 1973) and (Perrin 1977). These pairs of modes of vibration can have different frequencies and form a doublet, as described below in section 3.2.4. However, for the purposes of this work the doublet is treated as a single partial, whose frequency is taken as the frequency in the pair with the greater amplitude.

Figure 3-1 shows a typical spectrum of a bell sound. The bell is the old 9th bell at St Mary le Tower, Ipswich, cast by Warners in 1866. The analysis was done over 2s of sound. The vertical scale is logarithmic to emphasise the importance of the labelled partials. This bell has already been described briefly in the introduction.

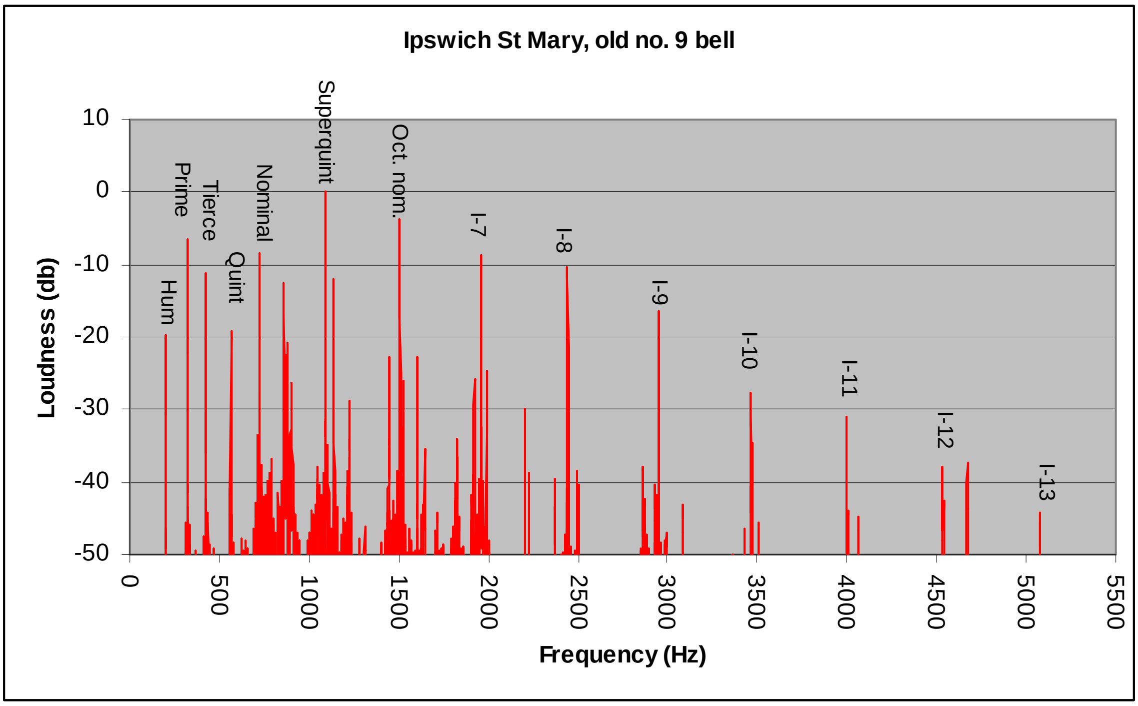


Figure 3-1 Spectrum in dB of Ipswich St Mary le Tower old 9th bell

The frequencies and musical relationships of the main partials are shown below in Table 3-1:

Partial	Freq (Hz)	Note	Ratio to lowest frequency	Ratio to nominal
Hum	201.9	G#(3) - 48	1.00	0.28
Prime	323.7	E(4) - 30	1.60	0.45
Tierce	424.2	G#(4) + 37	2.10	0.59
Quint	562.8	C#(5) + 26	2.79	0.78
Nominal	723.4	F#(5) - 38	3.58	1.00
Superquint	1091.4	C#(6) - 26	5.41	1.51
Octave Nominal	1507.3	F#(6) + 32	7.47	2.08
I-7	1959.3	B(6) - 13	9.70	2.71
I-8	2443.2	D#(7) - 31	12.10	3.38
I-9	2948.3	F#(7) - 6	14.60	4.08
I-10	3467.6	A(7) - 25	17.17	4.79
I-11	4000.8	B(7) + 22	19.82	5.53
I-12	4536.5	C#(8) + 39	22.47	6.27
I-13	5077.1	D#(8) + 34	25.15	7.02

Table 3-1 Main partials of Ipswich old 9th

The figure in parentheses after the note-name gives an indication of the octave, and the final figure is the deviation in cents from the equal tempered note based on A(4)=440Hz (one cent is 1/100th of a semitone). The cents calculation uses the standard formula (after Helmholtz 1877)

$$cents = \frac{1200}{\log_e(2)} \times \log_e\left(\frac{f_2}{f_1}\right) \cong 1731.234 \times \log_e\left(\frac{f_2}{f_1}\right). \qquad (3\text{-}1)$$

The ratios of the frequencies are given relative to the nominal because that partial determines the note name of the bell. This bell sounds F# when struck – it has a strike pitch of about 362Hz.

The partials in this bell do not form a harmonic series in relation to the lowest frequency (the hum). The relationship of the frequencies of nominal to superquint to octave nominal is a near-miss to a harmonic series with ratios of $\frac{2}{2} : \frac{3}{2} : \frac{4}{2}$ - a harmonic series with no fundamental. The strike pitch of this bell (as with all bells of Western shape) is about an octave below the nominal, i.e. somewhat flat of F#(4). There is no partial with this frequency in the sound of the bell. The prime is the closest in frequency to the strike, but as reported extensively in the literature and covered in the previous chapter, tests on bells with mistuned primes show that the prime does not determine the strike pitch of a bell. The most prominent partial, the superquint, does not determine the pitch heard. The absence of the strike pitch from the partial frequencies - the strike pitch paradox - leads to the explanation of strike pitch as a virtual pitch effect.

It is not known when bellfounders first realised that bells vibrate in multiple modes each with its own distinct frequency. The first written description known to the author dates from the 17th century (Beeckman 1633a). The partials with lower frequencies have traditional names, implying long knowledge of their existence, but those with higher frequencies, which are difficult to observe in practice but have a significant effect on bell timbre and pitch, have only been fully investigated in the last few decades.

A major innovation in bell tuning was the discovery that the relative frequencies of some of the lower partials (in particular the hum, prime, tierce and nominal) have a critical effect on the musical quality of a bell. In so-called true-harmonic tuning, the hum, prime and nominal are tuned in perfect octaves. To achieve this required both detailed changes in the shape of the bell, and the ability to tune the partials by removal of metal from the inside and sometimes the outside of the bell. This skill was first perfected by the Hemony brothers and Jacob Van Eyck in the early 17th century, lost, and then redeveloped at the end of the 19th century in the UK (Hibbert 2002). Modern bellfounders are able, through design of the shape of the bell and subsequent tuning, to control the relative frequencies of the lowest five partials. As well as hum, prime and nominal in perfect octaves, the tierce is put a minor third above and the quint a fifth above the strike pitch (i.e. a major 6th and a 4th below the nominal). The Ipswich bell given as an example here is not true-harmonic; the hum is about a tone sharp and the prime about a tone flat.

The tuning of bronze bells remains stable over long periods of time. Atmospheric pollution over a period of decades or centuries can cause corrosion of the bell's surface and alter the partial frequencies, especially in urban environments. However, this is unusual, and the author has verified partial frequency figures from recent recordings of bells tuned 50 and 100 years ago against the tuning figures noted by the bellfounder at the time of manufacture, and found a very good correspondence. It is likely that the partial frequencies of bells much older than this remain as the bellfounder intended. Steel bells on the other hand suffer from rapid corrosion from water vapour in the atmosphere if not protected by paint or some other coating.

The frequencies of bell partials are affected by temperature, flattening as the temperature rises. The temperature co-efficient of the frequencies in bronze bells is given in (Terhardt 1984) as -1.5×10^{-4} per degree C. This is equivalent to a fall of 2.6 cents in frequency per 10 deg. C. rise in temperature. Because all partials in all bells in a peal are changing in the same proportion, the bells will remain in tune at different temperatures, though rapid changes in environmental conditions causing different bells to be at different temperatures could give rise to tuning problems. Because all partials track together, and the primary interest in this research is the ratio of partials rather than their absolute values, temperature can be ignored for the purposes of this thesis.

3.2 Production and tuning of bells

3.2.1 The shape or profile of bells

Bells (at least for the purposes of the current research) are bronze or sometimes steel castings specifically shaped to produce a more or less musical sound. The bronze commonly used, known as bell metal, is an alloy of about 77% copper and 23% tin chosen for its musical properties. The earliest known bells were cast in China in the 19th to 16th centuries BC. Roman handbells dating back to the 1st century AD have been found in Britain, and there is documentary evidence of the casting and use of bells in Ireland and the mainland UK from the 5th century onwards.

For a full description of the processes used in bell manufacture and tuning, and the history of changes in technique and bell shape, see (Lehr 1965), (Elphick 1970), (Elphick 1988), (Price 1983), and (Jennings 2006).

Bells in the Western and Eastern traditions have evolved into quite different shapes. Eastern bells are more or less cylindrical or barrel-shaped with a domed top or crown, as Figure 3-2 shows (a modern temple bell in the Shiteno-ji, Osaka, Japan).

Figure 3-2 Modern Japanese bell

Figure 3-3 shows a modern Chinese bell (in the Great Bell Museum, Beijing):

Figure 3-3 Chinese bell cast in 1997

The shape of Western bells reached its current form in the early middle ages. Four examples of the evolution follow.

Figure 3-4 Bad Hersfeld bell (dated about 1050)

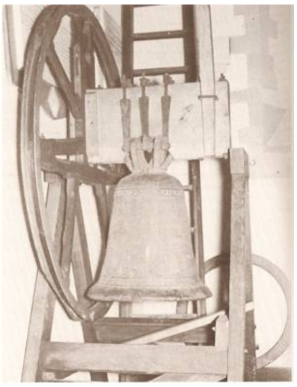


Figure 3-5 Caversfield bell (dated 1207 – 1219)

Figure 3-6 Southchurch bell (early 14th century)

Figure 3-7 Knightsbridge bell (dated 1892)

No written records exist of the design process which led to the final shape of Western bells, but comparisons of bells of different shape show that the musical quality of bells improves considerably as the shape changes towards that of a modern bell. It is likely that the sequence of shape changes resulted from trial-and-error modifications to try to improve the musical sound of bells.

Figure 3-8 below shows a cross-section of a bell casting:

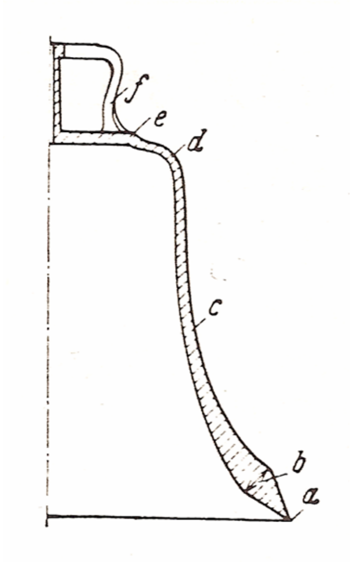


Figure 3-8 Diagram of bell showing constituent parts

The various parts of the bell are named as follows:

- a – lip or rim
- b- soundbow
- c- waist
- d -shoulder
- e - crown
- f - canons.

The canons were used traditionally to attach a bell to its support. Modern bells are cast with flat tops and bolted to the headstock or beam.

Figure 3-9 Australian bell with special profile

Recent work on bell design in Australia is reported on in (McLachlan & Cabrera 2002) and (McLachlan 2002). Finite element analysis was used to design bells with particular tonal qualities, which were then cast in moulds made using numerical controlled tools. The bells were designed on a number of different principles, such as to have harmonic partials or multiple pitches a planned interval apart. The profiles of these bells are complex as can been seen from Figure 3-9.

3.2.2 The numbering and names of bells in peals and carillons

Bells used in sets together are given numbers and names, and the terminology differs according to use and geographical location. Peals of bells hung for change-ringing (the common practice in the UK and sometimes in the US, Australia and elsewhere) normally comprise between 5 and 12 bells tuned to a diatonic scale. The smallest bell is called the treble and the largest the tenor. The bells are numbered in sequence from 1, the treble / smallest bell, down the scale to the tenor which has the highest number. If semitone bells are provided (to allow the use of a diatonic set with a smaller bell as the tenor) the semitones are designated sharp second, flat sixth etc.

In carillons – sets of at least 23 bells designed for the performance of music from a keyboard or clavier – the bells are numbered in the opposite way, so that the largest bell is number 1 and the smallest has the highest number. Semitone bells, which are normally provided in carillons to allow modulation between keys, are numbered in sequence rather than being described as sharp or flat. The lowest semitone is sometimes omitted in carillons as it is rarely needed in performance.

3.2.3 Casting of bells

Bells of traditional Western shape are cast in moulds comprising two parts, an inner mould or core, and an outer mould or cope. There are several ways in which these moulds are produced; a typical method is to use strickle boards shaped to the inner or outer profile of the bell, rotating around a circular pivot, to shape the mould which is usually formed from loam, a mixture of clay and other materials. Sometimes a false bell of wax is built upon the inner core and the outer cope formed around it (the lost wax process).

Figure 3-10 shows the core being built up using a strickle.

Figure 3-10 Constructing the core of a bell mould

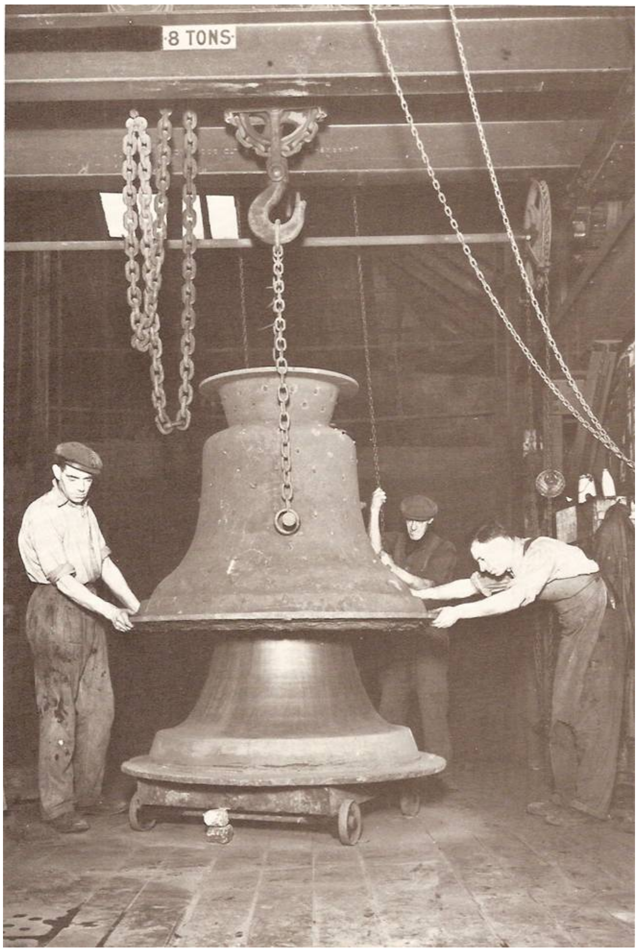


Figure 3-11 Closing core and cope of a bell mould

Figure 3-11 shows cope and core being fitted together prior to casting of the bell, and should give a sufficient idea of the process.

3.2.4 Tuning of bells

Fine control of the shape of the bell is critical to its sound, because of the effect on the relative frequencies of the various partials. The moulding and casting process is not precise, and although some bells emerge from the mould 'in tune with themselves' - these are known as maiden castings - the usual process is to err on the side of caution, and leave metal to be removed during a tuning process. Tuning of a bell has two objectives:

- To have the strike pitch of the bell (the note we hear) sound in tune with other bells in a ring or carillon - known as outer tuning
- To have the various partial frequencies of the bell in harmonious relationship with each other - known as inner tuning.

To tune a bell, metal is removed from the inside on a vertical lathe. The bell rotates and a cutting tool attached to the vertical post is used to remove the metal. During the tuning process, the frequencies of the important partials are measured, traditionally by beating against tuning forks, but nowadays using electronic methods. Figure 3-12 shows an example of a tuning machine in use.

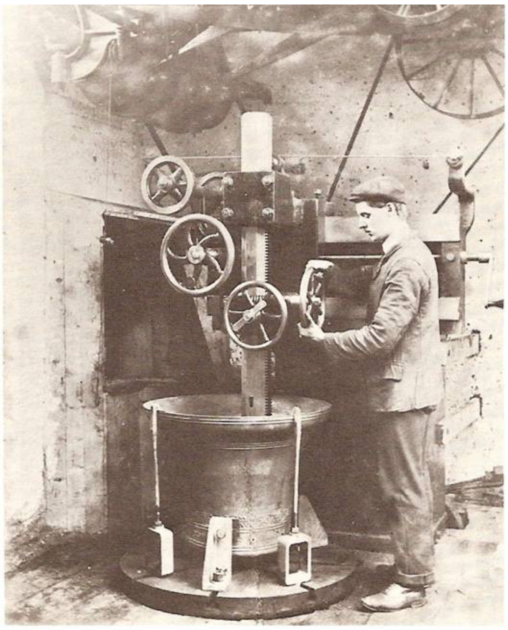


Figure 3-12 Bell tuning machine in use

Bell tuning is as much art as science, because removal of metal from a particular annular circle inside the bell affects the frequencies of multiple partials in different ways. Also, metal once removed cannot be replaced. Lehr (1965) provides details of experiments showing how each partial is affected by removal of metal at various points.

As a result, bell tuning is an iterative process. The bell is tested after each tuning cut to check on progress towards the desired result. Paradoxically, because of the interplay between stiffness and mass determining the frequency of each partial, removing metal generally lowers the pitch of a bell even though lighter bells of the same shape are higher in pitch.

3.2.5 The problem of doublets

One of the concerns in casting and tuning a bell is to ensure that it is symmetrical about a vertical axis through its centre. If this is not so, most of the vibrational modes in the bell split into two distinct modes with different frequencies.

A good illustration of this is given by a china mug as shown in Figure 3-13. If the rim of the mug is hit with a teaspoon above the handle, or at points 90, 180 and 270 degrees displaced from this (points A in Figure 3-13), a lower note is produced than if the mug is hit at points 45, 135, 225 and 315 degrees from the handle (points B in Figure 3-13). If the mug is hit at other points, both notes are heard. The simplest mode of vibration of the coffee mug rim has four nodes and four antinodes equally spaced around the rim. The extra mass of the handle makes the frequency lower if the handle is at an antinode (i.e. moving) than if it is stationary.

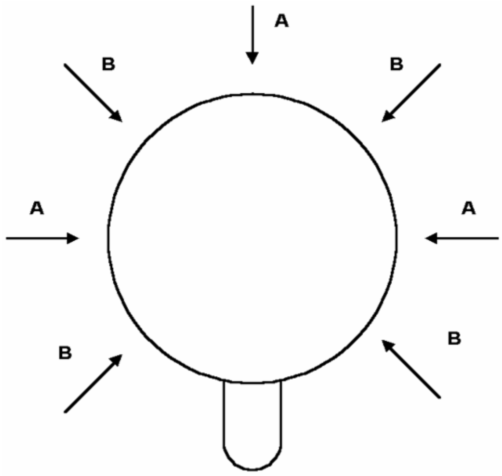


Figure 3-13 Diagram of a china mug showing doublets

In bells, the same effect exists if the bell is not exactly circular. The pair of modes is called a doublet. If the two vibrational modes are both active, as is usually the case unless the clapper happens to strike exactly on a node of one mode, the modes beat against each other producing a warbling sound. This warbling spoils the tone of the bell if too pronounced. Rayleigh (1890) describes the use of doublets in bells in an ingenious fashion to investigate the number of nodes of each vibrational mode around the bell. Robert Perrin carried out an analysis of doublets using group theory, reported in (Perrin 1977).

3.2.6 True-harmonic tuning

A major innovation in bell design and tuning was the introduction of true-harmonic tuning by the Hemonys, as rediscovered by the Taylor bellfoundry and others in the late 19th century. (Hibbert 2002) included as Appendix 6 gives an account of this. In true-harmonic tuning, certain key partials (the hum, prime and nominal) are tuned in perfect octaves, which brings about a considerable improvement in the sound of a bell. To achieve true-harmonic tuning, three things are necessary:

- To understand the contribution of the various partials to the overall sound of the bell
- To find a shape for the bell which brings the three partials into approximately the correct relationship
- To understand where metal should be removed from the inside of the bell to bring the three partials into the exact relationship.

In addition, the bellfounder must ensure that the strike pitch of the bell is 'in tune' with the other bells in the peal or carillon, and must control the relative frequencies of other partials (especially the tierce and the quint).

The tierce partial makes an important contribution to the overall sound of a bell. In most bells it is a minor third above the strike pitch (i.e. a major sixth below the nominal) and it is this partial which gives bells their characteristic ‘minor’ sound. Bellfounders control the frequency of the tierce by the shape of the bell (particularly in the area of the soundbow or lip) and most do not attempt to tune it on the tuning machine. Why this is so will be explained in chapter 5.

Some bellfounders also claim to 'tune' partials above the nominal, in particular trying to bring further partials into an octave relationship. The impossibility of doing this in bells with the traditional Western profile is also shown in chapter 5.

3.3 Partial amplitudes

A number of factors affect the amplitude of the various partials in the sound of a bell (as shown in the spectrum in Figure 3-1). These include:

- Mechanical characteristics of the bell, i.e. its shape and wall thickness and the composition of the metal
- The clapper material and the dynamics of the impact as the clapper hits the bell
- The acoustics of the room or building in which the bell is housed.

Perception of loudness of sounds in the ear also varies with frequency.

Bell towers typically have internal dimensions of the same order of magnitude as the wavelengths of the partials. The physical dimensions of the bell (mouth diameter and height) are also of the same order of magnitude as sound wavelengths. A bell swing-chimed or rung full circle as explained below is moving quickly within this enclosed space. As a result, the effect of room acoustics on the sound of bells is considerable. Some preliminary experiments were conducted on room acoustics and clappering as part of this research, but it was realised that the complexity of the subject was such that it could not be adequately dealt with in this thesis.

Bells of Western design are rung or sounded in three ways. In carillons, which consist of bells arranged to be played from a keyboard or automatic chiming mechanism, the bells are hung stationary and each is sounded with an internal or external clapper arranged to bounce away from the bell after the strike. For bells which are swing-chimed, a common practice in Europe, the bell is hung from a pivoted headstock and swung through an arc of typically less than 90 degrees either side of the mouth downwards position. The clapper, pivoted inside the crown of the bell, hits the bell near the top of each swing and again bounces clear. The clapper hits the bell harder than is the case for bells in a carillon. The Doppler effect of the moving bell is clearly audible, and can be seen in recordings as a broadening or splitting of lines in the spectrum.

In bells hung for full-circle ringing, the common usage in the UK, the bell swings through 360 degrees starting from rest in a mouth-upwards position. The clapper is again pivoted inside the crown of the bell and hits the bell as it rises towards its mouth-up position. Unlike swing chiming, the clapper lies against the rim of the bell as it rises towards the mouth-up position. Again, the Doppler effect of the moving bell is very evident.

Figure 3-14 Bell being rung full circle

Figure 3-14 shows a bell from Howden Minster in Yorkshire being rung full-circle. The bell is rising to the mouth-upwards position (turning clockwise) – it is pivoted on the bearings at either end of the blue headstock. The clapper has hit the bell and is now lying against the rim. The bell will come to rest mouth upwards and then begin to turn anti-clockwise, throwing the clapper ahead as it turns.

Practical experience of recording and investigating bells has shown that bells rung full-circle or swing-chimed have more prominent higher-frequency partials than bells which are chimed as in the carillon usage.

Experiments described in the literature and the work covered in section 8.3 show that differences in partial amplitude have a minor effect if any on virtual pitch shifts. However, to make the virtual pitch experiments as realistic as possible, measurements from a number of bell recordings were averaged to get typical values for relative amplitudes. As the main application of the experimental results is for bells rung full-circle, all the recordings used were of bells rung in this fashion. The recordings were taken with an AKG C1000S condenser microphone placed at varying distances from the bell. The output of the microphone was recorded via an M-Audio MobilePre USB pre-amplifier connected to a Toshiba Satellite Pro 4600 laptop. The bells were chosen to have a range of weights and tuning styles. The amplitudes for each bell are taken over the first 250ms after the clapper strike. This time period allows the amplitudes measured to be used in the virtual pitch tests as explained in Section 6.7.

Figure 3-15 shows the results of the average, as used in the experiments reported on later in the thesis. Each bell sound is normalised using the root-mean-square of all the partial amplitudes for that bell. The units of the amplitude in the figure are arbitrary; and are measured as voltages from a microphone, i.e. are sound pressure levels on a linear scale. The partials chosen are those which determine the strike pitch (as confirmed by the experiments in chapter 4), the lowest five partials (hum, prime, tierce, quint and nominal) which are traditionally tuned by bellfounders, and other prominent partials as seen for example in Figure 3-1 above.

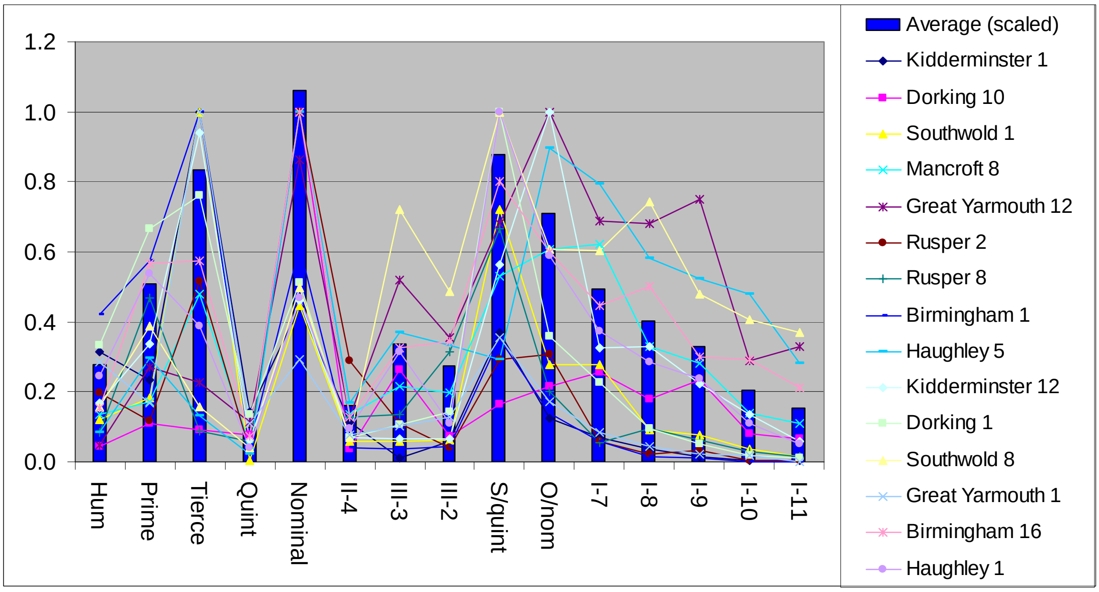


Figure 3-15 Relative partial amplitudes of 15 bells

Though these average amplitudes were used in the virtual pitch tests to make them as realistic as possible, it’s worth pointing out that in the virtual pitch experiments by Plomp

(1967a) and in the experiment described in section 8.3, it was found that partial amplitude did not significantly affect virtual pitch shifts.

3.4 Formation of strike pitch

The reports in the literature and the work of various researchers make it clear that the strike pitch of bells (the predominant note heard when a bell is struck) is a virtual pitch or missing fundamental effect, unless the bells are small, as explained below. What is required for this to occur is a set of partials forming an approximate harmonic series (i.e. with frequencies in the approximate ratio 2 : 3 : 4 etc.). A virtual pitch is then heard at about half the frequency of the lowest partial. An examination of the partial frequencies of the Ipswich bell as shown in Table 3-2 is illuminating in this regard:

		Ratios based on:								
	Freq.	Hum	Prime	Tierce	Quint	Nom'l	S'quint	Oct. Nom.	I-7	I-8
Hum	201.9	1.00								
Prime	323.7	1.60	1.00							
Tierce	424.2	2.10	1.31	1.00						
Quint	562.8	2.79	1.74	1.33	1.00					
Nominal	723.4	3.58	2.23	1.71	1.29	1.00				
S'quint	1091.4	5.41	3.37	2.57	1.94	1.51	1.00			
Oct. Nom.	1507.3	7.47	4.66	3.55	2.68	2.08	1.38	1.00		
I-7	1959.3	9.70	6.05	4.62	3.48	2.71	1.80	1.30	1.00	
I-8	2443.2	12.10	7.55	5.76	4.34	3.38	2.24	1.62	1.25	1.00
I-9	2948.3	14.60	9.11	6.95	5.24	4.08	2.70	1.96	1.50	1.21
I-10	3467.6	17.17	10.71	8.17	6.16	4.79	3.18	2.30	1.77	1.42
I-11	4000.8	19.82	12.36	9.43	7.11	5.53	3.67	2.65	2.04	1.64
I-12	4536.5	22.47	14.01	10.69	8.06	6.27	4.16	3.01	2.32	1.86
I-13	5077.1	25.15	15.68	11.97	9.02	7.02	4.65	3.37	2.59	2.08

Table 3-2 Partials of Ipswich old 9th showing harmonic series

The sets of partials fitting an approximate harmonic series are highlighted. The strike note of this bell is approximately 362Hz.

In bells of middling size (i.e. neither very big nor very small), the partials giving rise to the strike pitch are the nominal, superquint, octave nominal and potentially higher partials. In these bells, the strike pitch is about half the nominal frequency. In current practice, bellfounders assume the strike pitch is exactly half the nominal frequency, i.e. an octave below, and tune bells accordingly.

The effect of the strike pitch is dominant in the sound of a bell under most circumstances. The description of it as a virtual pitch suggests something subtle or ephemeral but this is not the case. The experiments reported on in this thesis further confirm the explanation of strike pitch as a virtual pitch effect, as already demonstrated by previous researchers.

3.4.1 Pitches of bells with different nominal frequencies

The discussion in section 2.4 on the dominance region for virtual pitch is of relevance to the perceived pitches of bells. Though authors differ on the details, there is agreement that there is a preferential range of frequencies for virtual pitches, possibly in the range 500 to 1500Hz. In bells of middling size, as explained in the previous section, the virtual pitch based on the nominal and partials above it falls into this region and the nominal determines the perceived pitch. In very large bells, the virtual pitch based on the nominal falls below the dominance region, and virtual pitches based on higher partials are also experienced. This effect, known as secondary strike, has traditionally puzzled bellfounders. The additional pitch, often a musical fourth above the pitch based on the nominal, can spoil a bell’s tone. For an illustration of this see the account of the dispute between Cyril Johnston (bellfounder) and Frederick Mayer (carillon consultant) regarding the bass bell at the Riverside Church, New York in (Johnston 2008 pp.89-92). Not all listeners experience secondary strike in big bells. Some writers have suggested that the secondary strike may be caused by metal composition or some other aspect of the design of large bells. However, recordings of bells of higher pitch not displaying secondary strike demonstrate it when slowed down, lowering the pitch while preserving the relative frequency of the partials.

In the case of small bells, when the nominal is high in frequency, all virtual pitches formed by the bell’s partials lie above the dominance region, and the pitch or pitches perceived in the bell are spectral pitches based on the frequency of the lowest partials. This change in pitch mechanism is a design problem in the manufacture of carillons, which often have very small bells with very high pitches. The partials which determine the pitch of the bells, and in consequence whether they sound in tune, change as the bells get smaller, and as will be seen, may differ from one listener to another. In some very light peals of change ringing bells, the pitch perception mechanism can change for a particular listener as the bells ring down the musical scale, giving the impression of an octave leap in the middle of the peal.

Though these effects are described in the literature, comprehensive experiments have not been carried out to quantify the impact of the virtual pitch dominance region on virtual pitch. Chapter 4 of this thesis reports on two sets of experiments conducted by the author to investigate this area. The first experiment explores the secondary strike effect in large bells. The second experiment investigates the effect of the dominance region and virtual and spectral pitch effects across a wide range of frequencies.

3.5 Strike pitch shifts

Today's bellfounders assume that the strike pitch of a bell (its musical note) is an exact octave below the nominal partial. The nominals of a set of bells are tuned to be exactly aligned with one another in the expectation that the bells will then sound in tune together. However, in reality other partials than the nominal affect the pitch which is perceived, to an extent that is clearly audible as mistuning.

In the author’s personal experience, lighter bells in peals can sound flat even though the nominals are in tune. For example, a peal of eight bells at St Edmund, Southwold, Suffolk was retuned by one of the UK's leading bellfoundries in 1990 with nominals tuned very exactly. When ringing on these bells I noticed that the higher pitched (treble) bells sounded distinctly flat or dull in tone compared with the lower pitched (tenor) bells. I carried out experiments by artificially sharpening the highest pitched bells (by shrinking the timebase of the individual bell recordings) and creating simulations of the bells rung together. In order for the treble bells to sound in tune to my ears, I had to sharpen the smallest by 35 cents (1/3 of a semitone) and the next smallest by 20 cents (1/5 of a semitone). These would be significant tuning discrepancies if experienced by all listeners. I discussed the tuning of the bells with the foundry who did the work and they remarked that the different timbres of the various bells had made tuning difficult.

Selected partial frequencies of the eight bells are shown in Table 3-3:

Bell	Founder, date	Nominal (Hz)	Nominal (cents)	Superquint (cents)	Oct. Nom. (cents)
1 (treble)	Moore, Holmes & Mackenzie, 1881	1419.5	1204	641	1140
2	Moore, Holmes & Mackenzie, 1881	1339.5	1103	640	1142
3	N. Dobson, 1820	1190.5	903	686	1235
4	John Darbie, 1668	1061.5	701	695	1243
5	John Darbie, 1668	946	501	702	1259
6	William Barker, 16th cent	892	400	679	1210
7	Brasyer of Norwich, 15/16th cent	793	195	689	1243
8 (tenor)	Dobson, 1828	708	0	689	1241

Table 3-3 Partials of the peal of 8 at Southwold

(Cents for the nominals are given relative to the nominal of bell number 8. Cents for the other partials are given relative to the nominal of that bell.)

Exact tuning of the nominals to equal temperament would give nominal cents of 1200, 1100, 900, 700, 500, 400, 200 and 0, so no bell's nominal is more than a few cents out and there are no nominal discrepancies that would lead to the degree of mistuning I hear in these bells. However, there is a very marked difference in the frequencies of the superquint and octave nominal between the smallest two and the largest six bells; the octave nominals in particular are typically 100 cents or a semitone flatter in the smallest two bells.

Another example of this effect is given by the peal of ten bells at St Mary Magdalen, Oxford. These bells are all by the same founder. Their partial frequencies are given in Table 3-4.

Bell	Founder, date	Nominal(Hz)	Nominal (cents)	Superquint (cents)	Oct. Nom. (cents)
1 (treble)	Taylor 2001	2224	1607.1	667.6	1194
2	Taylor 2001	1978	1404.1	685.2	1218.9
3	Taylor 2000	1749	1191.1	689.4	1232.8
4	Taylor 2000	1648.5	1088.7	686.7	1226.3
5	Taylor 1990	1477.5	899.1	700.6	1255.3
6	Taylor 1990	1320.5	704.6	701.3	1256.4
7	Taylor 1990	1172.5	498.8	714.9	1281.8
8	Taylor 1990	1108.5	401.6	712.6	1279
9	Taylor 1990	989	204.1	702.5	1265.3
10 (tenor)	Taylor 1988	879	0	701.3	1268.5

Table 3-4 Partials of the peal of 10 at St Mary Magdalen, Oxford

In a similar fashion to the Southwold bells, the upper partials in the smaller bells in this peal have flatter partials than the larger bells. The treble bell has an octave nominal 74.5 cents or ¾ of a semitone flatter than that of the heaviest bell. The smaller bells of this peal sound distinctly flat compared with the heavier bells even though their nominals are well tuned. I first noticed the apparent flatness in the sound of the small bells at St Mary Magdalen on walking past in the street while they were ringing. Experiments by the author similar to those performed on the Southwold bells suggest that the first bell should be sharpened by 25 cents and the second by 20 cents to sound ‘in tune’.

A third interesting example of this effect was provided by Carl Scott Zimmerman, a researcher into the history of American carillons, who provided the following anecdote (Zimmerman 2007): *‘I first heard of stretch tuning in 1967 in a very different context, when Frederick C. Mayer spoke informally to the GCNA Congress at Princeton University … For much of his working life he was also a highly respected organ designer and consultant, and this led to him becoming a carillon consultant as well. [He] spoke with enthusiasm and vigor about his involvement in the development of some of the great English-made carillons of the 1920s and 1930s, and especially about the importance of stretch tuning in the trebles of larger instruments (four octaves or more). I recall very distinctly Mayer’s stress on the fact that the perfectly tuned trebles of the very first large modern carillons sounded flat to the ear, making stretch tuning necessary to produce a good musical effect.’*

Examination of Southwold, Oxford and other peals of bells led me to the hypothesis that partials other than the nominal can have a significant effect on strike pitch and therefore whether a set of bells sounds in tune together. The pitch shift results described by Greenhough (1976) and Eggen & Houtsma (1986) lend weight to this hypothesis. The main purpose of the virtual pitch experiments described in this thesis is to investigate and fully quantify the effect in a way amenable to practical application.

3.5.1 Effect on upper partials of thickness and relative weight of bells

This flatness in the upper partials is caused by bells being cast thick and heavy for their note, as explained by Nigel Taylor, head tuner at Whitechapel Bellfoundry, who says in an email (Taylor 2004) *'I have established that flat upper partials are caused by scale of thickness and lip to soundbow profile'*. Casting bells heavy for their note is a common practice in small bells to improve their mechanical dynamics relative to the bigger bells, or in the case of carillon bells, to increase the volume of sound.

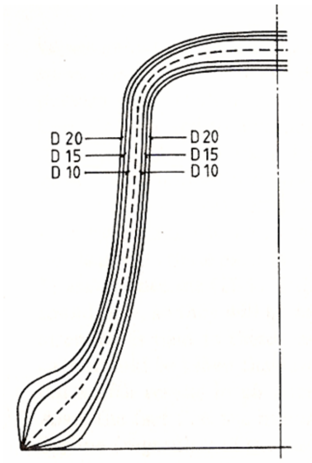


Figure 3-16 Diagram (after Lehr) showing bells with different thickness

André Lehr confirms this in an experiment described in (Lehr 1986) in which he cast three bells with the same overall shape but different wall thicknesses as shown in Figure 3-16 taken from his paper. The three bells are designated D10 (thin), D15 (medium) and D20 (thick).

The bells were not tuned. The frequencies for the nominal, superquint and octave nominal of each bell, represented as cents from the nominal, appear in Figure 3-17 below.

The difference in cents between the sharpest and flattest octave nominals in these bells is 80 cents.

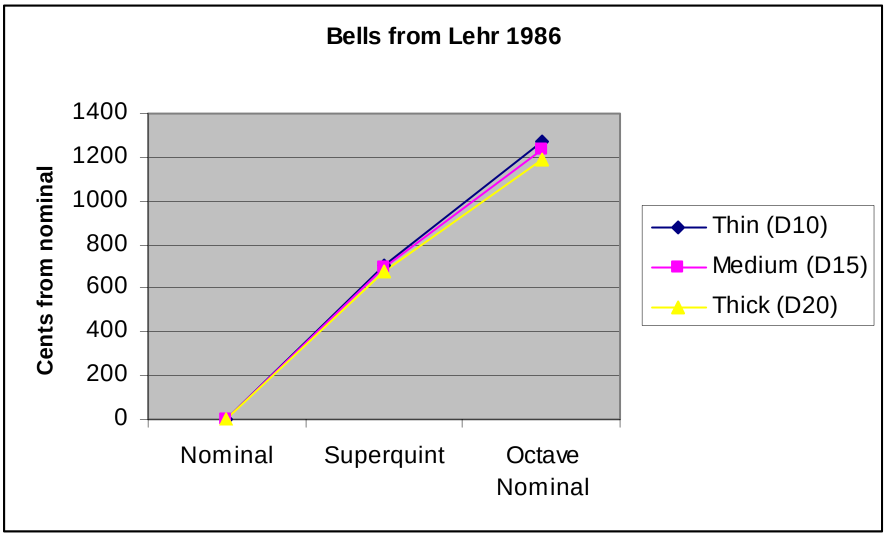


Figure 3-17 Upper partials of bells of different thickness in Lehr 1986

3.6 Effect of bell shape and weight on upper partial tuning

In this section, the effects described by Nigel Taylor and André Lehr are investigated in bells from the author’s database of tuning information. The investigation has two parts:

- The relationships between the main parameters (weight, diameter and nominal frequency) are established
- Deviations from the typical relationship are analysed for correlation against octave nominal tuning.

As will be seen from the work in chapter 5, the tuning of the octave nominal relative to the nominal acts as a good proxy for all the upper partials.

3.6.1 Weights, diameters and partial frequencies of 553 bells

The weights, diameters, nominal and octave nominal frequencies of all bells with this information available were extracted from the author's database of bell recordings. Full information was available for 531 bronze and 22 steel bells. Weights ranged from 30kg to 65 tonnes, nominal frequencies from 204Hz to 2,611Hz, and diameters from 34cm to 3.7m. Height information, which might also have proved to be significant, is not normally recorded for bells and so was not available for analysis.

In principle, the dependence of weight on the diameter should be a simple one. If all dimensions of a bell were to be scaled linearly, the weight should go as the cube of the diameter. For the nominal frequency, the standing wave pattern of the mode of vibration around the rim of the bell (with equally spaced, fixed nodes) suggests that if stiffness and mass were constant, frequency would vary inversely with circumference and therefore with diameter. Both of these relationships are explored in outline in (Lehr 1986).

Partial frequencies of bells can be measured with confidence from recordings. Diameters of bells can generally be measured accurately with a tape, unless the bell has been "skirted", i.e. has had its mouth trimmed (traditionally with a hammer and cold chisel) to raise its pitch. Weight information can be unreliable; weights are often estimated (and exaggerated!), bells with canons weigh more than equivalent bells without, and it is not practical to weigh bells to get confirmation of a reported figure. Therefore, weight will always be treated as the dependent variable in the analysis below.

The hypothetical relationships suggested above between weight, nominal frequency and diameter suggest that log plots should be used for investigation. Figures 3-18, 3-19 and 3-20 are scatter plots of $\log_e$(weight) against $\log_e$(nominal frequency), $\log_e$(diameter) against $\log_e$(nominal frequency), and $\log_e$(weight) against $\log_e$(diameter). Bronze and steel bells are plotted separately but as can be seen there is, quite remarkably, no significant distinction between bells made from the two materials and they will be handled together in the remainder of the analysis.

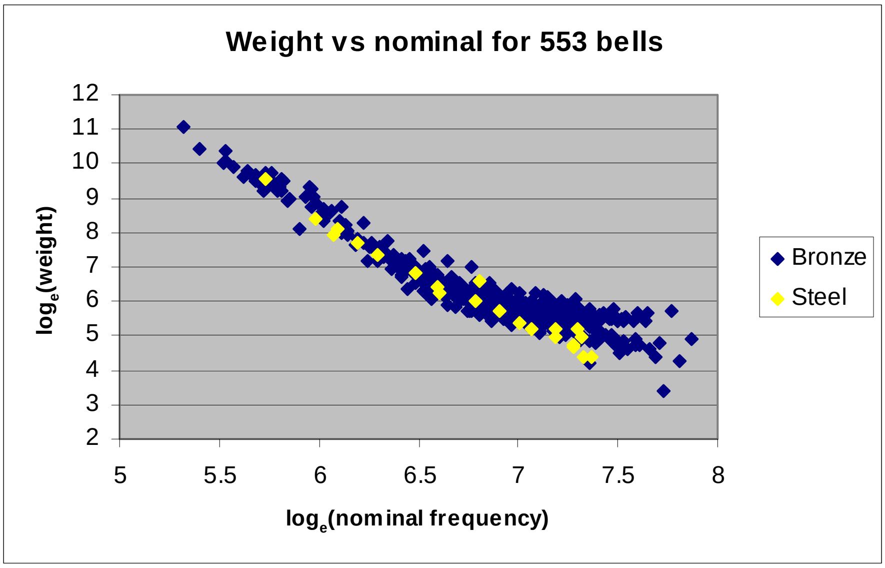


Figure 3-18 Weight and nominal frequency of 553 bells

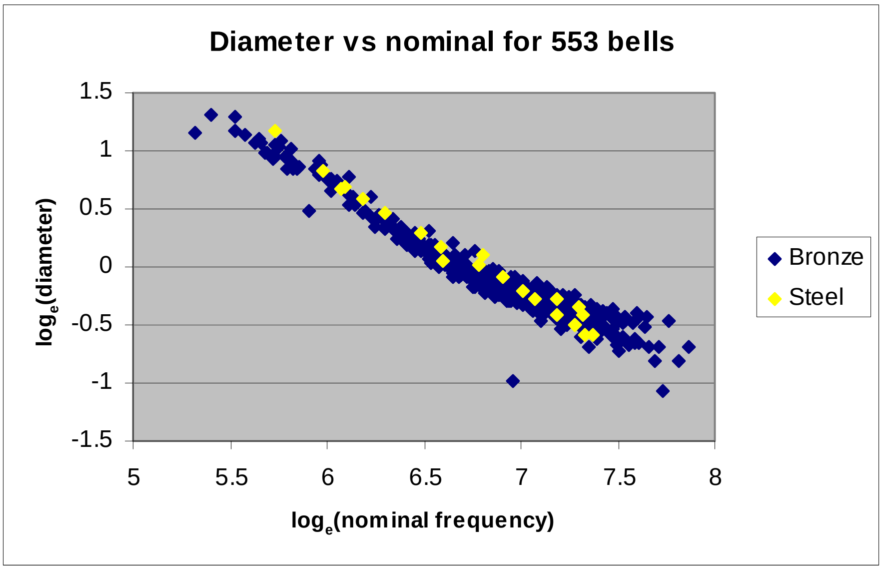


Figure 3-19 Diameter and nominal frequency of 553 bells

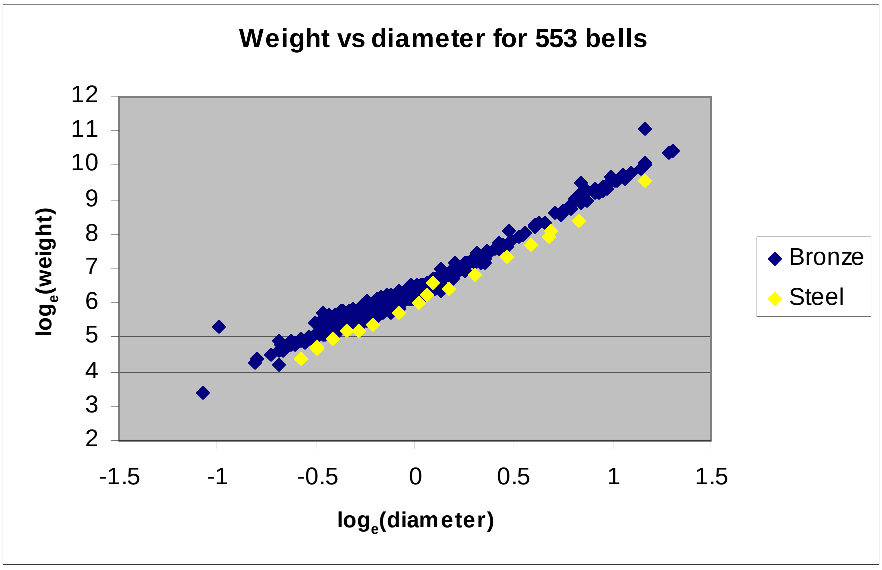


Figure 3-20 Weight and diameter of 553 bells

In all three cases, a broadly linear relationship is seen on the log plots. Figures 3-18 and 3-19 show that for bells with high nominal frequencies (i.e. small bells), both weight and diameter are higher in proportion than for large bells. This is normal bellfounding practice; small bells are cast proportionally heavier than large bells so that, for bells rung full circle, their rotational dynamics better match the larger bells. For chiming and carillon bells, small bells are cast proportionally heavier and larger to increase the loudness of their sound output. Figure 3-20 shows a close linear relationship on the log plot, though again there is a slight departure for smaller bells, in which the weight is slightly greater than a linear relationship would suggest.

Figures 3-18 and 3-19 both have two branches for small bells with higher nominal frequencies. Bells lying on the two branches were investigated. Those on the upper branch are the trebles of higher-numbered change ringing peals (twelves and sixteens) which are deliberately cast much heavier for mechanical reasons. Those on the lower branch were the trebles of lower-numbered change ringing peals (sixes and eights) and chiming bells.

In Figure 3-19, the four low outliers are, from top left to bottom right:

- The Uspensky bell in the Kremlin, Moscow
- The Hosannah bell from Freiburg, Germany
- A bell whose figures are given in (Lehr 1986) described as ‘vd Ghein 1615’
- The smaller of the two bells at Blackheath in Kent.

In Figure 3-20, the two high outliers are the Lehr and Uspensky bells. Figures for the Uspensky bell (65 tonnes in weight, 3.7m in diameter) are approximate. The Freiburg bell is an unusual shape and is discussed in some detail in section 4.2.4.

In order to quantify the relationship between the observed variables, and to establish the effect of the octave nominal tuning, three linear regressions were done on the three sets of variables. Because of the deviation from a linear relationship for small bells, the regression only included bells for which the relationship on Figures 3-18 and 3-19 appeared linear, i.e. bells for which $\log_e$(nominal frequency) was less than 6.8.

In the analysis below, w_i is the weight of a bell in kg, d_i is the diameter in metres, and $f_{n,i}$ and $f_{on,i}$ are the nominal and octave nominal frequencies in Hz. The ε values are error terms, and the β values are regression coefficients. The three models to be fitted (corresponding to Figures 3-18, 3-19 and 3-20) are:

$$\log_e(w_i) = \beta_0 + \beta_1 \log_e(f_{n,i}) + \varepsilon_{1,i} \tag{3-2}$$

$$\log_e(d_i) = \beta_2 + \beta_3 \log_e(f_{n,i}) + \varepsilon_{2,i} \tag{3-3}$$

$$\log_e(w_i) = \beta_4 + \beta_5 \log_e(d_i) + \varepsilon_{3,i} \tag{3-4}$$

Once the least squares fit has established values for the β's, then the dependent variables in each case can be adjusted for the primary independent variable by calculating the following residual values for each bell:

$\log_e(w_i) - \beta_0 - \beta_1 \log_e(f_{n,i})$ (the nominal-adjusted weight from Equation 3-2)

$\log_e(d_i) - \beta_2 - \beta_3 \log_e(f_{n,i})$ (the nominal-adjusted diameter from Equation 3-3)

$\log_e(w_i) - \beta_4 - \beta_5 \log_e(d_i)$ (the diameter-adjusted weight from Equation 3-4).

Chapter 5 shows that a good measure of the octave nominal tuning is the cents of this partial relative to the nominal. Following Equation 3-1, the octave nominal cents are calculated as $1731.234 \times \log_e\left({}^{f_{on,i}}\!\big/_{f_{n,i}} \right)$. Plots of the above residuals against the octave nominal cents should show whether departures from typical values of weight and diameter are correlated with octave nominal tuning.

Results of the regressions with 95% confidence intervals are shown in Table 3-5:

	Weight vs nominal	**Diameter vs nominal**	**Weight vs diameter**
Intercept	$\beta_0 = 28.53 \pm 0.67$	$B_2 = 6.95 \pm 0.18$	$B_4 = 6.29 \pm 0.03$
Primary effect	$\beta_1 = -3.33 \pm 0.10$	$B_3 = -1.04 \pm 0.03$	$B_5 = 3.19 \pm 0.06$

Table 3-5 Regression coefficients: weights, diameters and nominals

The value of β_3 is very close to -1, so that diameter and nominal frequency vary as the inverse of one another, as suggested above. From Equation 3-2, $d_i \propto f_{n,i}^{-1.04}$. The value of β_5 is close to 3, so that weight does vary roughly as the cube of diameter. From Equation 3-3, $w_i \propto d_i^{3.19}$.

Figures 3-21, 3-22 and 3-23 show the relationship between the adjusted weights and diameters and octave nominal tuning. Three significant outliers have been omitted from these plots so that the trend for the main mass of bells is easier to see. These outliers are:

- The Freiburg bell
- The vd Ghein bell from Lehr 1986
- Bell no. 2 from Little Somerford in Wiltshire. This bell is the thinnest known to the author in the UK: its estimated weight may be in error.

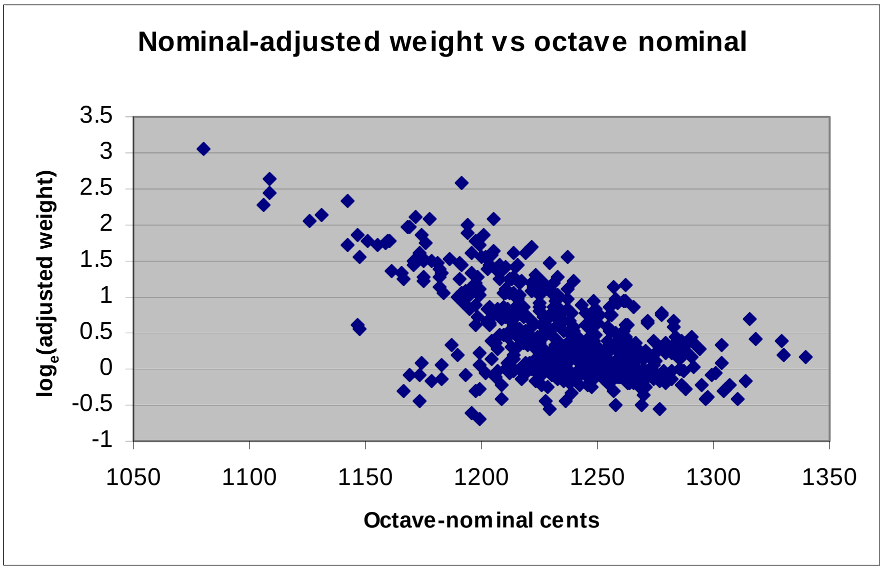


Figure 3-21 Relative weight and octave nominal

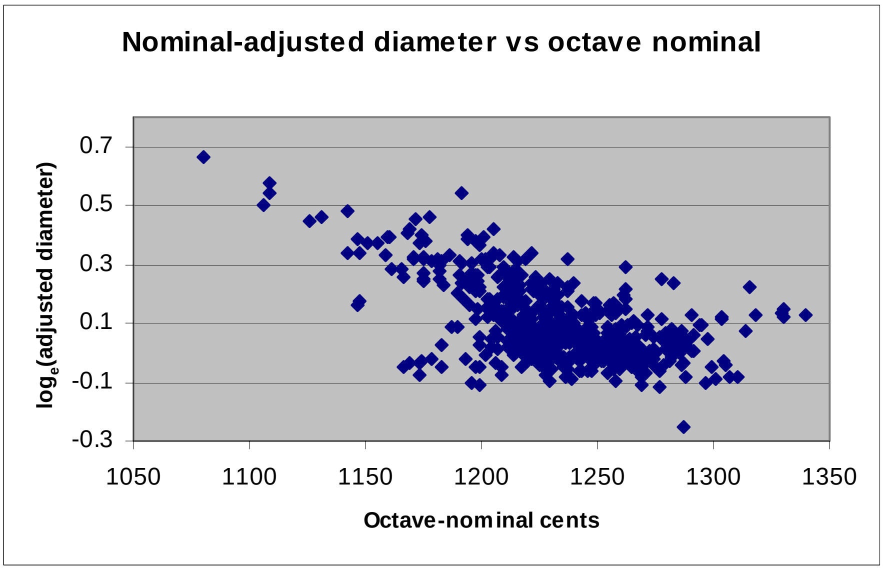


Figure 3-22 Relative diameter and octave nominal

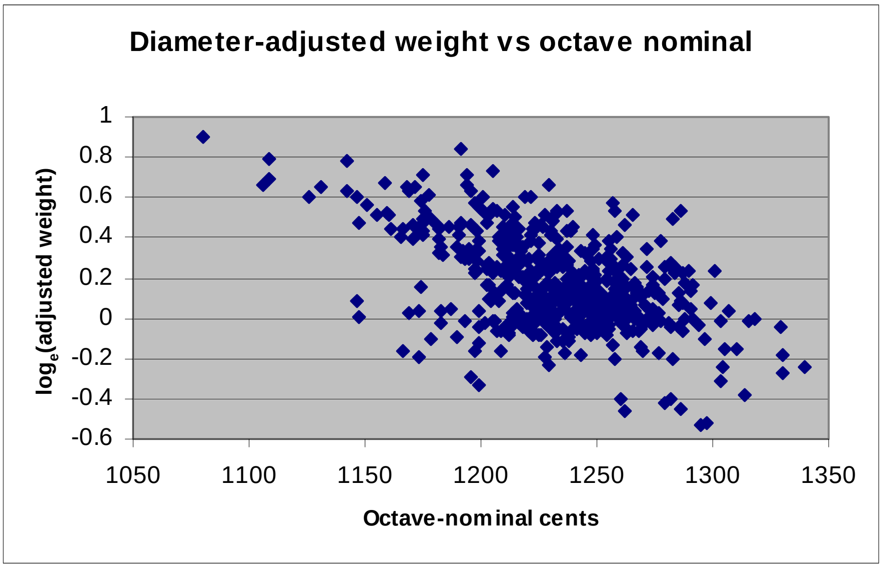


Figure 3-23 Relative weight and octave nominal

It will be seen in each case that there is a correlation between the adjusted diameters and weights and the octave nominal tuning. The correlation coefficients for the three scatter plots are as follows:

nominal-adjusted weight to octave nominal	-0.59
nominal-adjusted diameter to octave nominal	-0.61
diameter-adjusted weight to octave nominal	-0.38

Based on these results, one can say with reasonable confidence that increasing the relative weight of a bell while keeping the nominal at the same frequency (by thickening the bell and increasing its diameter) causes the octave nominal to flatten.

3.6.2 Weights and partial frequencies of 1,757 bells

The 553 bells investigated in the previous section were those for which diameters were available in the author's database. A total of 1,757 bells in the database (including the 553 already analysed) had information on weight. These bells comprised 1,732 bronze and 25 steel bells. A repeat of Figure 3-18 for this greater number of bells appears as Figure 3-24:

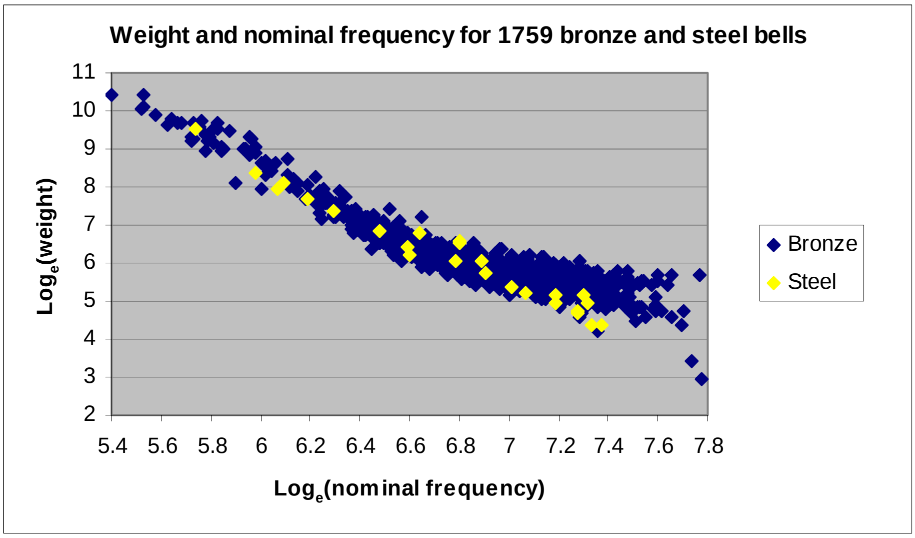


Figure 3-24 Weights and nominal frequencies of 1,759 bells

A repeat of Figure 3-21 for these bells, with yet again the regression done for bells with log$_e$(nominal) below 6.8, appears as Figure 3-25.

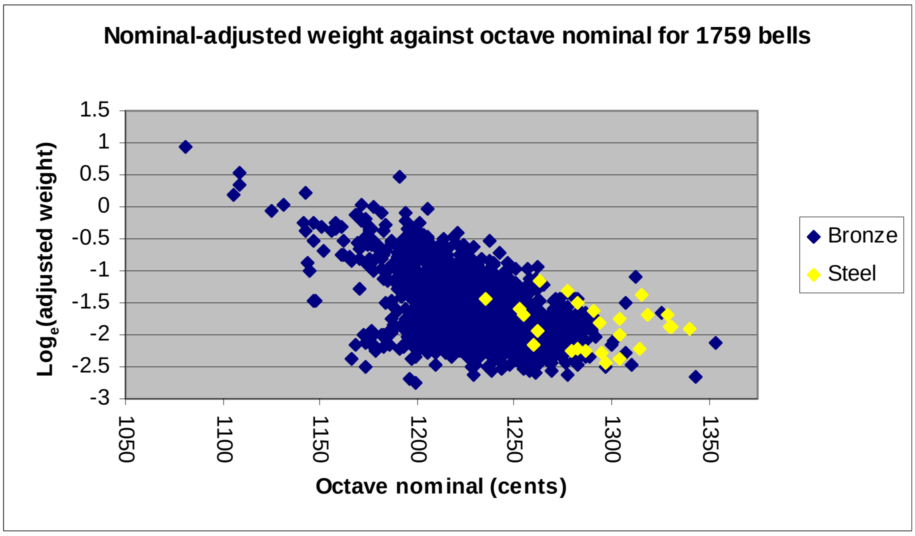


Figure 3-25 Relative weight and octave nominal for 1,759 bells

Clearly the relationship between relative weight and octave nominal tuning stands good for this greater number of bells.

It is sometimes said that bells which are not true-harmonic (i.e. which have non-octave hums and primes) are cast to a thicker scale than true harmonic bells. Figure 3-26 below shows the nominal-adjusted weight for the 1,759 bells plotted against hum tuning (represented as the cents of the hum relative to the nominal).

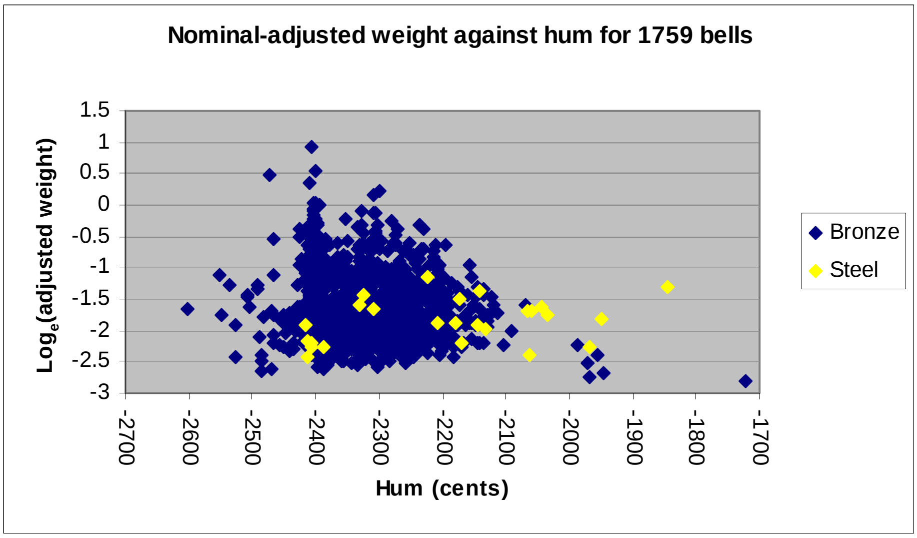


Figure 3-26 Relative weight and hum tuning for 1,759 bells

The grouping of true-harmonic bells with hums at roughly -2400 cents from the nominal can be clearly seen. The scatter plot shows no obvious relationship between adjusted weights and hum tuning. This is confirmed by the correlation coefficient of -0.115 between the two quantities, indicating no significant correlation.

The analysis of these two batches of bells confirms that flattened upper partials are indicated by the relative weight and thickness of bells, in confirmation of the statement made by Nigel Taylor and the experiment of André Lehr.

4 INVESTIGATION INTO THE PITCHES OF BELLS

4.1 Introduction

This chapter describes two experiments, conducted as part of this research, into the pitch or pitches perceived in bells. The first experiment investigates the phenomenon in large bells known as 'secondary strike' whereby some listeners perceive the bell to have a second strike pitch, often a musical fourth above the nominal pitch. The second and more significant experiment extends the technique developed for the first experiment to investigate what determines the pitches of bells across a range of frequencies.

Though the explanation of secondary strike explored and proved in the first experiment is already described in the literature, the experiment is more detailed than those conducted by previous researchers and provides an elegant confirmation that the pitches observed in bells are a virtual pitch effect.

The second experiment has not been conducted before and provides important new information on the pitch perception mechanisms in bells at different frequencies. The results of the second experiment confirm the existence of a dominant frequency region for virtual pitch effects.

Both experiment designs are based on the technique of ‘post vocalisation’ described and justified in (Terhardt and Seewann 1984), and further described in chapter 6 of this thesis. (Terhardt and Seewann 1984) report ambiguity over the octaves of pitches, and this is confirmed by the author’s experience, including that of conducting these experiments. Therefore, in both experiments described in this chapter, octave information is ignored by folding all pitches into a single octave. In addition, the need to circumvent octave ambiguity determined the choice of bell for use in the second experiment described in section 4.3 onwards.

4.2 Secondary strike in bells

As described in the previous chapter, bells typically have two sets of partials forming an approximate harmonic series, one based on the nominal and also including the superquint and octave nominal, and another based on partial I-7 and also including partials I-9 and I-11. In bells of a middle range of pitches and weight, the lower frequency series based on the nominal predominates, because these partials fall into the dominant range for pitch perception. In large, low pitched bells, the higher frequency series based on I-7 falls into the dominant region. The objective of the first experiment is to identify the pitch or pitches experienced in large bells by a number of test subjects, and prove or disprove the theory that bell pitches arise from virtual pitch effects from these sets of partials in the bell sound.

4.2.1 Secondary strike experiment design

25 bell recordings were selected from the author's collection. These bells were those with the lowest nominal frequencies, rejecting any recordings of very poor quality. The recordings were normalised to all have the same peak amplitude to eliminate any effects due to variation in loudness. The recordings were all monophonic.

The recordings were presented to the test subjects in random order. The random order, different for each test subject, was created using the Fisher-Kerr random shuffle documented and validated in section 7.2, but sorting 25 items rather than the 16 used for the main virtual pitch tests.

The tests were run from a Toshiba Satellite Pro 4600 laptop. The test sounds were played through the speakers built into the laptop. The same laptop was also configured to simultaneously record via an AKG C1000S condenser microphone linked to the laptop via an M-Audio MobilePre USB pre-amp. Recordings were made using CoolEdit sound editing software. The microphone was positioned so that it would simultaneously record both the test subject and the output of the laptop speakers. The whole test session with each subject was recorded.

Test subjects were selected who were confident and tuneful singers. No test subject other than the author had any serious knowledge of bell acoustics, nor of the hypothesis that a bell might be perceived to have more than one pitch. Test subjects did not hear one another taking the tests. The subject was asked to listen to each bell recording, and then sing the note that they heard. Subjects were able to ask for each recording to be played again if they didn’t identify the pitch on first hearing, though most subjects immediately identified a pitch on first hearing a recording. The recordings were played in the random sequence as previously described. The test subjects were encouraged to wait until the recording had played before singing the pitch heard (to avoid confusion in the subsequent analysis) though it was apparent that often the subject was singing or humming a note *sotto voce* to home in on a pitch while the recording played.

On occasion, subjects sang one note, changed their mind, and sang another. If this occurred the second was taken as the test result. On occasion, subjects sang the same note for consecutive bells. If the subjects themselves remarked on this they were asked to take the test again (and commonly sang a different note). If the subject did not remark on it the note sung was taken as a valid test result.

Subjects tended to sing the bell pitch in an octave they found comfortable – some subjects asked if it mattered which octave they sang in and were told that it didn’t, as it had already been decided to fold results into a single octave to avoid ambiguity. Subjects were allowed to pass on a test if they could not identify a pitch in the recording. No prompts were offered by the test administrator as to the result of each test. The whole set of tests for 25 recordings took 7 to 12 minutes for each subject to complete.

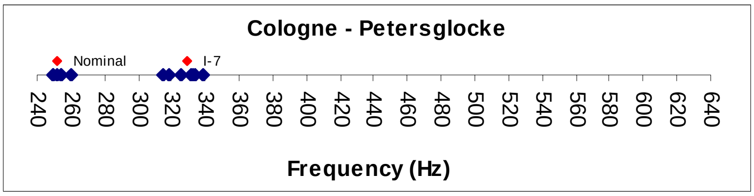
The tests were undertaken by 11 subjects, one of whom was the author. No test results were rejected. The author conducted two self-administered test runs. Because the author was aware that the bells would typically display two pitches, in one test run he sang the upper pitch heard, and in the other, the lower. One other test subject also said that he could hear two pitches, and was allowed to sing both in the tests. In both these cases, the two sets of results were taken as two test runs, i.e. there were 13 test runs in total.

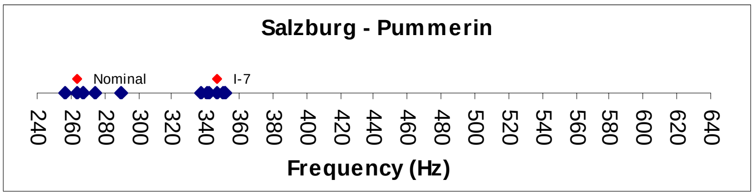
The recording of each test session was then analysed to identify the pitches sung (which in every case were taken as the lowest frequency or fundamental in the sung notes).

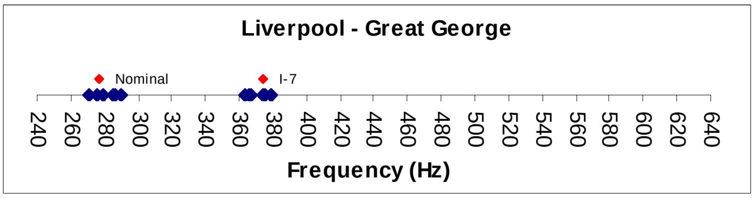
The recordings of bell sounds picked up in the test sessions were also checked to ensure the test results did not get out of sequence.

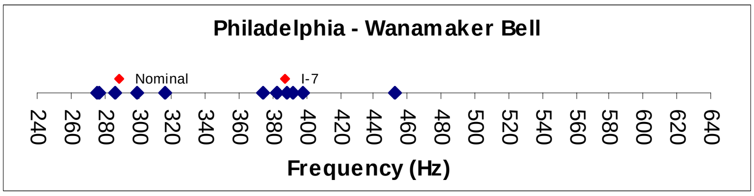
4.2.2 Analysis of secondary strike results

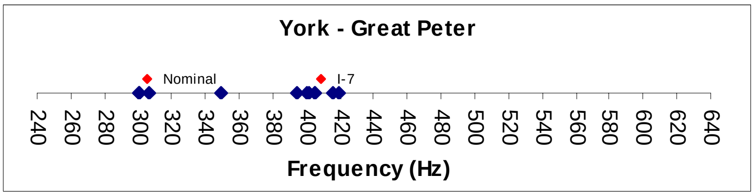
Therefore, for initial analysis the test results, and the partial frequencies of the bells used, were folded into a single octave by dividing or multiplying frequencies by powers of 2, so as to bring frequencies for each bell into a range from 1.5 semitones below the nominal to 10.5 semitones above. The pitches observed for each bell, together with the frequencies of its nominal and I-7 partials, were scatter-plotted as shown in Figure 4-1 below.

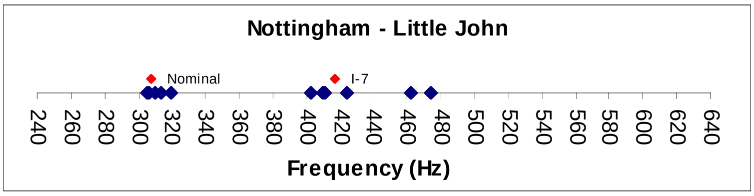

Cologne - Petersglocke
Nominal
I-7
240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540 560 580 600 620 640
Frequency (Hz)

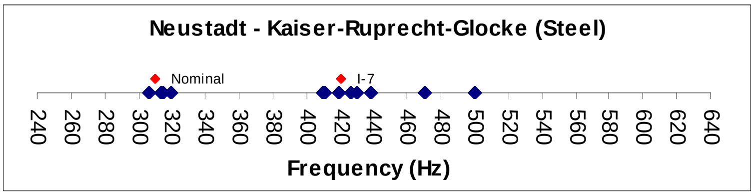

Salzburg - Pummerin
Nominal
I-7
240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540 560 580 600 620 640
Frequency (Hz)

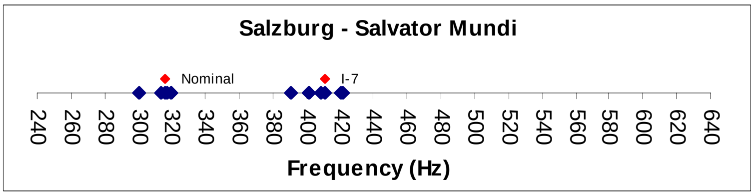

Liverpool - Great George
Nominal
I-7
240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540 560 580 600 620 640
Frequency (Hz)

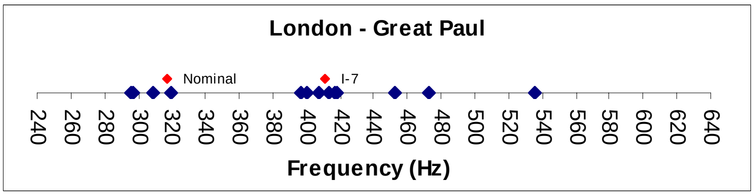

Philadelphia - Wanamaker Bell
Nominal
I-7
240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540 560 580 600 620 640
Frequency (Hz)

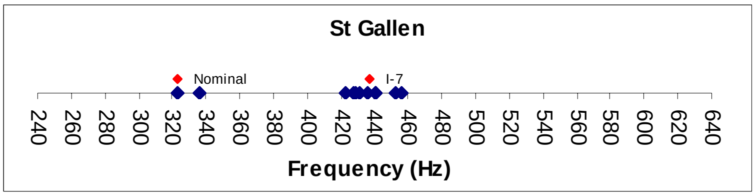

York - Great Peter
Nominal
I-7
240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540 560 580 600 620 640
Frequency (Hz)

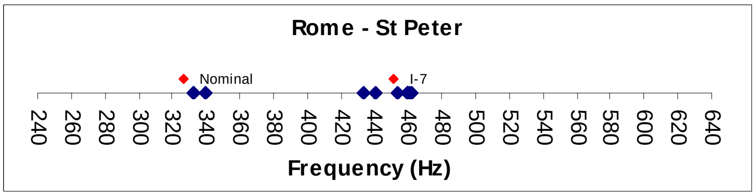

Nottingham - Little John
Nominal
I-7
240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540 560 580 600 620 640
Frequency (Hz)

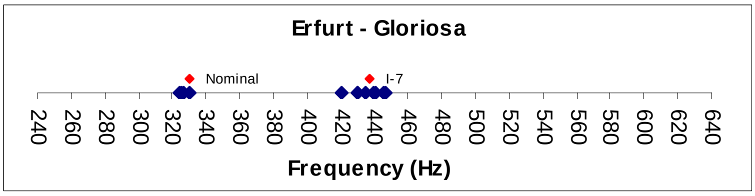

Neustadt - Kaiser-Ruprecht-Glocke (Steel)
Nominal
I-7
240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540 560 580 600 620 640
Frequency (Hz)

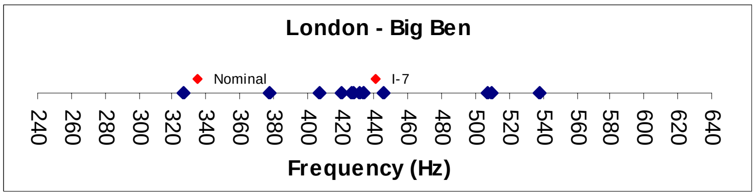

Salzburg - Salvator Mundi
Nominal
I-7
240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540 560 580 600 620 640
Frequency (Hz)

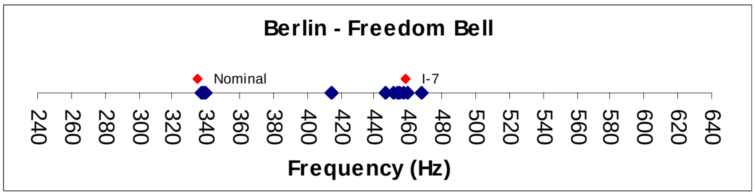

London - Great Paul
Nominal
I-7
240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540 560 580 600 620 640
Frequency (Hz)

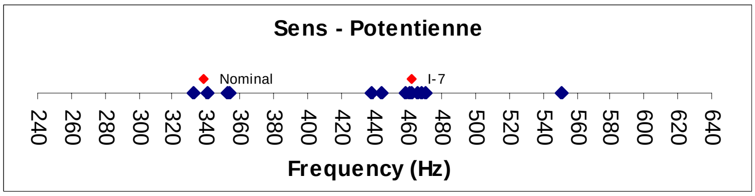

St Gallen
Nominal
I-7
240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540 560 580 600 620 640
Frequency (Hz)

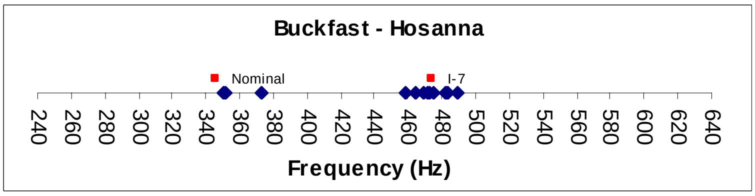

Rome - St Peter
Nominal
I-7
240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540 560 580 600 620 640
Frequency (Hz)

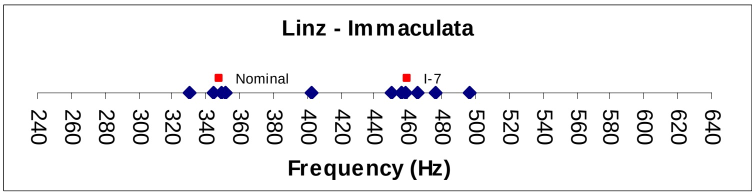

Erfurt - Gloriosa
Nominal
I-7
240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540 560 580 600 620 640
Frequency (Hz)

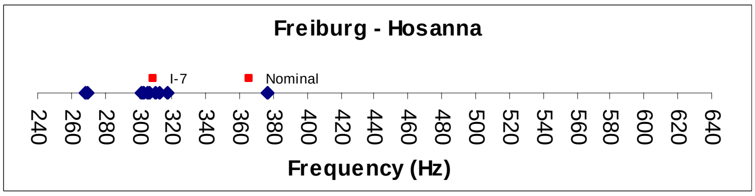

London - Big Ben
Nominal
I-7
240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540 560 580 600 620 640
Frequency (Hz)


Berlin - Freedom Bell
Nominal
I-7
240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540 560 580 600 620 640
Frequency (Hz)


Sens - Potentienne
Nominal
I-7
240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540 560 580 600 620 640
Frequency (Hz)


Buckfast - Hosanna
Nominal
I-7
240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540 560 580 600 620 640
Frequency (Hz)


Linz - Immaculata
Nominal
I-7
240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540 560 580 600 620 640
Frequency (Hz)


Freiburg - Hosanna
I-7
Nominal
240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540 560 580 600 620 640
Frequency (Hz)

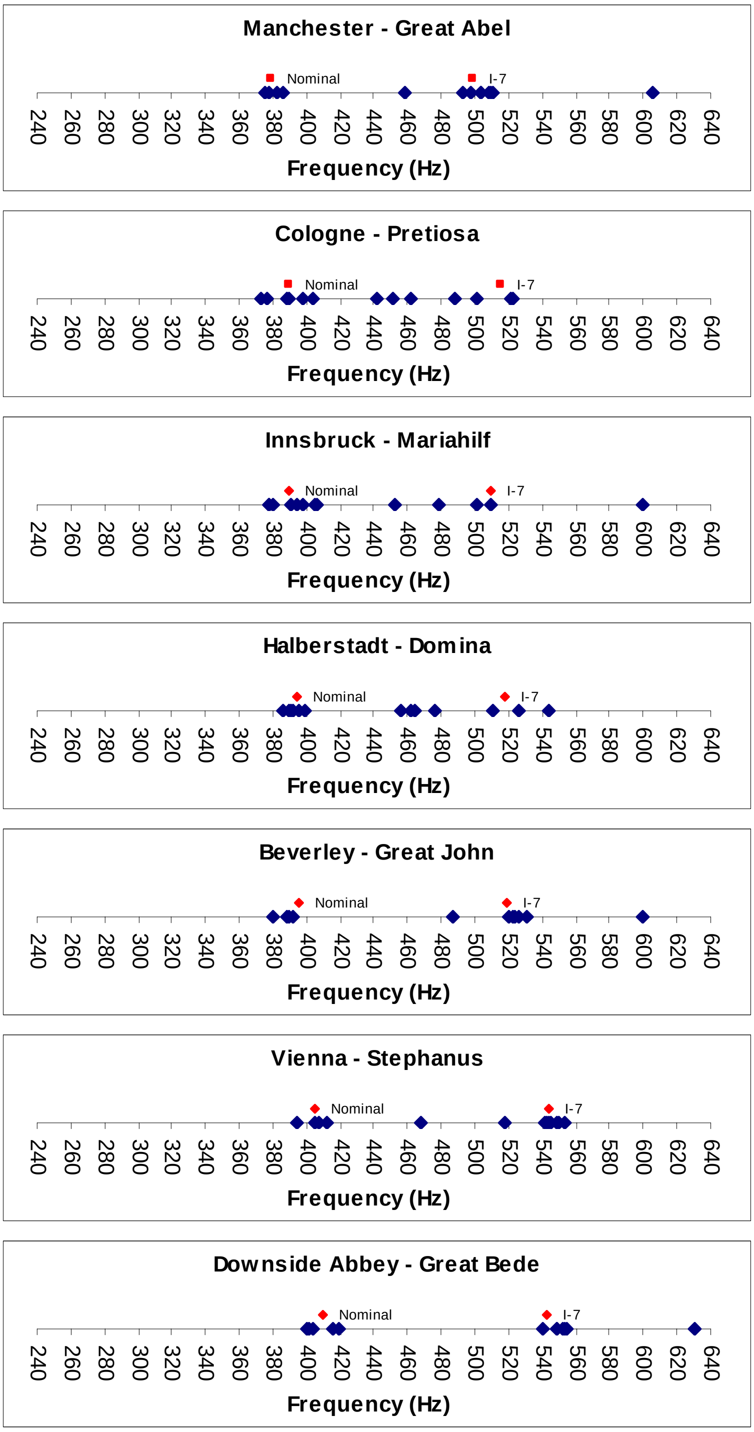


Figure 4-1 Results of secondary strike test

It is clear from these scatter plots that, for almost all bells, the pitches observed fall into two clusters located near the frequencies of the nominal and I-7 partials. Those bells for which not all pitches sung fell into these two clusters were often of poorer quality. No pitches sung were excluded from the analysis.

The bell ‘Hosanna’ from Freiburg, Germany had only one pitch sung at or near its nominal frequency. This bell has a very unusual partial structure and results for this bell are examined in more detail later in this chapter, where it is shown that its pitch results from an unusual mechanism. Results for this bell were excluded from the statistical analysis below.

Excluding the Freiburg bell, 89% of the 304 pitches observed were within 1.5 semitones of the nominal or I-7 frequencies, with 35% around the nominal and 54% around partial I-7. Figure 4-2 shows all the observed pitches plotted as cents relative to the nominal, folded into a single octave and divided into semitone bands (e.g. all the results with a frequency within +- 50 cents of the nominal appear in the band labelled ‘0’ in the figure). The clustering of the observed pitches around 0 cents (the nominal) and 500 cents (partial I-7) is obvious.

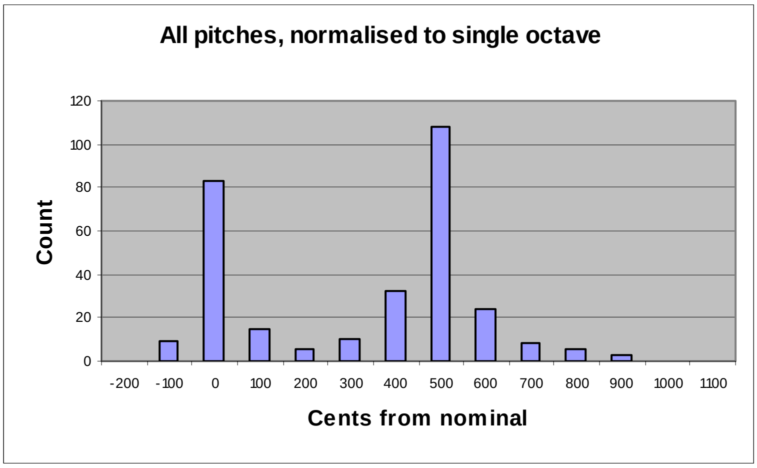


Figure 4-2 All pitches observed in secondary strike experiment

This method of plotting is slightly misleading as the relative frequency of partial I-7, approximately 500 cents above the nominal, varies from bell to bell. To avoid this problem, all frequencies within 1.5 semitones of the nominal or partial I-7 were separately plotted as above, but this time in half-semitone bands as shown in Figures 4-3 and 4-4. Note that the vertical scale is different in the two figures:

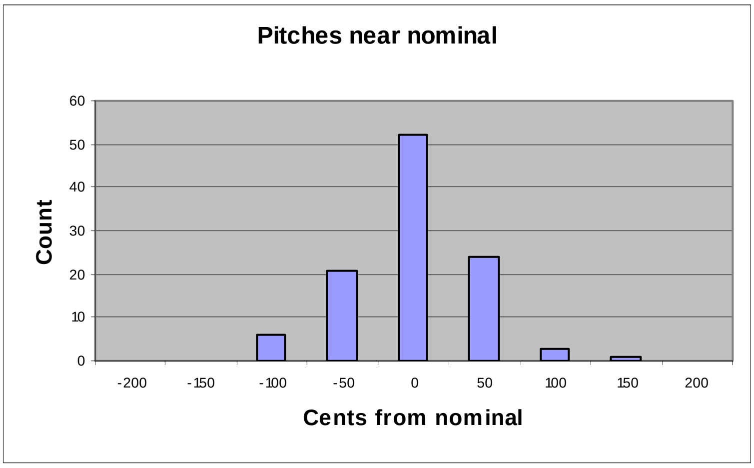


Figure 4-3 Pitches observed near nominal

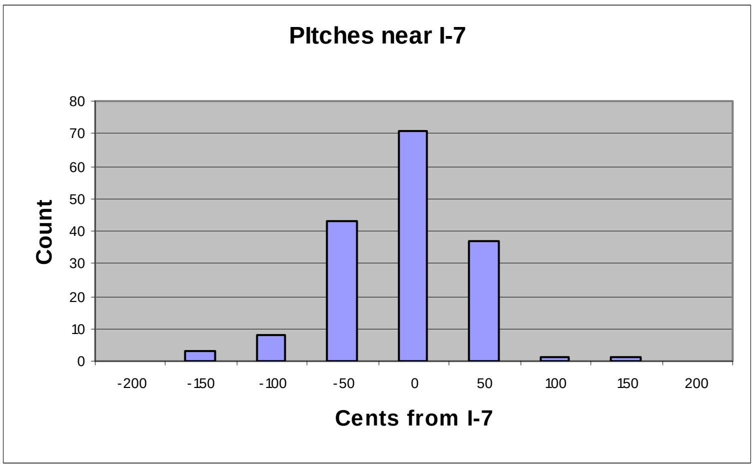


Figure 4-4 Pitches observed near partial I-7

To further investigate the results, normal probability plots were constructed for all the pitches near the nominal, and all near partial I-7. Figures 4-5 and 4-6 show the distributions of the variances from nominal and I-7 frequency respectively against the normal probability ranking for that variance:

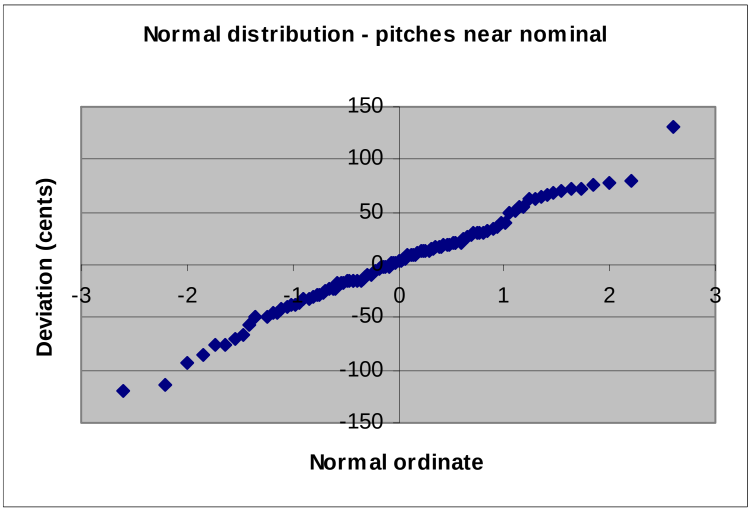


Figure 4-5 Normality check for sung pitches near nominal

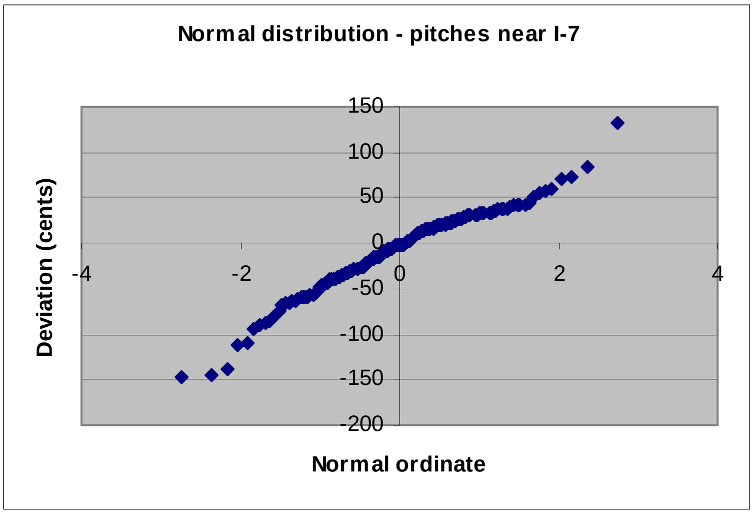


Figure 4-6 Normality check for sung pitches near partial I-7

These plots are broadly linear for variances of up to 100 cents in either direction, with some outliers beyond that. The inference drawn from these plots is that, for pitches sung within one semitone either side of the nominal and I-7 partial frequencies, deviations from a normal distribution are not significant and it is a justified assumption that the deviations are caused by experimental conditions and errors.

4.2.3 Origin of the pitches observed

Table 3-2 in section 3.4 in the previous chapter suggests that only the nominal and partial I-7 form the basis of a harmonic series among the partials of a bell. To confirm these two partials as the origin of the pitches observed in this experiment, analysis was carried out as follows. Average values for the two observed pitches for each bell were calculated by separately taking the average of all pitches within 1.5 semitones of the nominal and of the I-7 partial, in line with figures 4-3 and 4-4. As before, to avoid octave ambiguity, all observed pitches were folded into one octave.

Table 4-1 below shows the measured partial frequencies and observed pitches for all 25 bells used in this experiment. Note that, in line with section 1.3, some rows in the table have frequencies measured to less precision than others. The column headed ‘avg. low pitch’ is the average for each bell of the pitches clustered around the nominal. The column headed ‘avg. high pitch’ is the corresponding figure for the pitches clustered around partial I-7.

Bell	Nom.	Sup. - Quint	Oct. Nom.	I-7	I-8	I-9	I-11	avg. low pitch	avg. high pitch
Cologne - Petersglocke	252.3	375.3	510.4	658.0	808.8	966.6	1300.3	253.4	328.0
Salzburg - Pummerin	263.8	393.1	537.4	694.4	862.6	1038.7	1389.2	265.8	345.8
Liverpool - Great George	277.0	415.7	574.4	749.0	937.0	1133.5		281.1	371.9
Philadelphia - Wannamaker Bell	288.5	431	595	775.5	969.5	1173	1601	284.8	386.1
York - Great Peter	305.8	456.6	628.7	817.5	1019.0	1231.0	1671.0	301.9	405.6
Nottingham - Little John	308	461.5	639	834	1044.5	1266	1732	311.1	411.5
Neustadt - Kaiser Ruprecht	309.6	465.3	641.7	841.4	1064.4	1293.4		313.6	422.4
Salzburg - Salvator Mundi	316.2	468.4	637.4	822.4	1012.0	1210.8	1602.2	314.3	408.0
London - Great Paul	317.2	467.8	636.2	821.8	1017.6	1215.2	1633.8	305.3	408.6
St Gallen	322.5	488	670.5	874	1091	1314	1775	329.6	437.7
Rome - St Peter	326.9	497.4	693.0	901.8	1125.1	1360.4	1838.6	335.3	454.2
Erfurt - Gloriosa	329.8	489.8	675.6	874.8	1086.4	1313.6	1771.6	326.5	435.9
London - Big Ben	335.1	494.3	676.8	882.2	1087.0	1304.4	1746.0	327.0	428.1
Berlin - Freedom Bell	335.2	506.6	699.9	917.8	1149.5	1387.9	1897.4	338.6	456.0
Sens - Potentienne	338.5	508.2	711.5	923.0	1155.0	1389.0	1867.5	345.1	458.1
Buckfast - Hosanna	346.0	523.2	725.2	947.4	1187.9	1436.5	1971.5	358.3	474.9
Linz - Immaculata	348.2	577.0	713.4	919.5	1138.0	1360.5	1799.0	345.5	465.5
Freiburg - Hosanna	365.8	615.8	912.0	1234.5	1582.5	1974.5	2755.0	377.0	307.3
Manchester - Great Abel	378.5	562.1	770.3	996.6	1236.1	1485.5	2005.8	380.5	498.0
Cologne - Pretiosa	389.7	581.4	797.4	1030.4	1275.7	1532.3	2051.0	388.2	508.3
Innsbruck - Mariahilf	389.0	580.2	787.5	1018.5	1256.5	1510.5	1992.0	391.5	496.0
Halberstadt - Domina	394.0	585.2	803.2	1036.0	1279.0	1527.0	2015.5	392.1	514.1
Beverley - Great John	395	586	802.5	1039	1291.5	1555	2100.5	387.4	518.3
Vienna - Stephanus	405.4	605.4	836.0	1088.6	1365.9	1640.6	2220.6	404.8	542.9
Downside Abbey - Great Bede	409.4	608.6	835.6	1085.4	1352.8	1631.6	2214.0	408.2	549.5

Table 4-1 Measured frequencies and observed pitches for 25 heavy bells

The next table (Table 4-2) shows the ratio of all the measured partial frequencies to the average observed low pitch:

Bell	Nom.	Sup. - Quint	Oct. Nom.	I-7	I-8	I-9	I-11
Cologne - Petersglocke	1.00	1.48	2.01	2.60	3.19	3.82	5.13
Salzburg - Pummerin	0.99	1.48	2.02	2.61	3.24	3.91	5.23
Liverpool - Great George	0.99	1.48	2.04	2.66	3.33	4.03	
Philadelphia - Wannamaker Bell	1.01	1.51	2.09	2.72	3.40	4.12	5.62
York - Great Peter	1.01	1.51	2.08	2.71	3.38	4.08	5.54
Nottingham - Little John	0.99	1.48	2.05	2.68	3.36	4.07	5.57
Neustadt - Kaiser Ruprecht	0.99	1.48	2.05	2.68	3.39	4.12	
Salzburg - Salvator Mundi	1.01	1.49	2.03	2.62	3.22	3.85	5.10
London - Great Paul	1.04	1.53	2.08	2.69	3.33	3.98	5.35
St Gallen	0.98	1.48	2.03	2.65	3.31	3.99	5.39
Rome - St Peter	0.97	1.48	2.07	2.69	3.36	4.06	5.48
Erfurt - Gloriosa	1.01	1.50	2.07	2.68	3.33	4.02	5.43
London - Big Ben	1.02	1.51	2.07	2.70	3.32	3.99	5.34
Berlin - Freedom Bell	0.99	1.50	2.07	2.71	3.40	4.10	5.60
Sens - Potentienne	0.98	1.47	2.06	2.67	3.35	4.03	5.41
Buckfast - Hosanna	0.97	1.46	2.02	2.64	3.32	4.01	5.50
Linz - Immaculata	1.01	1.67	2.06	2.66	3.29	3.94	5.21
Freiburg - Hosanna	0.97	1.63	2.42	3.27	4.20	5.24	7.31
Manchester - Great Abel	0.99	1.48	2.02	2.62	3.25	3.90	5.27
Cologne - Pretiosa	1.00	1.50	2.05	2.65	3.29	3.95	5.28
Innsbruck - Mariahilf	0.99	1.48	2.01	2.60	3.21	3.86	5.09
Halberstadt - Domina	1.00	1.49	2.05	2.64	3.26	3.89	5.14
Beverley - Great John	1.02	1.51	2.07	2.68	3.33	4.01	5.42
Vienna - Stephanus	1.00	1.50	2.07	2.69	3.37	4.05	5.49
Downside Abbey - Great Bede	1.00	1.49	2.05	2.66	3.31	4.00	5.42

Table 4-2 Ratio of partial frequencies to lower observed pitch in 25 heavy bells

It is to be noted that in every case the nominal, superquint and octave nominal partials have ratios very close to 1 : 1.5 : 2 with the low pitch (apart from the Freiburg bell which is dealt with as a special case below). The other high partials do not show integer ratios in this way. The assumption that the lower pitch observed in these bells is a virtual pitch derived from nominal, superquint and octave nominal is supported by this data for every bell except the Freiburg bell.

Table 4-3 below shows the ratio of the measured partial frequencies to the higher average observed pitch:

Bell	**Nominal**	**Super-quint**	**Octave Nominal**	**I-7**	**I-8**	**I-9**	**I-11**
Cologne - Petersglocke	0.77	1.14	1.56	2.01	2.47	2.95	3.96
Salzburg - Pummerin	0.76	1.14	1.55	2.01	2.49	3.00	4.02
Liverpool - Great George	0.74	1.12	1.54	2.01	2.52	3.05	
Philadelphia - Wannamaker Bell	0.75	1.12	1.54	2.01	2.51	3.04	4.15
York - Great Peter	0.75	1.13	1.55	2.02	2.51	3.03	4.12
Nottingham - Little John	0.75	1.12	1.55	2.03	2.54	3.08	4.21
Neustadt - Kaiser Ruprecht	0.73	1.10	1.52	1.99	2.52	3.06	
Salzburg - Salvator Mundi	0.77	1.15	1.56	2.02	2.48	2.97	3.93
London - Great Paul	0.78	1.14	1.56	2.01	2.49	2.97	4.00
St Gallen	0.74	1.11	1.53	2.00	2.49	3.00	4.06
Rome - St Peter	0.72	1.10	1.53	1.99	2.48	3.00	4.05
Erfurt - Gloriosa	0.76	1.12	1.55	2.01	2.49	3.01	4.06
London - Big Ben	0.78	1.15	1.58	2.06	2.54	3.05	4.08
Berlin - Freedom Bell	0.74	1.11	1.53	2.01	2.52	3.04	4.16
Sens - Potentienne	0.74	1.11	1.55	2.01	2.52	3.03	4.08
Buckfast - Hosanna	0.73	1.10	1.53	2.00	2.50	3.03	4.15
Linz - Immaculata	0.75	1.24	1.53	1.98	2.44	2.92	3.86
Freiburg - Hosanna	1.19	2.00	2.97	4.02	5.15	6.43	8.96
Manchester - Great Abel	0.76	1.13	1.55	2.00	2.48	2.98	4.03
Cologne - Pretiosa	0.77	1.14	1.57	2.03	2.51	3.01	4.04
Innsbruck - Mariahilf	0.78	1.17	1.59	2.05	2.53	3.05	4.02
Halberstadt - Domina	0.77	1.14	1.56	2.02	2.49	2.97	3.92
Beverley - Great John	0.76	1.13	1.55	2.00	2.49	3.00	4.05
Vienna - Stephanus	0.75	1.12	1.54	2.01	2.52	3.02	4.09
Downside Abbey - Great Bede	0.75	1.11	1.52	1.98	2.46	2.97	4.03

Table 4-3 Ratio of partial frequencies to higher observed pitch in 25 heavy bells

For every bell except the Freiburg bell, the frequencies of partials I-7, I-9 and I-11 are very close to the ratios 2 : 3 : 4 with the higher observed pitch. No other partials show integer ratios. The assumption that the higher observed pitch is a virtual pitch based on partials I-7, I-9 and I-11 is supported by this data for every bell except for the Freiburg bell.

In the Freiburg bell, the superquint, octave nominal and I-7 partials are in the ratio 2 : 3 : 4 and are the likely generators of a virtual pitch. This is most unusual as explained below.

4.2.4 The bell 'Hosanna' from Freiburg in Germany

This bell, cast in 1258 and weighing 3,290kg, provides an extreme and convincing example of the virtual pitch explanation for the strike pitch. The bell has an archaic design, as will be seen from Figure 4-7: it is shaped like a large flowerpot with a heavy rim. The technical term for this shape is long-waisted.

Figure 4-7 Bell 'Hosanna' (Freiburg) dated 1258

The note of this bell is quoted as Eb, and the listening tests by test subjects documented above give an average perceived pitch of 307.3 Hz (which is 21 cents flat of Eb in concert pitch). This pitch is unusually high for a bell of this weight. The main partials of the bell do not follow the typical arrangement due to the archaic shape of the bell.

Despite the atypical arrangement of partials, the bell by general opinion has a sweet tone (presumably why it has survived in use for so long). The partials of the bell are shown in Table 4-4:

Frequency (Hz)	Partial	Note +- cents
135.4	Hum	Db -40
267.4	Prime	C +37
346.4	Tierce	F -14
365.8	Nominal	F# -19
615.8	Superquint	Eb -18
912.0	Octave Nominal	Bb -38
1231.6	I-7	Eb -18
1582.4	I-8	G +15
1961.8	I-9	B -12
2356.0	I-10	D +4
2758.0	I-11	F -22
3166.0	I-12	G +16
3579.2	I-12	A +28

Table 4-4 Partials of Freiburg bell

There is nothing significant in the sound at 307 Hz. But also, quite importantly, the half nominal, which in a normal bell would be the strike pitch, is an F# which is not near the pitch actually observed.

The actual pitch generation mechanism becomes immediately clear if the ratios of the partial frequencies are calculated as shown in Table 4-5:

Freq	Ratios				
135.4	1.00				
267.4	1.97	1.00			
346.4	2.56	1.30	1.00		
365.8	2.70	1.37	1.06	1.00	
615.8	4.55	2.30	1.78	1.68	1.00
912.0	6.74	3.41	2.63	2.49	1.48
1231.6	9.10	4.61	3.56	3.37	2.00
1582.4	11.69	5.92	4.57	4.33	2.57
1961.8	14.49	7.34	5.66	5.36	3.19
2356.0	17.40	8.81	6.80	6.44	3.83
2758.0	20.37	10.31	7.96	7.54	4.48
3166.0	23.38	11.84	9.14	8.66	5.14
3579.2	26.43	13.39	10.33	9.78	5.81

Table 4-5 Harmonic series in partials of Freiburg bell

The lowest harmonic series involves the sequence of partials starting at 615.8 Hz, highlighted in grey in the table above. The first three partials of this series with ratios close to 1 : 1.5 : 2 are harmonic and give rise to a virtual pitch of 615.8 / 2 = 307.9 Hz which corresponds with the 307.3 Hz measured in the experiment. Therefore, the pitch of this bell is determined by its superquint. The author has not so far found any other bell with this characteristic, which is successfully predicted by the virtual pitch theory.

4.2.5 Conclusions from secondary strike experiment

This experiment gives confirmation that virtual pitch theory provides a very good explanation for the pitches heard by listeners in large bells. The pitches heard by the vast majority of test subjects (89% of all pitches experienced, in bells of all ages and designs including the Freiburg bell) are correctly predicted by the virtual pitch theory.

4.3 Variation of pitch perception mechanisms with frequency

The success of the previous experiment on low-pitched bells suggested that an experiment looking at pitch perception across a broad range of frequencies would be illuminating. Such an experiment was devised and conducted. The objective of the experiment was to determine, for a range of test subjects, which partials determine perceived pitch at different frequencies. The hypothesis was that, due to Fletcher-Munson effects or the dominance region for virtual pitch perception (as explained in section 3.4.1 of this thesis), different partials will dominate pitch perception at different frequencies.

4.3.1 Design and setup of experiment

It was decided that the test sound to be used at the various frequencies should be derived from a single bell sound, digitally transposed to multiple frequencies, to allow comparison of results. Transposition by re-sampling and interpolation preserves both the relative frequency and relative amplitude of all the partials.

It is not possible to use a true-harmonic bell for this experiment, as due to the octave ambiguity documented above it would be impossible to distinguish between hum, prime and nominal. On the other hand, for an old-style bell with sharp hum and flat prime, it should be possible to distinguish pitches derived from the three partials despite octave ambiguity. In addition, it is desirable that the chosen bell recording should have partial amplitudes broadly in line with the averages derived in section 3.3 (so that in this sense it is a ‘typical’ bell).

The author’s collection of bell recordings was searched for a bell with the following characteristics:

- Hum broadly a tone sharp of two octaves below the nominal
- Prime broadly a tone flat of one octave below the nominal
- Partial amplitudes similar to the average values
- Recording of good quality.

The bell selected was the old 9th bell at St Mary-le-Tower, Ipswich, cast by John Warner in 1866. Some important partials of the bell are shown in Table 4-6. It will be seen that these partials are nicely spread across the octave, to allow them to be easily distinguished.

Partial	Frequency	Cents to nominal	Cents (single octave)
Hum	201.9	-2209.3	190.7
Prime	323.7	-1392.1	-192.1
Tierce	424.2	-924.0	276.0
Quint	562.8	-434.6	765.4
Nominal	723.4	0	0
I-7	1959.3	1724.9	524.9

Table 4-6 Partials of St Mary le Tower 9th

The bell's spectrum is as shown in Figure 4-8.

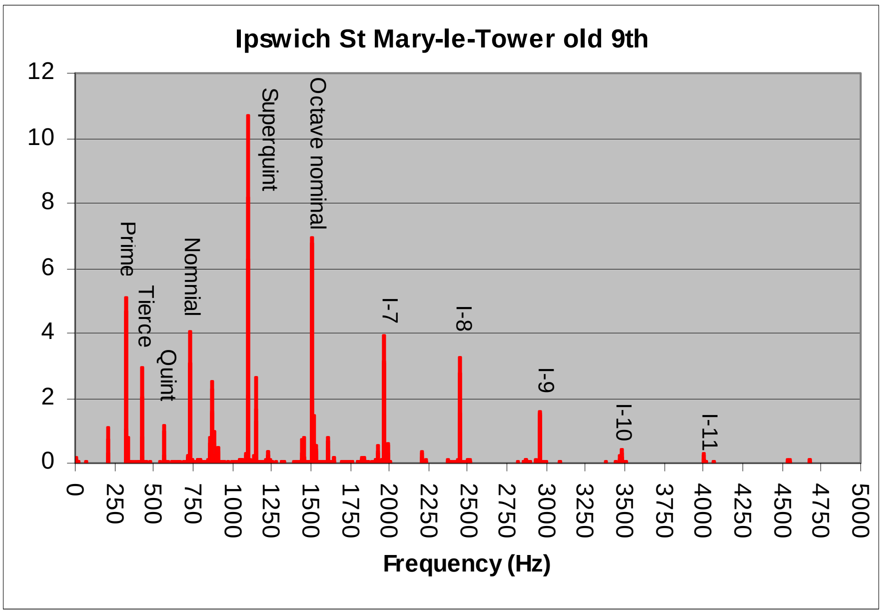


Figure 4-8 Spectrum of Ipswich St Mary-le-Tower 9th

It will be seen by comparison with Figure 3-15 in section 3.3 that the tierce and nominal are weaker and the prime is stronger than average. All in all, though, the shape of the spectrum is fairly typical.

As regards the frequencies at which to conduct the test, it was decided to use the same nominal frequencies as used in the virtual pitch experiments presented later in this thesis to allow comparison of results. The frequency range was extended upwards to explore which partials determine pitch at elevated frequencies. The nominal frequencies were spaced 1/3 of an octave apart and were as in Table 4-7, giving 12 test sounds in total:

Nominal frequencies (Hz)
315.0
396.9
500.0
630.0
793.7
1000.0
1259.9
1587.4
2000.0
2519.8
3174.8
4000.0

Table 4-7 Nominal frequencies for pitch perception test

The original recording of the selected bell was digitally manipulated to create each test waveform using the 'Resample Stretch' facility of Adobe Audition 1.5, which stretches or shrinks the timebase of the sound to shift frequencies and amplitudes in proportion.

4.3.2 Conduct and results of experiment

This experiment was conducted in an identical fashion to the previous experiment, except of course that the random sequences were for 12 sounds rather than 25. Selection criteria for the test subjects (confident and tuneful singers) were the same as before; some but not all subjects participated in both experiments. The tests were conducted and the results analysed in an identical fashion, using the same equipment as for the first experiment. Section 4.2.1 above gives details.

A total of 17 subjects carried out the tests, one of whom was the author. Unlike the previous tests, subjects were not allowed to sing more than one pitch but were forced to decide (leading one subject to say, for one test sound ‘But there’s so many notes!’). As before, subjects were allowed to pass on a test if they genuinely could not decide on a single pitch. Each subject took about 5 or 6 minutes to complete the 12 tests.

The recordings taken for each test were analysed by measuring the fundamental frequency of the sung note. For each test recording, the measured frequencies and the bell partials were folded into a single octave (by division or multiplication by powers of two). Measured frequencies were assigned to bell partials by finding the nearest partial to the measured frequency, and making the assignment provided the two were less than 1.5 semitones apart (the same interval as for the previous experiment). 190 out of 204 measured pitches or 93% of measured pitches were assigned to a partial by this method. Narrowing the interval to 0.75 of a semitone caused this to drop to 169 matches out of 204 measurements (83%), i.e. the width of match is not critical.

A normal probability plot of the deviation of the sung pitches from the nearest partial (assigned using the above approach, and ignoring those pitches not near one of the chosen partials) is given below as Figure 4-9:

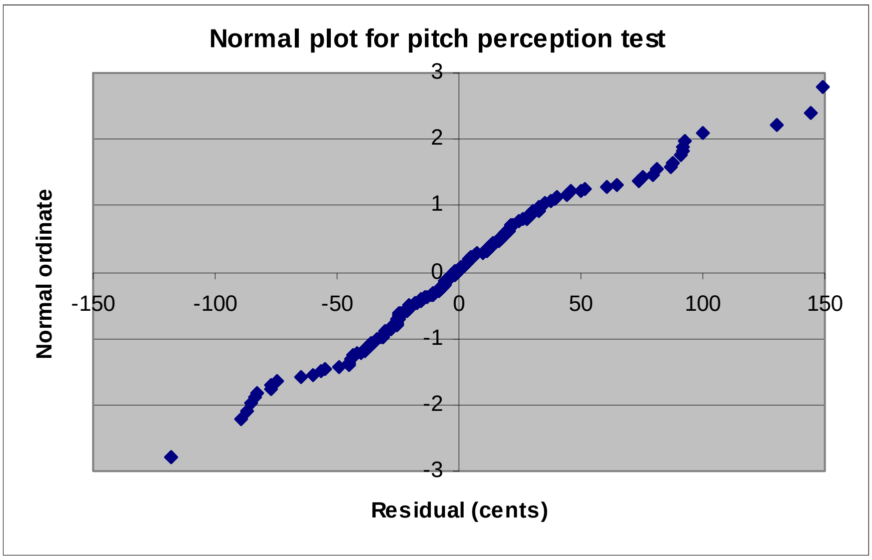


Figure 4-9 Normal plot of results from pitch perception test

Based on this plot, there is no reason not to suppose that the residuals are normally distributed, i.e. the match to the chosen partials plus human error in pitching the notes is an adequate explanation for the results seen.

Numbers of assignments to partials at each frequency were counted and plotted as in Figure 4-10 below. Measured frequencies not assigned to a partial do not appear in this plot.

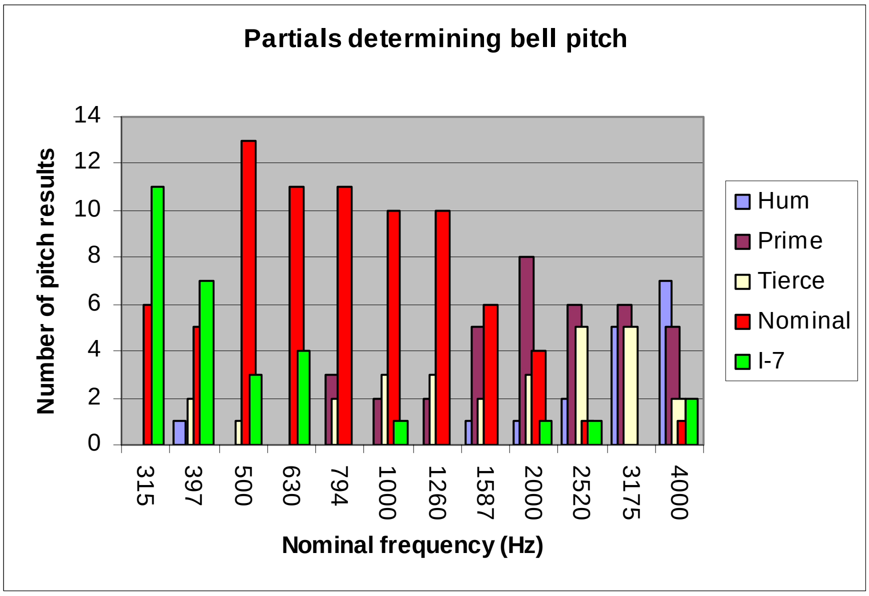


Figure 4-10 Partials determining bell pitch at different frequencies

The results can be considered in three frequency bands:

- Below 500Hz, this experiment duplicates the results of the previous one: most pitches heard are virtual pitches based on either partial I-7 or the nominal, with I-7 rather more frequent
- In the range 500 to 1600Hz, the majority of pitches are virtual pitches based on the nominal
- At 2000Hz and above, partials such as the prime, tierce and hum play the leading role in pitch determination. Pitches based on these partials are probably spectral pitches, as there is no suitable harmonic series in the bell partials to support a virtual pitch model.

These results support a dominant region for virtual pitch perception of broadly 500Hz to 1600Hz. If the nominal frequency is lower than this range, virtual pitch effects due to a higher partial (I-7) predominate. If the nominal frequency is higher than this range, lower frequency partials determine the pitch. Which partial determines the pitch at these higher frequencies is probably dependent both on partial amplitude and where the individual partials lie in the frequency spectrum. From Figure 4-10 it can be see that at nominals of 2000Hz, 2520Hz and 3175Hz the prime has a slight edge over the hum (the prime is rather louder). At a nominal of 4000Hz, the hum has a slight edge over the prime. The frequencies of the various partials for these sounds are as follows:

Nominal	Hum	Prime	dominant partial
1587	443.0	710.5	Prime
2000	558.0	895.0	Prime
2520	703.5	1127.5	Prime
3175	886.0	1420.5	Prime
4000	1116.5	1789.5	Hum

Table 4-8 Dominant partials in high frequency bell sounds

As the pitches formed from these partials are probably spectral pitches, one can assume that equal loudness or Fletcher-Munson effects favour the hum at higher frequencies even though the prime is louder because the prime has moved into an area of lesser sensitivity.

It is notable that in the dominant virtual pitch region, the nominal determines pitch via a virtual pitch mechanism in preference to higher amplitude partials via a spectral pitch mechanism. To quote section 2.3 of (Terhardt & Seewann 1984): *'This perception mode [virtual pitch] in acoustic communication with language and music is not the exception, but the rule.'* The test conditions, with the test subjects able to listen to each sound as many times and for as long as they desired, favour spectral pitch over virtual pitch formation, further showing the power of the virtual pitch effect.

4.4 Summary of pitch experiments

These two experiments taken together confirm two significant things:

- That virtual pitches provide a robust explanation of pitch perception in bells at nominal frequencies of 1600Hz and below
- The existence of a dominance region for virtual pitch perception, and its impact on pitch perception in bells.

The analysis of which partials determine pitch at different frequencies is new work, proving for bells the inferences in the literature based on work in other domains.

4.5 The timbre of bells

Timbre is that quality of a sound or complex tone that allows us to distinguish sounds of the same pitch as being from different instruments or sources. Timbre is determined primarily by the frequencies, amplitudes and envelopes of the partials in a complex tone. Unlike pitch, there is not a definition of timbre that aids its measurement. It is clear from experience that timbre is multi-dimensional. When considering the timbre of bells, adjectives are used such as sweet, harsh, shrill, tuneful, growling, and even "opening like a rose", though the meaning of the last term is a little obscure.

In fact, pitch is also a timbral effect; the changes in upper partial spacing researched in the virtual pitch experiments in this thesis bring about a significant change in the timbre of the sound. What distinguishes pitch from other timbral effects is the capability we have of ordering sounds as higher or lower and assigning a measurement of pitch to that ordering.

It is well established that certain factors in bell design and tuning have a known effect on timbre. For example, true-harmonic tuning of the lower partials (i.e. that the hum, prime and nominal form perfect octaves) is usually considered to give an improvement in sound quality over bells which have the lower partials ‘out of tune’ with themselves (so-called old style tuning). Quality of metal has a known effect on sound quality; impure bronze has a deadening effect on a bell’s tone, probably due to rapid decay of some of the important partials. As the experiments in this thesis prove, the upper partials make a significant contribution; bells with flatter upper partials sound flat or dull in tone.

The effect on timbre of true-harmonic as compared to old-style tuning of the lower partials for bells of different nominal frequencies can be explained by reference to the experiment results given earlier in this chapter, in terms of the dominance region for virtual pitch.

For bells with low frequency nominals (below 500Hz), the lowest partials, especially the hum, are so low as to make it hard for the human ear to judge their frequency. Even if the lower partials are badly out of tune, the effect on the overall timbre is not great. For these bells, timbre is mostly determined by the primary and secondary strike, together with partials above the nominal.

For bells with high frequency nominals (above 1500Hz), there is competition between the virtual pitch based on the nominal and spectral pitches based on the lower partials. If the bell is true-harmonic, i.e. hum and prime are in octaves with the nominal, the virtual and spectral pitches will align, leading to a harmonious sound. If the bell is not true-harmonic, the various pitches will conflict with each other and the overall effect is discordant. This effect is especially noticeable in small bells with sharp hums, leading to an unpleasant and shrill timbre. Although much more investigation is needed, the dominance region for virtual pitch thus provides a straightforward and convincing explanation for the phenomena heard in practice.

Beyond these effects there is much still to explore; modern bells from different founders with various shapes and designs do not sound alike even though the tuning of their main partials is the same. The research to explain this in terms of partial frequencies, amplitudes and decays remains to be done. The first step may be to establish a measurement scale for these timbral affects such as exists for pitch.

5 PARTIAL CLASSIFICATION SCHEMES AND FREQUENCIES

5.1 Introduction

This chapter documents investigations into the following areas:

- A comparison of the partial classification schemes proposed by Lehr and Perrin, reconciled with a recording of the actual bell investigated by Perrin
- A simple model for the relative frequencies of the upper partials of bells which is a new discovery, and considerably simplifies the experiments carried out
- Before and after tuning figures for the Berlin Freedom bell illustrate this model and the effect of bell tuning on upper partials
- A model of upper partial frequencies previously proposed by Perrin and Rossing (based on Chladni's law) which is found not to explain the effects giving rise to variations in virtual pitch.

This work was originally undertaken just with the aim of simplifying the virtual pitch tests. As it progressed, substantial new insights emerged not previously reported in the literature. The reconciliation between Lehr's and Perrin's classification schemes based on analysis of the sound of the actual bell used by Perrin has not been done before. The investigation into upper partial frequencies and the model developed for their relationship is new work, and provides a robust explanation of some effects in bell design not previously understood.

5.2 Partial classification schemes

The high relative amplitude of certain partials is shown in the spectral diagram at the start of chapter 3 (Figure 3.1), and their contribution to the formation of virtual pitches has been explained. What has not yet been covered is the naming of the partials, the modes of vibration involved, and why some partials tend to stand out in the sound of all bells.

The five partials lowest in frequency (the hum, prime, tierce, quint and nominal) are easy to observe - they can be heard independently by musical listeners after a little practice and are easy to beat against tuning forks so their frequency can be measured. The hum is the lowest frequency partial and lasts the longest in a freely vibrating bell. The prime (also known as the fundamental, extra or second partial) is co-incident with the strike pitch in a true-harmonic bell but can be easily heard if it is not an exact octave below the nominal. The tierce and quint form approximately the third (usually minor) and fifth of a triad with the strike pitch. The nominal is so called because it determines the note name of the bell (the strike pitch is about an octave below the nominal in almost all bells). The big step forward in true-harmonic tuning was the recognition of the bad effect that mistuning of the low partials relative to the nominal had on the sound quality of a bell, and the subsequent devising of tuning methods and changes to the bell shape to bring them under control.

The higher frequency partials are not easy to observe. They can be stimulated by striking the bell away from the rim (often they are referred to as 'crown partials' not because the crown is especially important in their formation, but because they can be stimulated without the lower partials by striking a bell near its crown). They are high pitched and of shorter duration than the hum and other low partials, and even measuring their frequency with a tuning fork is difficult except in big bells. Some attempts have been made to tune them but, as shown below, fundamental characteristics of the Western bell shape limit what can be done. It was only the advent of electronic methods of sound and spectrum analysis that made investigation of the higher partials practical, even though they are key to the formation of the strike pitch.

There are three classic studies of bell partials using modern techniques. The first, (Lehr 1965), was grounded in the practice of bell founding and tuning and studied two bells, finding 19 partials in one and 32 in the other. Lehr classified the partials according to the number of nodal meridians (stationary points around the circumference of the bell) and the behaviour of the partial on the tuning machine (which is tantamount to classifying by the number of nodal circles between the rim and the crown of the bell).

In a later paper, Lehr (1986) describes a more comprehensive investigation of a bell using electronic methods, and identified 42 partials, which he again, in most cases, classified by number of nodal meridians and circles. He identified a number of partials, which he calls extensional vibrations, which involve longitudinal stretching of the bell material in a direction tangential to the surface (rather than flexing or twisting, which are the common modes of vibration). Lehr makes the point that because the extensional modes have an antinode at the crown of the bell, they are damped when the bell is bolted to a support and are not of great acoustical significance.

The third work, reported on in (Perrin, Charnley & DePont 1983) and subsequently updated in (Perrin, Charnley, Banu & Rossing 1985), (Perrin & Charnley 1987a) and (Perrin & Charnley 1987b) is certainly the most comprehensive investigation ever done into the modes of vibration of a bell. Over a period of years they identified 134 modes of vibration in a bell provided by Taylor's bellfoundry, measuring the frequencies and identifying nodal meridians and circles using electronic stimulators and sensors attached to the bell. They identified four different types of vibrational modes:

- Those where the rim deforms from a circle without stretching (identified as Rim Inextensional Radial or RIR)
- Those where the rim remains circular but moves axially, or with a twisting motion (identified as Rim Axial or RA)
- Those where the rim is at rest and the body only of the bell flexes (identified as R=m where m is a quantum number related to the number of nodal circles)
- Various extensional modes (e.g. the one identified as alpha in Table 5-1).

To assist in the analysis, DePont built a computer model of the bell using finite element analysis which was used to confirm the vibrational characteristics of modes already investigated and predict the existence of others still to be found. Perrin et al proposed an alternate classification scheme to that of Lehr, still using numbers of nodal meridians and circles but including the additional information not available to Lehr on the type of vibration. What was not fully considered during the course of the work was the acoustical significance of the many modes of vibration, i.e. which modes make an appreciable contribution to the bell's sound.

The bell used by Perrin et al is preserved in the museum at Taylor's bellfoundry in Loughborough. A photograph appears as figure 5-1 below of the bell, still suspended from the frame used for the work reported in (Perrin, Charnley & DePont 1983).

Figure 5-1 The bell investigated by Perrin and Charnley

A high quality recording of this bell taken by the author provided the information on acoustical significance of vibrational modes missing from the original work, and allowed the classification schemes of Lehr and Perrin to be reconciled. This reconciliation makes three things possible for the first time:

- Cross-referencing the classification schemes of Lehr and Perrin
- Understanding the details of the vibrational modes studied by Lehr
- Understanding the physical basis for the prominence of certain partials in the sound of bells.

The analysis of the recording of the 'Perrin' bell, as for all bells studied in this research, was done using the author's Wavanal software. This software analyses the spectra of sound files digitised on a PC and displays the results in such a way as to facilitate acoustical analysis of bells.

Bell partial frequencies are temperature dependant (they flatten slightly with increasing temperature). A previous poor quality recording of the 'Perrin' bell gave exact correspondence with Perrin's results when analysed with Wavanal. The high quality recording, taken on a warm September afternoon (6th Sept 2006), showed lowered frequencies compared with the Perrin analysis. A regression fit was done of frequencies measured with Wavanal against the reported results to determine the temperature shift.

A regression analysis with zero intercept was used. If y_i are the values measured by Perrin, and x_i the values measured with Wavanal, then assuming a model $y_i = m \cdot x_i + \varepsilon_i$ where ε_i is an error value with expectation value 0, then the error sum of squares is $SS = \sum_i (y_i - m \cdot x_i)^2$ and the value of m giving minimum SS is $m = \sum_i x_i y_i \Big/ \sum_i x_i^2$.

The calculated correction factor was 1.000422, which is a shift of 0.7309 cents. The temperature coefficient of bell partials is reported as 1.5 x 10^{-4} per deg. C. (for example see Terhardt & Seewann 1984 section 2.1), so this shift implies a temperature difference of 2.8 deg. C between Taylor's museum and the environment in which the Perrin analysis was done, which I am told was temperature controlled. The figures reported below are after the correction has been applied.

As explained in section 3.2.5 on doublets, modes of vibration of a bell split into pairs because of asymmetry in the bell. Perrin et al investigated both halves of each pair and reported their frequency results as the frequency of the upper, and the difference between upper and lower. In the recording, either or both halves of the doublet may be present (depending on where the clapper hits compared with nodal positions). When Perrin reported a split doublet but only one half could be found in the sound, the detected frequency was assigned to the upper or lower of the doublet pair so as to minimise the correction factor.

Once the analysis of the bell recording had been reconciled with Perrin's reported results, the combined analysis and partial classification was further reconciled against the analysis done by Lehr of his two bells. The various partials were aligned by:

- Matching the number of nodal meridians reported by the two authors
- Matching the number of nodal circles (though Perrin explains in a later paper that sometimes the number of circles is over-estimated near the crown due to the low amplitude)
- Matching the relative frequencies measured in cents from the nominal for all three bells.

The measured spectrum of the bell (analysed over a period of 1s following the clapper strike) appears below as Figure 5-2, followed by the detailed reconciliation of recording, Perrin et al's analysis, and Lehr's results as Table 5-1. Many more partials were identified in the recording than those apparent in Figure 5-2.

Note that in Table 5-1, only those partials where amplitude could be detected in the recording after a diligent search are listed. Many of the PCD partials and all the extensional partials from (Lehr 1986) are missing because of this. Conveniently, this avoided some practical difficulties in matching the PCD and Lehr classifications where the correspondence was ambiguous. All the partials detected in the recording were found in the PCD analysis and could unambiguously be allocated a classification from both schemes. The highlighted rows are those for the partials most visible in the spectrum.

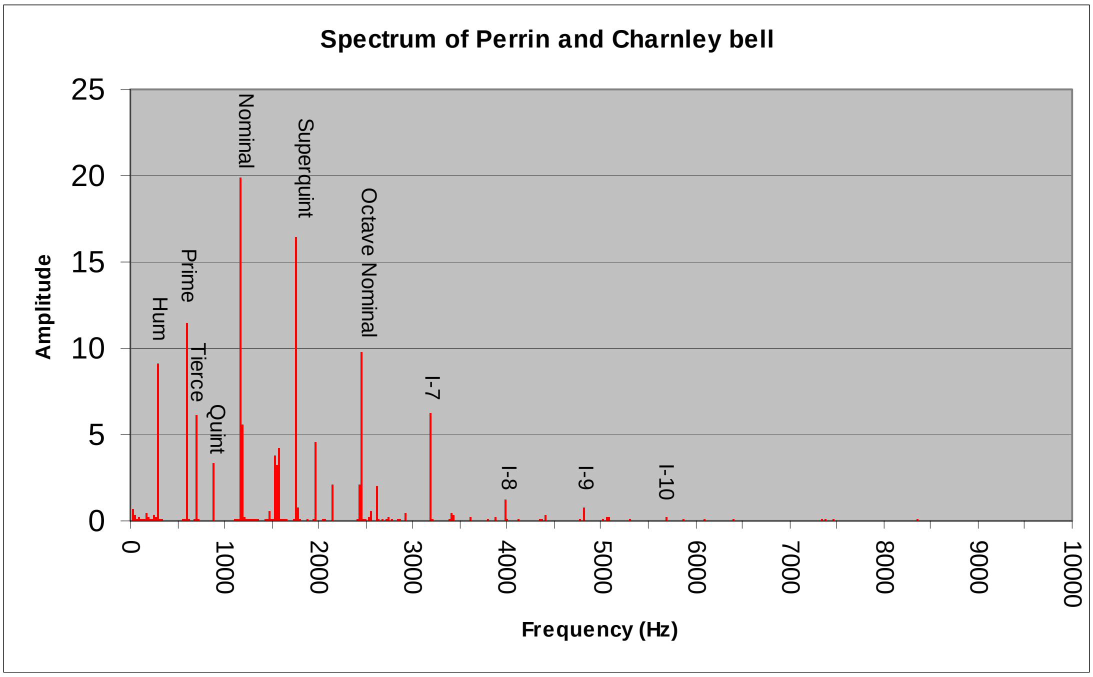


Figure 5-2 Spectrum of bell investigated by Perrin and Charnley

Recording		PCD Measurements			PCD	Nodal	PCD	Lehr Group							Lehr 1965 table 2		Lehr 1965 table 3		Lehr 1986 table 1	
Lower	Upper	Lower	Upper	splitting	class.	mer'dns	Cents	I	II	III	IV	V	VI	VII	Freq	Cents	Freq	Cents	Freq	Cents
	292.7		292.72	0	RIR	4	-2401.7	I-2							205.1	-2400.4	211.4	-2400.0	170	-2405.1
	585.9		585.92	0	R=1	4	-1200.2		II-2						410.3	-1200.0	422.6	-1200.8	341	1200.0
	692.9		692.94	0	RIR	6	-909.8	I-3							491.3	-888.1	505.4	-891.1	409	-885.2
882.3		882.21	882.53	0.32	R=1	6	-491.1		II-3						614.4	-501.0	630.6	-507.9	512	-496.4
1171.2		1171.4	1172	0.6	RIR	8	0.0	I-4							820.6	0.0	845.6	0.0	682	0.0
1472.2		1472.5	1472.8	0.3	R=1	8	395.5		II-4						1058	439.9	1082	426.8	876	433.4
	1526.7		1525.6	0	R=2	6	456.5			III-3					1076	469.1	1087	434.8	908	495.5
1559.0	1560.1	1559.2	1559.8	0.6	R=2	4	494.9			III-2					1089	489.9	1100	455.3	890	460.8
1760.8		1761.1	1764.3	3.2	RIR	10	708.1	I-5							1232	703.5	1251	678.0	1026	707.0
1948.8	1949.3		1948.9	0	R=2	8	880.4			III-4					1371	888.6	1398	870.4	1132	877.2
	2039.9		2040.3	0	R=2	2	959.8													
2145.7		2145.6	2146.8	1.2	R=1	10	1047.9		II-5						1582	1136.4	1597	1100.8	1289	1102.1
2436.5	2440.8	2436.9	2441.2	4.3	RIR	12	1270.3	I-6							1706	1267.0	1751	1260.2	1423	1273.3
	2483.8		2485.7	0	R=3	4	1301.6				IV-2				1735	1296.2	1773	1281.8	1425	1275.7
2537.3		2537.6	2540.3	2.7	RA	4	1339.2													
	2550.5	2545.8	2550.4	4.6	R=3	6	1346.1				IV-3				1772	1332.8	1782	1290.5	1450	1305.9
	2616.2	2616.3	2617.8	1.5	R=2	10	1391.3			III-5					1854	1411.1	1914	1414.3	1528	1396.6
	2721.1	2716.6	2718.1	1.5	alpha	2	1456.4													
	2832.5	2828.6	2832.5	3.9	R=3	8	1527.7				IV-4				1980	1524.9	2009	1498.1	1623	1501.0
2857.1		2858.2	2858.5	0.3	R=3	2	1543.5													
2920.3	2921.5	2920.7	2921.6	0.9	R=1	12	1581.3		II-6						2162	1677.1	2167	1629.2	1767	1648.2
	3183.9	3176	3183.1	7.1	RIR	14	1729.8	I-7							2241	1739.3	2290	1724.8	1866	1742.5
	3375.6		3376.8	0	R=3	10	1832.0				IV-5				2378	1842.0	2438	1833.2	1951	1819.6
3402.7		3403	3403.5	0.5	RA	6	1845.7													
	3431.1	3431.3	3433.4	2.1	R=2	12	1860.8			III-6							2534	1900.0	2021	1880.7
3603.3		3603.6	3607.1	3.5	R=4	6	1946.2					V-3					2505	1880.1	2130	1971.6
3795.3	3796.9	3795.6	3797.6	2	R=1	14	2035.3		II-7								2823	2087.0	2272	2083.3
3863.2	3866.7	3863.5	3867	3.5	R=4	8	2066.7					V-4					2738	2034.1	2244	2061.9
3977.8	3982.0	3978.4	3982	3.6	RIR	16	2117.4	I-8									2869	2115.0	2330	2127.0
4125.3		4125.7	4126.8	1.1	R=3	12	2179.3				IV-6						3022	2204.9	2393	2173.2
	4341.2		4340	0	R=2	14	2266.5			III-7							3233	2321.8	2575	2300.1
4366.4	4402.1	4368	4401	33	RA	8	2290.6													
	4545.9	4539	4543	4	R=6	4	2345.6							VII-2			3301	2357.8		
	4764.6	4759.4	4765.3	5.9	R=1	16	2428.3		II-8								3544	2480.8		

Recording		PCD Measurements			PCD	Nodal	PCD	Lehr Group							Lehr 1965 table 2		Lehr 1965 table 3		Lehr 1986 table 1	
Lower	Upper	Lower	Upper	splitting	class.	mer'dns	Cents	I	II	III	IV	V	VI	VII	Freq	Cents	Freq	Cents	Freq	Cents
4813.9	4815.3		4813.5	0	RIR	18	2445.7	I-9									3484	2451.2		
	5004.9	5002.5	5005.5	3	R=3	14	2513.4				IV-7						3721	2565.2		
5063.4		5064.5	5076	11.5	R=4	12	2537.7					V-6					3644	2529.0		
5296.0		5295	5298	3	RA	10	2611.8													
5557.3		5556	5563	7	R=5	10	2696.3						VI-5				3792	2597.9		
	5677.5	5674	5676	2	RIR	20	2731.1	I-10												
5877.0		5877	5878	1	R=4	14	2791.6													
	6009.0	6002	6006	4	R=3	16	2828.9													
	6102.8	6101	6105	4	R=5	12	2857.2													
	6401.9	6394	6402	8	RA	12	2939.5													
6555.3	6558.3	6555	6559	4	RIR	22	2981.4	I-11												
	6657.8	6655	6657	2	R=6	10	3007.1													
	6818.3	6789	6817	28	R=4	16	3048.2													
6918.9		6919	6921	2	R=5	14	3074.4													
7326.3	7329.5		7329	0	R=6	12	3173.6													
7373.7		7373	7377	4	RA	14	3184.9													
	7448.5	7441	7448	7	RIR	24	3201.5	I-12												
	7497.0		7499	0	R=7	10	3213.3													
7546.0		7547	7550	3	R=2	20	3225.0													
	7808.3	7805	7812	7	R=5	16	3284.1													
7869.3		7870	7874	4	R=4	18	3297.7													
	8143.4	8139	8142	3	R=6	14	3355.7													
	8235.5	8213	8236	23	R=3	20	3375.6													
	8344.5	8338	8343	5	RIR	26	3397.9	I-13												
	8364.5	8348	8363	15	RA	16	3402.1													
8565.6	8571.6	8567	8573	6	alpha	10	3445.0													
8805.7		8807	8812	5	R=5	18	3492.6													
9030.8		9029	9034	5	R=6	16	3535.7													
	9252.4	9249	9254	5	RIR	28	3577.3	I-14												

Table 5-1 Partial classifications of Perrin and Lehr compared with bell recording

The classification schemes reconcile as follows:

- The number appended to Lehr's group classification is half the nodal meridians as reported by Perrin.
- Lehr's group I are Perrin's RIR partials (whose significance is explained below)
- Lehr's group numbers II, III, IV etc. are one greater than Perrin's R=x numbers (the number of nodal circles)
- Above the nominal, the prominent partials are Perrin's RIR / Lehr's group I.

The partials are ordered by their frequency in the Perrin bell. At higher frequencies, the Lehr ordering begins to diverge from this, which is brought about by differing bell designs: different partial groups advance at different rates up the frequency spectrum in different bells.

In Perrin's classification scheme, RIR partials (Lehr's group I) are those which have a radial antinode at or near the rim of the bell. These are maximally stimulated by the impact of the clapper - it is clear why these partials tend to be the most obvious in the spectrum. In the plot for the Perrin bell, RIR partials up to I-9 are visible; in the spectrum of the Ipswich bell in Figure 3-1, RIR partials up to I-13 are visible. These RIR partials are the partials which are responsible for formation of the primary and secondary strike.

Perrin's RA partials arise from vibrational modes in which the rim doesn’t flex, but rather vibrates along or twists about the axis of the bell, causing flexure of the walls of the bell. Several of these modes can be detected in the sound, e.g. the partials at 2537Hz, 3403Hz and 4366Hz. None of these were detected by Lehr.

The prime and quint are in a different Lehr group to the hum, tierce and nominal which are all group I / RIR partials. This means their nodal circles follow a different pattern; prime and quint can be controlled by the bellfounder and tuner independently of the group I partials. The vibrational modes giving rise to the hum, tierce and nominal only differ in the number of nodal meridians, and separate control of these three partials to produce a true-harmonic bell requires considerable skill and knowledge on the part of the bell designer.

Much more of interest can be found by further study of this reconciliation and comparison against Perrin's and Lehr's papers but will not be further covered here.

The excellent agreement between the results of the Wavanal analysis and Perrin et al's painstaking analysis of the same bell justifies the use of Wavanal for bell sound analysis.

5.2.1 Identification of partials in recordings

The key conclusion from this part of the work is that the RIR / group 1 partials:

- Are those most stimulated by the clapper blow
- Have relative frequencies determined in part by the exact shape of the rim / soundbow of the bell (because they have a radial antinode there)
- Are the partials which primarily determine the strike pitch of the bell through virtual pitch effects.

Lehr (1986) confirms this from practical experience, saying that as partials of his group I all have a node in the waist, they are expected to dominate the total bell sound because the clapper always strikes the sound bow.

The prominent nature of the group I partials (and their roughly equal spacing, as explained below) makes it practical to identify them uniquely in recordings of bells. This was used extensively in the analysis of 2,066 bells described below. It is not possible to uniquely identify the partials in other groups in this way.

5.3 Frequencies of partials

The investigation into strike pitch to this point has established the hypothesis that strike pitch, although roughly an octave below the nominal, is affected by the frequencies of other partials. To investigate the effect of all appreciable partials seems a daunting task. An experiment with factors at two levels (see chapter 7 for an explanation of this) for just the eleven rim partials plus prime and tierce would require $2^{13} = 1024$ tests at one nominal frequency. This test set would have to be rerun at several nominal frequencies throughout the audible range to establish the effects at different frequencies, giving an infeasible number (8,000 to 10,000) of tests for an individual test subject to perform.

In looking for ways to reduce the number of tests, correlation between upper partials was investigated with quite remarkable results. Figure 5-3 below shows the group I / RIR partials for four specially chosen bells (having very flat or very sharp upper partials), together with the average of 2,066 bells:

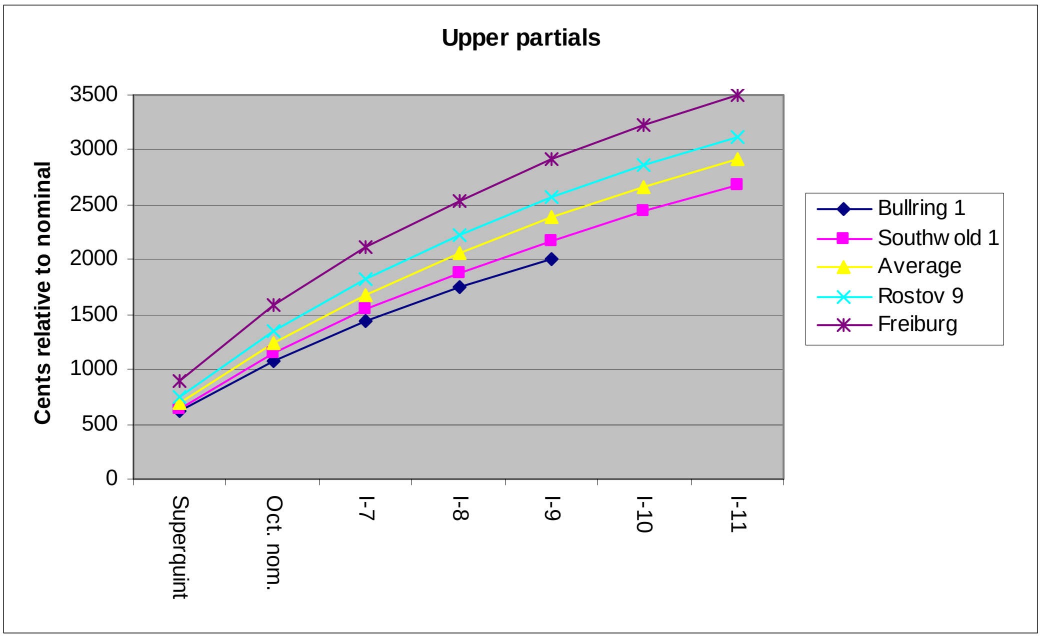


Figure 5-3 Upper partials of bells compared

As can be seen, there is a relationship between all the upper partials. A good way to visualise this relationship is to imagine the spectrum of the bell drawn on a sheet of rubber, which if stretched spreads the upper partials apart while maintaining the relationships between them. As explained in sections 3.5.1 and 3.6 above, the sharpness or flatness of the upper partials is due to the thickness of the bell, especially in the area of the soundbow - thin bells give sharp upper partials, thick bells give flat upper partials.

This remarkable correlation between the upper partials in bells has not to date been reported in the literature. On the contrary, it is thought by some professionals in the trade that independent tuning of upper partials is possible. The relationship displayed in the plot above suggests this is not practical.

To fully investigate the relationship, the tuning figures of 2,066 bells from the author's collection of bell recordings and from Lehr's and Perrin's papers were analysed. The partials from hum to octave nominal were analysed for all the bells. The upper partials from I-7 to I-11 were analysed for 888 of the 2,066 bells chosen as described below.

The analysis was done in two ways:

- A heuristic investigation looking for a practical way to represent the relationship between the partials
- A theoretical investigation based on a proposal by Perrin and Rossing to fit a modified version of Chladni's law to the tuning figures.

The first investigation was very successful; a single parameter to represent the tuning of all upper partials to a good degree of accuracy was identified, which enormously simplifies experiments in strike pitch. The second investigation was partially successful; the modified version of Chladni's law was found to predict average upper partial tuning well, but did not give a good way to represent the variations in upper partial tuning which it is hypothesised give rise to strike pitch shifts.

5.3.1 Relationship between partial frequencies

The 2,066 bell recordings analysed for this investigation have been accumulated over a number of years on an opportunistic basis. Recently, specific recordings have been acquired to explore the extremes of parameters identified in the analysis. The bells span the following ranges:

- 15 different countries
- Weights ranging from 19 kg to 38 tonnes
- Casting dates ranging from 1285 to today
- Nominal frequencies from 222Hz to 6,425Hz, a range of just less than 6 octaves
- 45 steel bells
- A few bells with major-third tierces

- 1,303 of the bells were not true-harmonic, i.e. 63% of the total
- The bells are the work of at least 100 different founders.
- The bell used by Perrin for the research described above, and the 13 bells used as examples in Lehr's two papers, some of which were untuned, are included.

Tonally, the bells ranged from very good to very poor. All bells for which the partials could be reliably measured were included, i.e. no bell once investigated was omitted from the analysis. The 888 bells whose higher partials were analysed were chosen to explore the extremes of weight, tonal quality and upper partial tuning.

Due to recording quality and microphone response (for example, some Russian bells were taken from a CD on public sale, and several of the bells were analysed from video recordings) not every partial could be measured successfully. Table 5-2 below gives the count of values for each partial investigated. The analysis took missing data into account.

Partial	**Hum**	**Prime**	**Tierce**	**Quint**	**Nom'l**	**S'quint**	**Oct. nom.**	**I-7**	**I-8**	**I-9**	**I-10**	**I-11**
Count	2056	2059	2059	1781	2066	2066	2066	888	855	804	720	558

Table 5-2 Count of partial frequencies investigated

The tuning figures for all the bells were captured in Hertz, and then converted into cents relative to the nominal, to bring the figures for all bells of whatever size into the same range. The cents calculation used the standard formula given in Equation 3-1.

Trials of other ways to normalise the figures, such as taking the simple ratio (partial frequency) / (nominal frequency) proved not to show the good correlations reported below. A plausible reason for the need for a logarithmic relationship arises from the analysis of the Chladni model below; for the time being it should be accepted as a necessary transformation to make the relationships linear.

The results of Pearson linear correlations using Excel function CORREL(range1, range2) between all the partial cents are given in Table 5-3 below. The nominal is not included, as it has already been eliminated by the conversion to cents. A correlation coefficient of +1 or -1 suggests perfect linearity between the two ranges, a coefficient close to zero suggests no relationship, or at least not a linear one.

	hum	**prime**	**tierce**	**quint**	**s’quint**	**o’nom.**	**I-7**	**I-8**	**I-9**	**I-10**	**I-11**
hum	1	-0.206	0.464	0.631	-0.184	-0.147	-0.054	-0.032	-0.043	-0.024	-0.043
prime	-0.206	1	0.391	-0.066	0.051	0.124	0.174	0.217	0.253	0.301	0.349
tierce	0.464	0.391	1	0.205	-0.277	-0.208	-0.078	-0.040	0.003	0.051	0.126
quint	0.631	-0.066	0.205	1	-0.152	-0.133	-0.090	-0.079	-0.103	-0.129	-0.199
superquint	-0.184	0.051	-0.277	-0.152	1	0.961	0.933	0.892	0.852	0.817	0.774
oct. nom.	-0.147	0.124	-0.208	-0.133	0.961	1	0.986	0.966	0.939	0.908	0.871
I-7	-0.054	0.174	-0.078	-0.090	0.933	0.986	1	0.991	0.976	0.955	0.926
I-8	-0.032	0.217	-0.040	-0.079	0.892	0.966	0.991	1	0.993	0.983	0.966
I-9	-0.043	0.253	0.003	-0.103	0.852	0.939	0.976	0.993	1	0.994	0.985
I-10	-0.024	0.301	0.051	-0.129	0.817	0.908	0.955	0.983	0.994	1	0.995
I-11	-0.043	0.349	0.126	-0.199	0.774	0.871	0.926	0.966	0.985	0.995	1
Sum^2	1.291	1.416	0.556	0.730	5.611	6.329	6.581	6.643	6.571	6.440	6.238

Table 5-3 Correlation between major partials in 2,066 bells

This correlation matrix is symmetric, the correlation of range A against range B is identically equal to the reverse. It is clear from the figures that:

- Partials from superquint upwards are highly positively correlated
- Hum, prime, tierce and quint are moderately correlated with one another but not strongly correlated with the other partials.

5.3.2 Correlation of upper partials

The high degree of correlation between all the upper partials suggests that a single quantity can be used to parameterise all the upper partials for a given bell - one of the partials being an obvious choice for this parameter. To see the 'total correlation' for a partial we need to sum the squares of the individual coefficients, which is done in the last line of the table above. It will be seen that partial I-8 is the most highly correlated of all the partials, closely followed by I-7, I-9, I-10 and the octave nominal. However, due to recording quality, frequency response of microphones etc. the higher partials are often difficult to identify, and the final choice of a partial to stand as parameter for all was the octave nominal, which in the author's experience can always be measured in a bell.

To understand more fully the relationship between the cents values for the upper partials, they were plotted as scatter diagrams against the octave nominal (chosen for the reason explained above). Values for the steel bells (which I expected to be anomalous because of the very unpleasant tone of some steel bells) are picked out in a different colour. Figure 5-4 is the scatter diagram for the superquint:

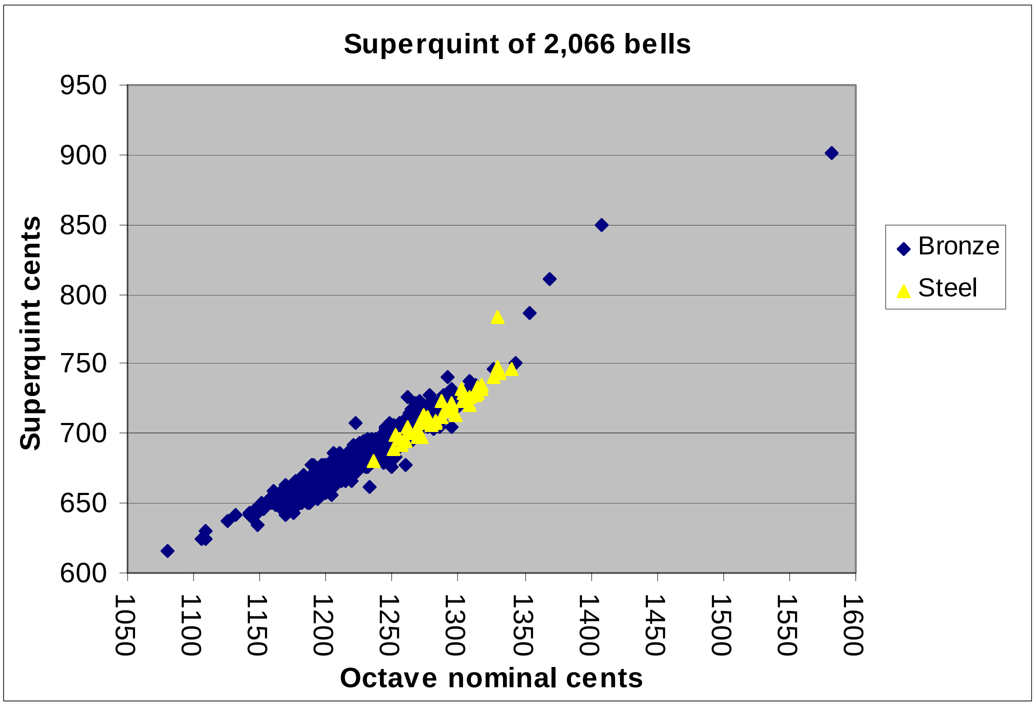


Figure 5-4 Scatter plot of superquint vs. octave nominal tuning for 2,066 bells

There are five significant outliers. From the top right they are:

- The bell from Freiburg described in section 4.2.4, with a very archaic profile and unique acoustics
- Bell no. 2 from Little Somerford in Wiltshire, cast by John Tosier in 1725. This is the thinnest UK bell in my collection of recordings
- A bell of normal Western shape which was experimentally tuned to be thin
- A bell cast by Van den Ghein in 1615 whose tuning figures are given in (Lehr 1986). I do not so far have a recording of this bell
- Bell no. 2 from Staveley in Yorkshire, a steel bell cast by Naylor Vickers in 1864 – again, very light in weight for its nominal.

The Somerford, Staveley and experimental bells are all tonally very poor. They sound quite unusual because the virtual pitch formed by the upper partials is significantly sharper than the nominal. All five bells are included in the numerical analysis in the rest of this chapter, but the first three are omitted from the charts below to allow a better focus on more typical bells. Scatter plots for all the upper partials on this basis follow (Figures 5-5 to 5-10). The purple line superimposed on each scatter plot is a linear regression.

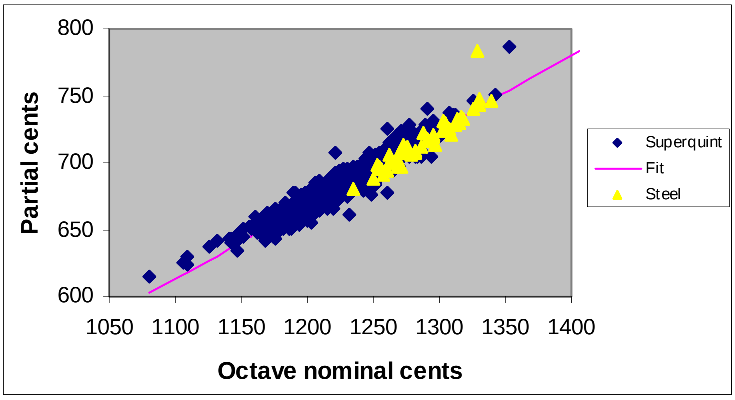


Figure 5-5 Scatter plot of superquint against octave nominal

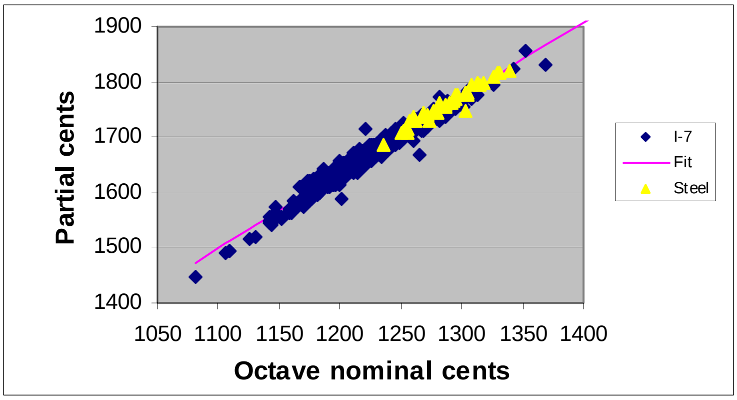


Figure 5-6 Scatter plot of partial I7 against octave nominal

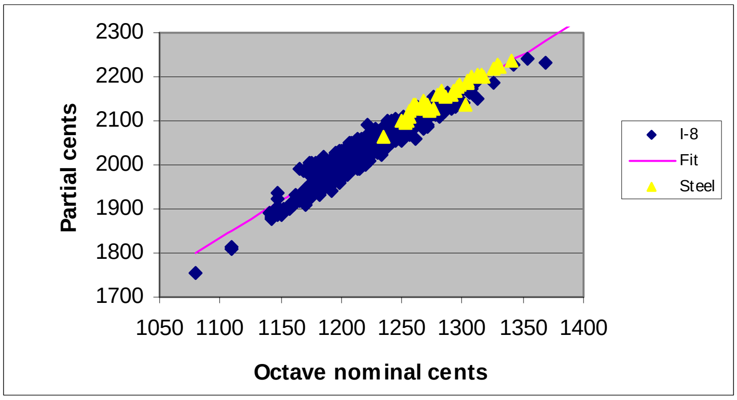


Figure 5-7 Scatter plot of partial I8 against octave nominal

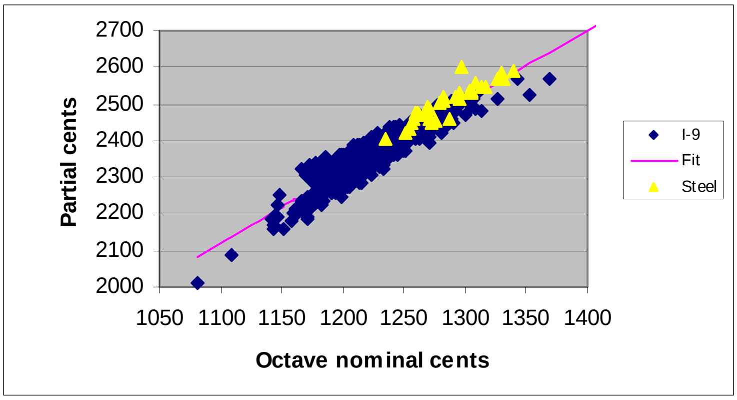


Figure 5-8 Scatter plot of partial I9 against octave nominal

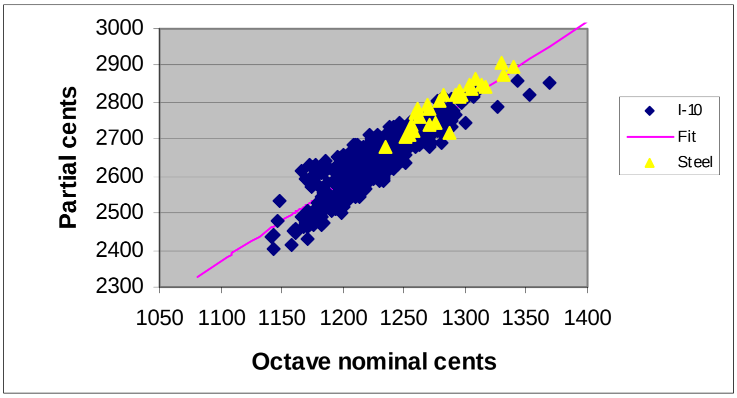


Figure 5-9 Scatter plot of partial I10 against octave nominal

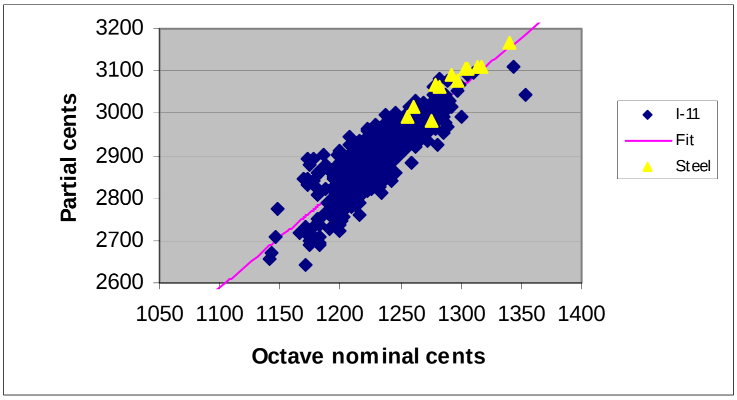


Figure 5-10 Scatter plot of partial I11 against octave nominal

The relationships are clearly linear, as the high value of the correlation coefficients would suggest, though the scatter in the results increases in the higher partials.

A linear regression of each of the partial cents against the octave nominal cents was performed using as a model $c_p = c_{on} \cdot m + c$ where c_p is the partial cents, c_{on} is the octave nominal cents, and m and c are the fitted parameters. These parameters allow the cents of any of the upper partials to be calculated if the octave nominal cents is known. Results of the regression fit appear in Table 5-4, together with a 95% confidence range for m and c:

Partial	C	m	Residual std. dev.
Superquint	-0.175 ± 0.720	0.557 ± 0.001	4.86
Octave Nominal	0.0	1.0	0
I-7	0.186 ± 1.389	1.363 ± 0.001	8.87
I-8	0.106 ± 2.740	1.668 ± 0.002	17.28
I-9	-0.203 ± 4.607	1.929 ± 0.004	26.28
I-10	-1.307 ± 6.508	2.156 ± 0.005	34.76
I-11	-9.003 ± 12.199	2.362 ± 0.010	42.34

Table 5-4 Regression coefficients for upper partials

The significance of the standard deviation of the residuals will become clear in the discussion below about the modified Chladni model. Figure 5-11 shows a plot of residuals (i.e. the difference between fitted and measured cents values for the upper partials):

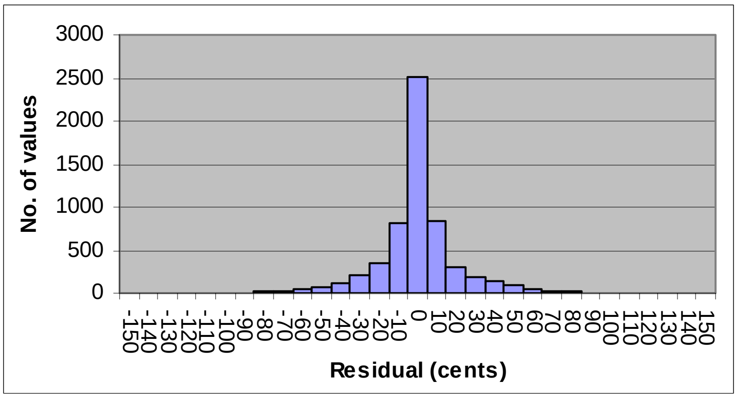


Figure 5-11 Residuals of upper partials regression fit

It will be seen that there are significant residuals, arising from the greater scatter in the higher partials as shown in the scatter plots. The residuals do not follow a normal distribution, as the normal probability plot in Figure 5-12 shows.

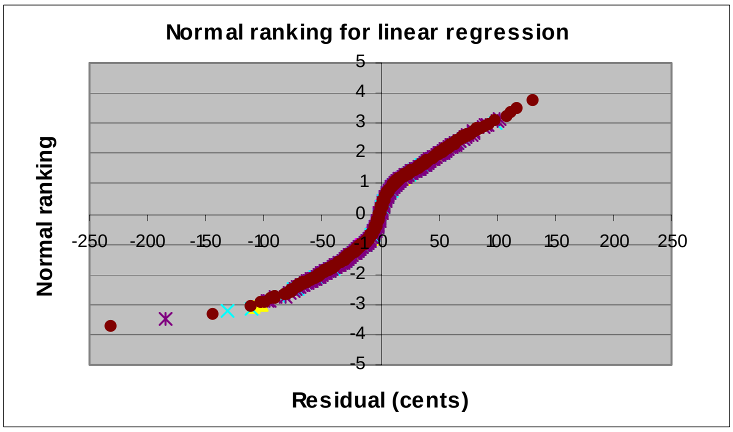


Figure 5-12 Normal probability plot of regression residuals

The S-shape of the normality curve shows that the tails of the distribution are much more extended than to be expected from a normal distribution. All the partials follow the same distribution. In fact, due to the way that cents are calculated, the figures involve the ratio of two quantities with measurement errors; the partial of interest and the nominal. This means that it is to be expected that the values follow a Lorentz or Cauchy distribution. To illustrate this, Figure 5-13 below shows the distribution of the residuals, as before, but with two curves superimposed:

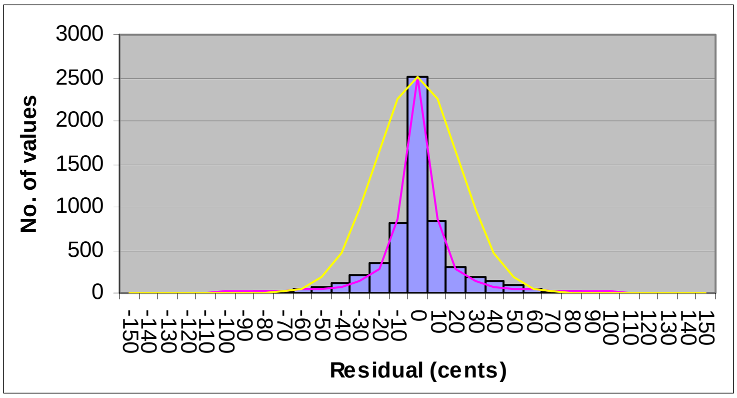


Figure 5-13 Upper partials residuals with Normal and Lorentz distributions

The yellow curve is a normal distribution, with standard deviation calculated from the residuals. The red curve is a Lorentz distribution, with the width at half-height fitted to the residuals. If n_0 is the number of residuals in the central bar, x is the deviation from zero (i.e. the horizontal axis in the plot) and w is the width of the distribution at half height, the formula for the Lorentz distribution plotted above is $\frac{n_0}{1+{x^2}/{w^2}}$. The value of w was obtained by using the Excel Solve facility to minimise the difference between calculated and measured values for the distribution. A half-width of 7.22 gave the best fit to the results.

The fact that the residuals fit an approximate Lorentz distribution is of more than academic interest; the statistics of this distribution are unusual in that quantities such as the mean and standard distribution cannot be calculated algebraically. Therefore, any statistical tests on these residuals must be treated with great caution.

Returning to Figure 5-12, the four most extreme outliers on the left, and the outlier on the right are as follows:

- Partials I-10 and I-11 of the Freiburg bell

- Partial I-11 of the Van den Ghein bell from (Lehr 1986)
- Partial I-11 of the 6th bell at Castleton, Derbyshire (a bell with a very squat profile).

The significance of this analysis is considerable; as explained earlier, it shows a remarkable correlation in the frequencies of upper partials of bells. This result is important for two reasons:

- In practice, once the relationship of any one of the upper partials to the nominal is set by the bell design and tuning, the frequencies of the other upper partials are determined
- This enormously simplifies the experiments to be done as part of this research. When testing the effect of the upper partials on strike pitch, a single parameter (the relative frequency of octave nominal to nominal) can be varied, and the frequencies of the remaining upper partials calculated using the regression results.

5.3.3 Correlation of hum and tierce

The correlation matrix for all partials suggests that cents for hum and tierce are relatively uncorrelated with those for the higher partials.

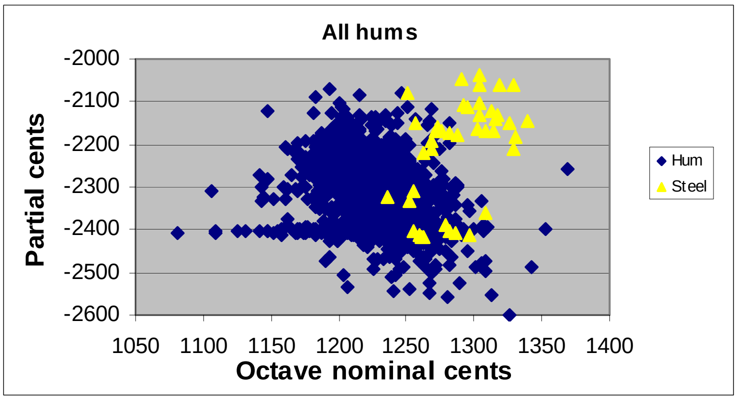


Figure 5-14 Scatter plot of hum against octave nominal

For the hums, plotted in Figure 5-14, two things are apparent: the hum tuning of steel bells is anomalous - sharp compared with their bronze equivalents. Steel bells are noted for their poor tone and this is one of the reasons. Secondly, there is a horizontal line of values at -2400 cents. This hum frequency, two octaves below the nominal, is characteristic of true-harmonic tuning. If the true-harmonic bells are removed, the scatter plot including the steel bells is as follows in Figure 5-15:

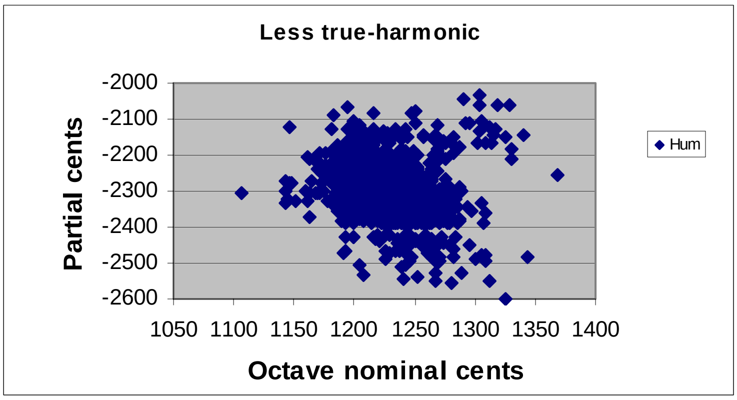


Figure 5-15 Scatter plot of non-true harmonic hums against octave nominal

There is very little if any correlation (the correlation coefficient for the 1300 bells in Figure 5-15 is -0.160). What correlation there is slopes the other way than for the higher partials because the hum is the other side of the nominal; as the upper partials go up relative to the nominal, the hum goes down. The significance of this observation is that special changes to the 'natural' shape of the bell have to be made to achieve a true-harmonic hum for all values of the upper partials.

A scatter plot of tierces follows as Figure 5-16:

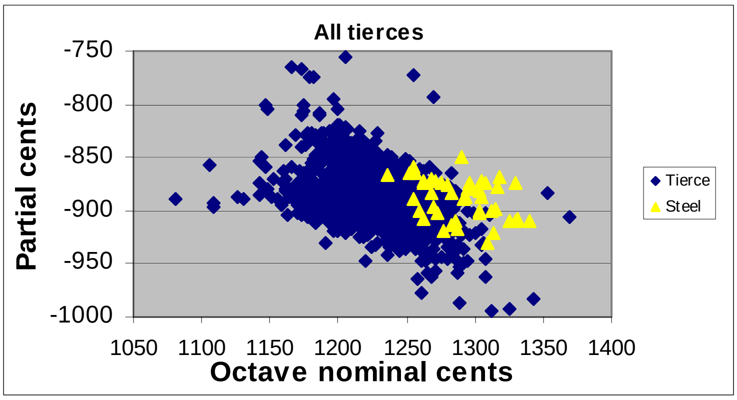


Figure 5-16 Scatter plot of tierce against octave nominal

The steel bells have tierces typical of those of bronze bells. There is a cluster of values for all bells in the range -880 to -900 cents which again represents true harmonic tuning. There appears to be a general negative correlation in the above plot but in fact the co-efficient for the 2,059 bells in the plot is -0.208 which is quite weak. The correlation with the hum is much stronger at 0.464. In practice, bellfounders treat the tierce as determined by the shape of the bell (especially in the area of the soundbow) and not particularly susceptible to tuning.

The lack of correlation between hum, tierce and octave nominal means that the effect of these partials needs to be independently examined in the virtual pitch tests.

5.3.4 Frequencies of prime and quint partials

The prime and quint belong to a different partial group than those considered so far; they are the lowest two partials in Lehr's group II. Table 5-5 below is an extract from the correlation matrix in Table 5-3 for these two partials and the octave nominal for the 2,059 bells with a frequency for the prime and the 1,781 bells with a quint:

	prime	quint	oct. nom.
prime	1	-0.066	0.124
quint	-0.066	1	-0.133
oct. nom.	0.124	-0.133	1

Table 5-5 Correlation of prime and quint

There is clearly little correlation between any of these partials. Figure 5-17 is the scatter plot of all Primes against the Octave Nominal:

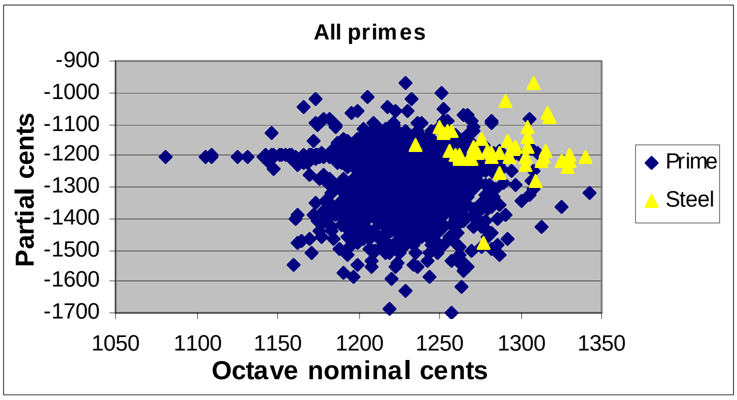


Figure 5-17 Scatter plot of prime against octave nominal

The horizontal line for the prime at -1200 cents (an octave below the nominal) shows the values for bells tuned according to true-harmonic principles. As with the tierce, the shape / design of bells was changed to bring the prime to an exact octave below the nominal. Changes to the internal shape of the bell inside the shoulder particularly affect the prime.

The plot for the quints (Figure 5-18 below) shows a similar but less prominent line at -500 cents, a fifth above the strike pitch, which is the target value for the quint in modern bells.

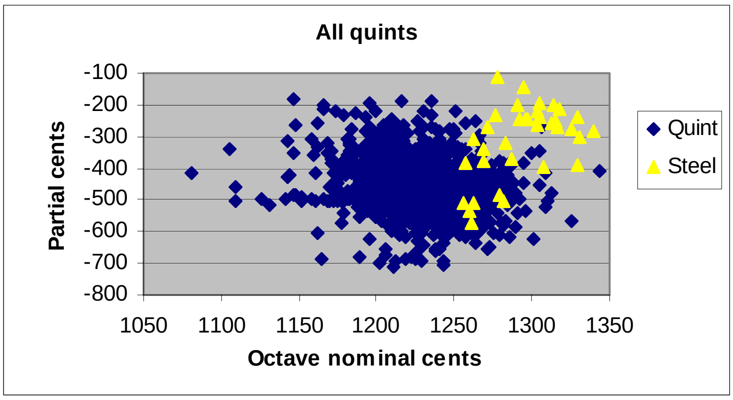


Figure 5-18 Scatter plot of quint against octave nominal

The lack of correlation between primes, quints and the group I partials means that the effect of these partials on strike pitch also needs to be independently investigated in the experiments.

5.3.5 The Berlin Freedom bell

This bell, weighing 9.66 tonnes, was cast and tuned by Gillett and Johnston in August 1950 and hung at the Rathaus Schöneberg in Berlin following a tour of the United States. Figure 5-19 is a picture of the bell following its rehanging in 2001:

Figure 5-19 The Berlin Freedom bell

The tuning of the bell was experimental and provides a good example of the impossibility of independent tuning of upper partials. The following quote from (Offen 2007) gives the background: *I well remember Wally Spragett telling me of his adventures tuning this bell. It was one of G & J's early ventures into serious upper partial tuning ... I seem to remember Wally saying that, because of the experiment, they made a bish of the tuning ..., but as the bell was never to be rung with any other, they let it go - would have been a rather expensive mistake had the bell needed recasting like the Wanamaker Bell a few decades earlier!*

The tuning figures for this bell were provided to me by Alan Buswell from the Gillett and Johnston tuning books. A subset of the figures is shown in Table 5-6 below. The ‘As cast’, ‘Ideal’ and ‘As tuned’ frequencies are taken from the tuning books; the ‘Measured’ figures are those obtained from recent recordings. The measured figures are an aggregate of four recordings, all taken by video camera from the ground. The figures have been adjusted to compensate for temperature and recorder speed so as to minimise the squared differences from the Gillett and Johnston figures.

Partial	**As cast**		**Ideal**		**As tuned**		**Measured**	
	Freq.	**Cents**	**Freq.**	**Cents**	**Freq.**	**Cents**	**Freq.**	**Cents**
Hum	93.1	-2350	81.5	-2400	82.1	-2436	81.3	-2453
Prime	190.3	-1112	163	-1200	165.0	-1227	163.9	-1241
Tierce	219.0	-869	195.5	-885	199.5	-899	198.9	-906
Quint	287.4	-399	244.5	-498	248.9	-516	248.7	-519
Nominal	361.8	0	326	0	335.2	0	335.5	0
S'quint	536.5	682	489	702	504.5	708	504.6	706
Oct.nom.	738.0	1234	652	1200	699.8	1274	699.1	1271
I7	959.0	1688	869	1697	917.2	1743	914.5	1736
I8	1186	2056			1142.3	2123	1144.1	2124
I9	1433	2383	1304	2400	1388.5	2461	1389.9	2460
I10	1688	2667			1640.5	2749	1641.6	2749
I11	1948	2915			1904	3007	1897.0	2999
I12	2195	3121			2153	3220	2156.2	3221
I13					2413	3417	2418.8	3420
I14	2721	3493			2677	3597	2681.3	3598
I15	2993	3658					3008.1	3797
I16	3268	3810			3224	3919		
I17	3534	3946			3493	4058		
I18	3799	4071			3757	4184		

Table 5-6 Partials of Berlin Freedom bell

The Gillett and Johnston tuning books did not name all the upper partials. Identification of them was done by the author by comparison against the recordings. Due to the large size and low partial frequencies of this bell, it was possible to detect many more upper partials than would normally be possible.

The cents for the ‘Ideal’ figures show that the experiment was an attempt to tune the superquint, octave nominal, I7 and I9 partials to a perfect fifth, octave, octave + perfect fourth and double octave above the nominal. The analysis in section 5.3.2 above shows that this tuning is not possible in a bell of normal shape. In the process of trying to achieve this tuning, over 1.4 tonnes of metal was cut out of the bell; its as-cast weight from the tuning books was 11.075 tonnes. Unfortunately, as can be seen from the ‘As tuned’ figures, not only were the desired intervals not achieved in the upper partials, but the hum and prime got out of control and both ended up rather flat.

Figure 5-20 shows the relative tuning of the upper partials before and after the tuning work. The ‘measured’ figures for the top three partials are the ‘as tuned’ figures from the tuning books, as these partials could not be detected in any of the recordings. Comparison with Figure 5-3 shows that, although the spacing of the upper partials increased because of the thinning of the bell, the overall relationship between them was unchanged by the tuning work.

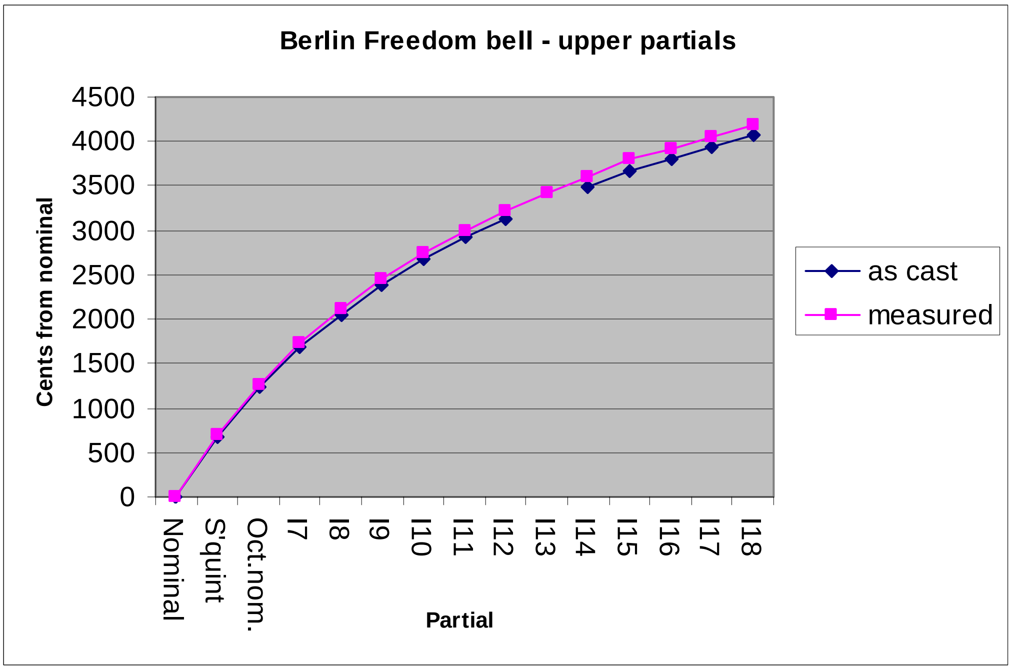


Figure 5-20 Upper partials of Berlin Freedom bell

As a consequence of the thinning of the bell and the spacing out of the upper partials during the extensive tuning, the secondary strike of this bell determined by partial I-7 is audibly higher than a perfect fourth about the primary strike, which has an unfortunate effect on the bell's tone.

The analysis of this bell shows two things:

- The impossibility of independent tuning of the upper partials despite the extensive tuning experiments
- That removing metal from a bell increases the spacing of the upper partials, in confirmation of the analysis in section 3.6.

5.4 Chladni's law and the frequencies of group I partials

In the end of the 18th century, Ernst Chladni carried out investigations into the modes of vibration of flat plates. He stimulated plates mounted at their centre into vibration using a violin bow on their free edges, and used sand scattered on the surface of the plates to show nodal lines and circles for various modes of vibration – the so-called Chladni patterns. His results were first published in 1787. In circular plates, with nodal lines forming either circles about the centre of the plate or diameters through it, Chladni discovered that the frequency of vibration increased by roughly the same amount for every one additional circular node as for two additional diametric nodes, so that if m is the number of diametric nodes and n the number of circular nodes, the frequency of vibration is related to $m + 2n$. Rayleigh (1894) showed that at higher frequencies the frequency $f_{m,n}$ was given by $f_{m,n} = C(m + 2n)^2$ where C is a constant. He called this result Chladni's law.

In two papers (Perrin, Charnley, Banu & Rossing 1985) and (Perrin & Charnley 1987b), Robert Perrin and his collaborators attempted to apply a modified version of Chladni's law to the partials of the test bell reported in (Perrin, Charnley & DePont 1983). Chladni and Rayleigh did their work on flat plates and it is a considerable generalisation to extend it to a vibrating body with a shape as complex as a bell. However, reasonable results were achieved with the following relationship:

$$f_{m,n} = C(m + bn)^p$$

where $f_{m,n}$ is the frequency of a partial with $2m$ nodal meridians and n nodal circles, and C, b and p are constants. The challenge is to find if this relationship applies to the bells analysed in this study, and whether single values for the constants are possible for all the bells – Perrin speculated that different values might be required. In the course of this investigation, it becomes clear why looking at partial frequencies in cents makes the relationships in the previous section linear.

For the RIR / group I partials, $n = 1$ and the above relationship simplifies to $f_m = C(m + b)^p$ for these partials. The remainder of this section assumes group I partials only.

There is an immediate problem with this relationship, because if the three constants are fixed for all bells, the only parameter that can affect partial frequency is the number of nodal meridians, whereas we know from the previous section that the group I partials can shrink and stretch according to the bell design. So, we should not expect to find single values for the three constants for all bells.

If for simplicity we define $K = {}^{1200}\!/_{\log_e(2)} \cong 1731.234$ in line with Equation 3-1, then if f_n is the nominal frequency, the cents c_m for the partial with $2m$ nodal meridians and frequency f_m is

$$c_m = K \cdot \log_e\left(\frac{f_m}{f_n}\right) = K \cdot \log_e(f_m) - K \cdot \log_e(f_n)$$

and from the proposed Chladni relationship

$$\log_e(f_m) = \log_e(C) + p \cdot \log_e(m + b)$$

so that

$$c_m = K \cdot p \cdot \log_e(m+b) + K \cdot \log_e(C) - K \cdot \log_e(f_n)$$

It is now clear, if the Chladni model holds, why using cents from the nominal as a measure of upper partial frequency makes the relationship linear. A plot of c_m against $\log_e(m+b)$ should be a straight line for an appropriate value of b, with slope $K \cdot p$ and intercept $K \cdot \log_e(C) - K \cdot \log_e(f_n) = K \cdot \log_e\left(\frac{C}{f_n}\right)$. Unfortunately the relationship is not linear in b, and it is necessary to use the Excel Solve facility on b to iterate to a value giving the best straight-line fit. The value of c_m was plotted against $\log_e(m+b)$ for all 2,066 bells. The plot follows as Figure 5-21:

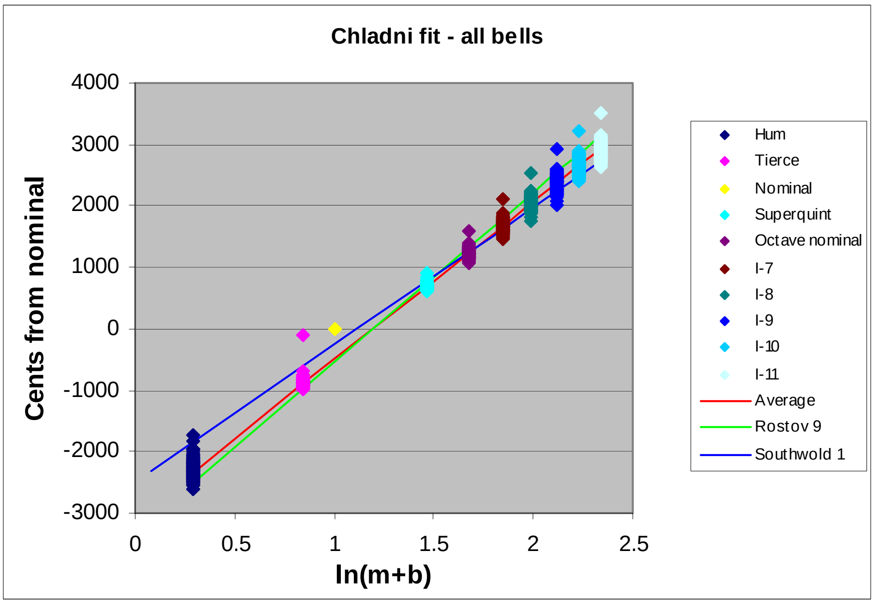


Figure 5-21 Plot of modified Chladni model for 2,066 bells

As can be seen, straight lines result. The spread of values for each partial arises because of the variation of partials in individual bells. The high outliers are the Freiburg bell. The red line shows the average of all bells in the study, and the green and blue lines two bells with extreme upper partials - Rostov 9 with very sharp, and Southwold 1 with very flat, upper partials. Three separate goal seeks and regressions were done to optimise the fit for the average of all bells, for Rostov 9 and for Southwold 1. The regression parameters are shown in Table 5-7, and the residuals for all three regression fits to show the quality of the fit in Table 5-8:

Quantity	**Average bell**	**Rostov 9**	**Southwold 1**
b	-0.668	-0.659	-0.913
p	1.483	1.597	1.297
$K \cdot \log_e\left(C/f_n\right)$	-3068	-3310	-2517

Table 5-7 Regression fit of modified Chladni model

Partial	**Average bell residuals**	**Rostov 9 residuals**	**Southwold 1 residuals**	**Fit to all bells individually residual std. dev.**
Hum	-0.44	12.79	-1.08	84.9
Tierce	10.62	-24.76	9.40	30.2
Nominal	-20.53	-25.20	-14.51	0.00
superquint	-6.61	1.52	-3.97	17.6
octave nominal	6.24	20.79	6.93	30.0
I-7	10.72	28.48	8.12	53.8
I-8	11.11	25.41	2.04	66.5
I-9	6.08	11.91	-6.48	76.6
I-10	-4.22	-11.01	2.15	83.0
I-11	-12.97	-39.93	-2.61	85.1

Table 5-8 Residuals from modified Chladni model

These results show that the proposed relationship based on Chladni's law is a good fit to an average of the bells investigated in this study, and to Southwold 1 (with no residual more than 20 cents), and a moderate fit to Rostov 9. However, different values of p, b and C for each bell are needed to achieve this fit. The fit to the nominal is poor in all cases, especially for the average bell. There is no one parameter to express the relationship of the upper partials with this model in the way that the octave nominal acts as a parameter in the linear model discussed above. Therefore the Chladni model is not a good one for the purposes of experiments in the effect of partials on strike pitch, or for categorising bells.

The standard deviation of the residuals when the Chladni model is applied individually to each of the 2,066 bells (using the values of p, b and C derived for the average bell) is rather more than for the linear model considered earlier, as the comparison in Table 5-9 shows:

Partial	**Chladni residual std. dev.**	**Linear residual std. dev.**
superquint	17.6	4.56
octave nominal	30.0	0
I-7	53.8	8.36
I-8	66.5	16.90
I-9	76.6	26.02
I-10	83.0	34.91
I-11	85.1	42.35

Table 5-9 Comparison of Chladni and linear models

Figure 5-22 is a histogram of the residuals against the model for each individual bell:

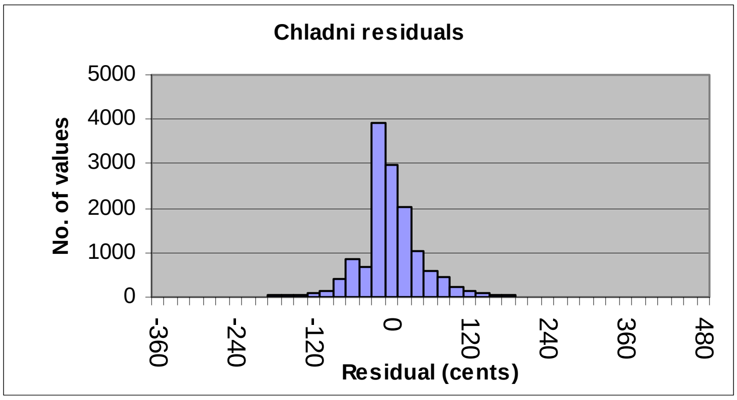


Figure 5-22 Residuals from modified Chladni model

The spread of the residuals is much more than that for the simple linear model.

Figure 5-23 gives a normal probability plot of the residuals:

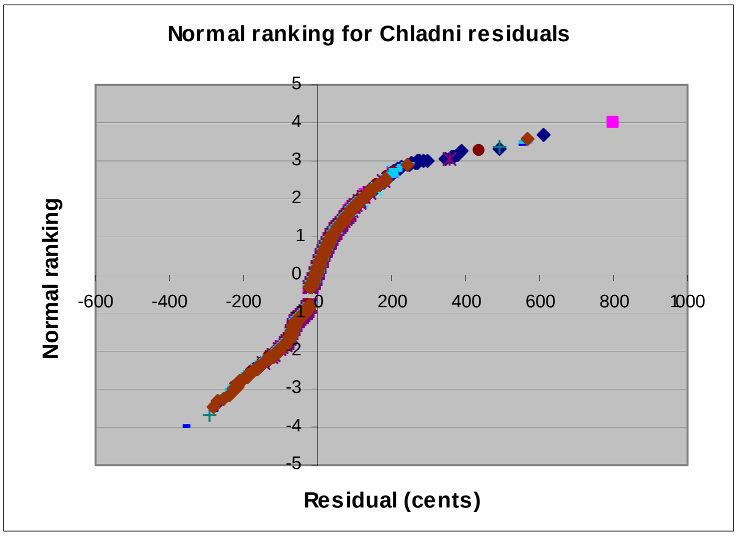


Figure 5-23 Normal probability plot of Chladni residuals

This confirms that the spread of residuals is between 2.5 and 4 times that of the linear model. The outliers are as follows:

- At top right, the hum, tierce, I-7, I-8, I-9, I-10 and I-11 partials from the Freiburg bell
- At the bottom left, partials I-8 and I-9 from the no. 1 bell at St Martin, Birmingham (bells with a very thick profile and very flat upper partials).

The conclusions from this work are as follows:

- The modified Chladni model proposed by Perrin and Rossing is a good model for the average of many bells

- There is no easy way to parameterise the model to represent the shrink and stretch of upper partials experienced in practice in individual bells
- The spread of residuals for the Chladni model is several times that for the simple linear model
- As a result, the Chladni model will be not used in the experiments on strike pitch shift.

5.5 Summary

The results of this investigation into the tuning figures of 2,066 bells can be summarised in a single plot (Figure 5-24 below) showing the 12 most significant partials, related to the nominal frequency and plotted against the octave nominal, with regression lines; the Freiburg bell has been omitted from the charts but is included in the regressions:

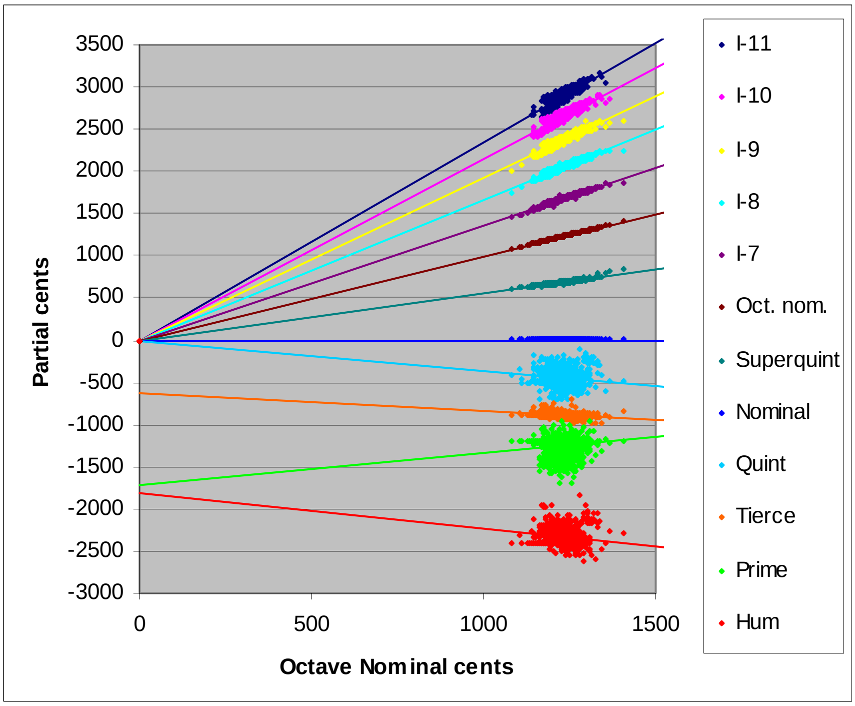


Figure 5-24 Overview of correlation of all partials

There is a remarkable and quite unexpected linear relationship between the tuning of all the group-I partials above the nominal, such that if the frequency of one of them relative to the nominal is known, the frequency of all the others can be predicted to a good degree of accuracy.

No such relationship exists for the partials below the nominal; frequencies of these lower partials can be chosen independently of that of the upper partials, within broad limits, by changes to the details of the bell’s shape.

6 PRACTICAL ISSUES IN PITCH MEASUREMENT

6.1 Introduction

This chapter describes a number of different candidate designs for the virtual pitch tests reported on in this thesis, and gives justification for the choice of approach used. The extensive trials used to validate the approach and choose the test parameters are described. Literature references to a number of studies in pitch determination are given which further justify the choice of experimental method. Issues concerning equipment used to conduct the tests and musical skills of the test subjects are also covered.

The textbook definition of pitch, e.g. from (Moore 2004) *"that attribute of auditory sensation in terms of which sounds may be ordered on a musical scale"* mandates comparison by listeners rather than the absolute measurement of a physical quantity. It will be seen from the experiment results that the effects for different listeners vary - there is no absolute value for the pitch of a sound independent of a particular listener's ability to judge it. Four different comparison methods were considered for this research:

- Comparison of bell sounds against sine tones, either from tuning forks or electronically generated
- The method of post-vocalisation described in (Terhardt 1984)
- Simulation of bell-ringing, i.e. a number of bells rung together in sequence with the tuning of one of them varied
- Comparison of bell sounds against other complex tones with similar timbre.

The following sections describe in more detail the pros and cons of each of the four comparison methods.

6.2 Methods of pitch measurement

6.2.1 Comparison against sine tones

This is the classic method of pitch estimation – references to the literature are given in section 2.6.2. It has the advantage of simplicity of execution, especially using electronically generated sine tones. The test procedure involves adjusting the frequency of a reference sine tone until the user judges that it best matches the pitch of a complex tone. The reference tone and the complex tone may either be played together or in sequence. The frequency of the sine tone after adjustment gives a direct measure of the pitch being estimated.

This method however has three disadvantages:

- The difference in timbre between test and reference tones can be quite confusing
- The pure timbre (i.e. single frequency) of the reference tone encourages spectral rather than virtual listening; the test subject can match the reference tone to an individual partial in the complex tone rather than the perceived pitch
- Variation of perceived pitch of a sine tone with its amplitude introduces uncertainty: should the sine tone have the same amplitude as the complex tone or of one of its individual components?.

Some of the experiments reported on in (Terhardt & Seewann 1984) use this method and the confusion it poses for the test subject are clear from the results obtained.

6.2.2 Post-vocalisation

In this method of pitch estimation, the test subject hears the complex tone whose pitch is to be ascertained and, after the complex tone has ended, hums or sings the pitch they have heard. The pitch of the note hummed or sung is then measured in a second stage of the test. In the work reported on in (Terhardt & Seewann 1984), the test subject's vocalisation was recorded and its pitch estimated afterwards.

This method of pitch estimation ensures that what is measured is a user's holistic perception of the sound, rather than a match against an individual partial. The method is very effective – the author routinely uses this method to establish the pitch of bells by listening to the bell sound, humming its pitch and adjusting a sine tone to match the pitch of the hum. This method was also successfully used for the investigations into bell pitch documented in chapter 4. However, the method has a number of disadvantages:

- The pitches that can be measured are limited to the vocal range of the test subject
- It is not practical to automate the tests due to the need to capture the recording of the test subject
- Due the complexity of the procedure, close supervision of the tests is needed.

6.2.3 Simulation of bell-ringing

This test procedure involves creating multiple examples of simulated change-ringing on a set of bells, with the tuning of one or more bells in each example changed. The test subject is asked in which example of change ringing the bells whose tuning has changed sounds most ‘in tune’. The simulated change-ringing is easy to generate in software.

This method has the advantage that timbral effects are eliminated; the sounds being compared are bell sounds with very similar timbres. Also, the results are directly applicable to the tuning of bells in peals; in particular the environment and duration of the bell sound being listened to is exactly that encountered in practice. This method was successfully used by the author during early investigations into bell pitch, in particular the work done on the bells at Southwold and Oxford reported on in section 3.5.

However, the method has some severe disadvantages:

- The procedure is very difficult to explain to anyone unfamiliar with change ringing
- Because of this, it is extremely unlikely that tests could be carried out without a long period of training for test subjects
- It is probable that test subjects without reasonable musical experience would fail to carry out the tests effectively.

6.2.4 Comparison of bell-like sounds

This test procedure was devised as a way of combining the best from the previous three methods. In this procedure, simulated bell-like sounds with particular characteristics are compared against reference bell-like sounds. In principle it would be possible to provide a reference sound whose characteristic frequencies could be changed under the control of the test subject until a best match for pitch is obtained. In practice, because of the time it would take to generate the reference sounds ‘on the fly’, the comparison takes place against a set of reference sounds generated beforehand, spaced over the expected range of pitches. The test subject selects the reference sound which sounds nearest in pitch to the test sound. The duration of the sounds and the partial frequencies and relative intensities are based on typical values found in bells.

The advantages of this test method are that:

- The sounds being compared are very similar in timbre
- The short duration and complex partial structure of the sounds forces virtual rather than spectral pitch comparison
- The test environment of the sounds is similar to that for bells being rung in changes
- All the test sounds can be generated before any testing starts, and the tests are easy to automate in software.

A potential disadvantage of the test technique is that only a limited range of pitches are available to the user as test results. To some extent this is a reasonable restriction; some sounds including those used in these tests have ambiguous or multi-valued pitches. In practice, in a peal of bells, pitches of other bells in the peal set expectations as to the pitch of an individual bell.

Extensive trials of the test technique with a number of test subjects, reported on below, have shown that the technique gives robust and repeatable results. These trials were also used to establish various parameters for the final tests conducted for this research.

Recent work on pitch shift effects in piano strings reported in (Järveläinen 2003), published after the above method was devised by the author, has used similar techniques to good effect.

6.2.5 Summary of comparison between test techniques

Table 6-1 summarizes the advantages and disadvantages of the various test techniques:

Test technique	Advantages	Disadvantages
Comparison against sine tones	• easy to automate • easy test procedure to explain	• differences in timbre can be confusing • test subject can match spectral frequencies rather than virtual pitch • amplitude of sine tone affects results
Post-vocalisation	• easy for subjects to carry out	• pitches limited to vocal range of test subject • complicated test procedure • test subjects require training and supervision • difficult to automate
Simulated bell-ringing	• sounds to be compared have similar timbre • test environment is realistic	• procedure very difficult to explain to non-ringers • significant training required for test subjects • test subjects without musical training will find the tests difficult
Comparison against bell-like reference sounds	• sounds to be compared have similar timbre • test environment is realistic • tests force virtual rather than spectral listening • test procedure easy to automate	• range of pitches is constrained by test setup (though this is a realistic restriction)

Table 6-1 Summary of pitch measurement techniques

Further insight into the choice of test technique is given in section 9.8 where the precision of the results from experiments in this thesis using the two chosen techniques (post-vocalisation and comparison against bell-like sounds) is compared.

6.3 Details of chosen test design

The statistical design of the tests and analysis of the test results is described in the next chapter. The test arrangements were designed around a four-factor analysis of variance to ensure that analysis of the test results would yield statistically valid results. A four-factor analysis allows the effect of four independent variables to be established. The previous chapter explains how newly established relationships between partials in bells allows bell sounds comprising many more than four partials to be used in the tests.

The practical arrangement devised after various trials involved the test subject matching each test sound against one of 16 references sounds. At a particular nominal frequency, blocks of 16 tests were used (exploring all combinations of two values of four chosen factors) and the reference sounds were kept the same for the block of 16 tests at that nominal frequency so that the test subject became familiar with them as the block of tests proceeded.

The reference sounds were spaced an equal number of cents apart. Listeners with musical ability can distinguish pitches as close as 5 cents (Moore 2004 p. 197/8), so for initial trials the reference tones were set 5 cents apart, giving a total pitch range of 15 x 5 = 75 cents. As trials on my own and other's ears proceeded it was discovered that a pitch variation greater than this was possible across all four factors at lower frequencies and the spacing was widened to encompass the full range of user response at these lower frequencies.

It was expected that the effect of the various partials would change at different frequencies, as already demonstrated in chapter 4, and this proved to be the case. Nominals of church bells encountered in practice range from about 250Hz (a bell of 24 tonnes in weight) to several kilohertz (bells weighing tens of kg only). However, above 2,000Hz pitch perception is generally spectral, not virtual. Therefore, a set of tests was constructed with nominals spaced 1/3 of an octave apart from 315Hz to 2000Hz, a range of 3 octaves comprising 9 blocks of tests in all, to allow the virtual pitch effect to be explored across the range of frequencies. Each block of tests at a given nominal frequency comprised a unique set of 16 test and 16 reference sounds.

To make the tests easy to administer, they were automated in software and designed to run across the internet or from a local web server. The only requirement on the user's PC or Macintosh is that it has internet access, a standard web browser and means of sound reproduction. For security reasons all the software runs on the server so that user activity cannot affect its operation. Test results are either displayed on the screen for printing, emailed, or saved on the server for later access as selected by a configuration parameter. For performance reasons all the sounds used in the tests are generated beforehand using software driven from test data files.

During the extensive trials of the test method (involving hundreds of test runs by the author and collaborators) the following issues were explored and catered for in the test method:

- Quality of the sound reproduction equipment in the test subject's PC
- Musical experience of the test subject
- Which partials affect virtual pitch perception
- Ranges of the various test parameters
- Documentation for the tests, in particular ensuring that the tests can be completed successfully by a subject relying on the description on the web pages.

The following sections document the results of these trials and give an overview of the software used to generate the sounds and run the tests.

6.4 Quality of sound reproduction equipment in the test PC

This issue is obviously important given that a range of different equipment is to be used, some of which is outside the test administrator's control. However, the typical situations in which bells are installed are not a hi-fi environment; the bells are metres or tens of metres from the listener; with a sound path involving walls, floors and multiple reflections from other buildings; and often listened to in an urban environment with a high level of background noise. Only effects which can survive these circumstances are of practical importance.

There are two factors in sound reproduction equipment which could affect the outcome of virtual pitch tests: accuracy of the internal clock or data rate in the reproduction equipment, and frequency response.

Accuracy of internal clocks was immediately eliminated as a concern. Experience of many recordings of bells taken on suspect equipment shows that although variations of a few percent in recorder speed or data rate are encountered, all partial frequencies within the sound are affected by an equal factor so that the interval relationship of the partials is not affected. The errors observed in PC sound-cards are less than 0.5% which represents an interval of less than 10 cents. Trials have shown that virtual pitch effects only vary slowly with centre frequency, so that an error in centre frequency of this magnitude is not material.

Frequency reproduction is a concern. The frequencies in the test files range from just less than 80Hz to just above 11,000Hz. To test the effect of equipment on results, tests were run by the author on a range of equipment including:

- High quality (Sennheiser HD555) headphones
- Consumer headphones (Goodmans Pro CD 3100)
- The internal speakers on an HP desktop PC
- The speakers on an HP laptop base unit (docking station)
- No-name (i.e. cheap) Walkman earpieces
- Cheap loudspeakers on a Dell desktop PC.

The full set of tests (9 sets of 16 tests) were run by the author on all six types of equipment, i.e. 864 tests in total. The results were analysed using the four-factor approach described in the next chapter. The nine tests employed are as listed in Table 6-2:

Test i/d	Nominal frequency of test sound
3t	314.98Hz
4t	396.85Hz
5t	500.00Hz
6t	629.96Hz
8t	793.70Hz
10t	1000.00Hz
13t	1259.92Hz
15t	1587.40Hz
20t	2000.00Hz

Table 6-2 List of virtual pitch tests

The statistical analysis described in chapter 7 calculates the residuals of the test results after application of a model based on the four factors independently. Figure 6-1 shows the standard deviation of the residuals of the sixteen tests on each of equipment type at each frequency:

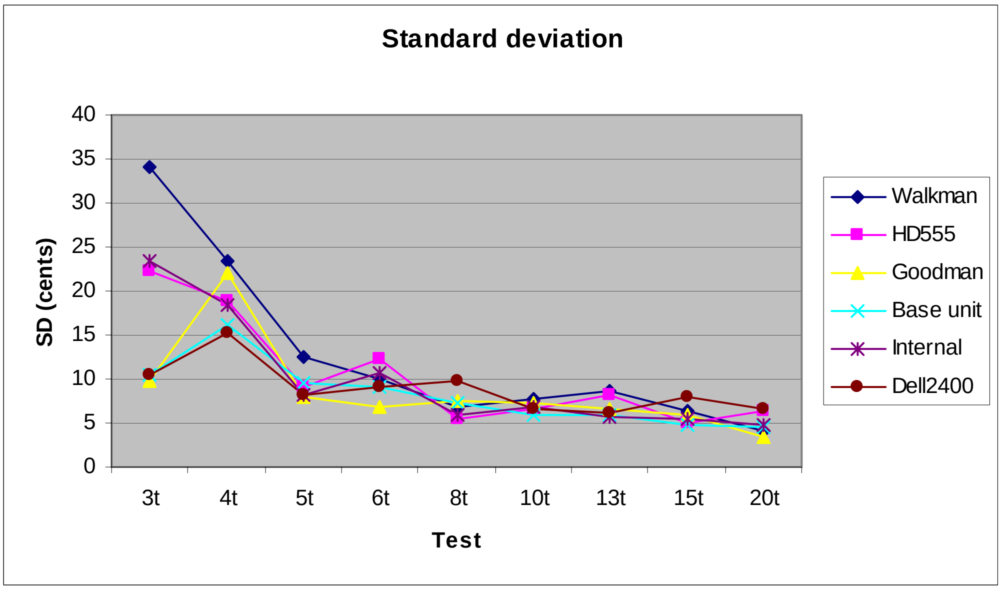


Figure 6-1 SD of virtual pitch results with different equipment

It can be seen that for the higher frequency tests, the residual standard deviation for all equipment types is comparable. For test 3t, this is not the case. This test, and test 4t to some extent, are difficult to perform because the test sounds display the secondary strike effect and there is ambiguity as to the pitch which is heard.

Interestingly the 'best' quality equipment (the HD555 headphones) does not give the lowest variance in the test results. This may be because the good low frequency reproduction of these headphones increases the ambiguity in the pitches heard. On the other hand, it is commonly suggested in hi-fi magazines that even the best headphones give inaccurate reproduction of sound compared with loudspeakers because of the artificially close proximity of the sound source to the ears, and because lower frequencies from loudspeakers enter the aural system through other parts of the body that the ear ducts. It is said by organ tuners that the pitch of low-pitched organ pipes is inaudible when too close to them.

To further explore the distribution of the residuals, a normal probability plot was constructed and is given as Figure 6-2.

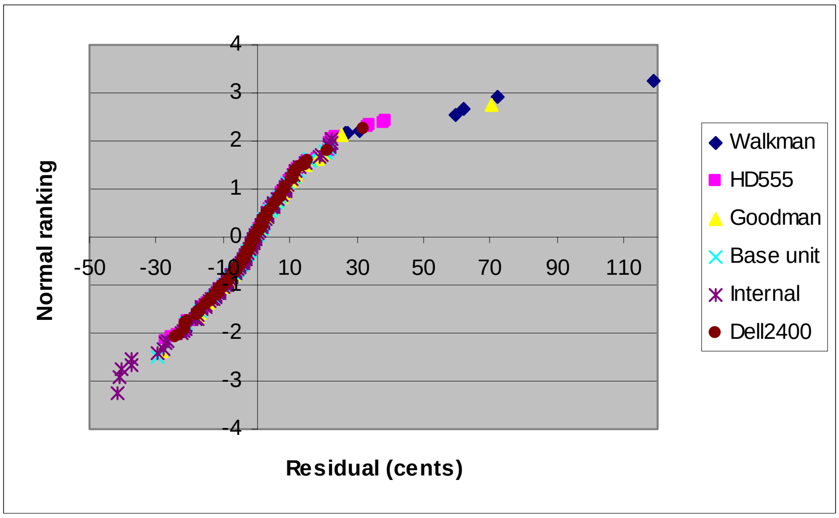


Figure 6-2 Normal probability plot of virtual pitch tests on different equipment

The residuals are roughly normally distributed (the centre part of the curve is roughly linear) but with some very significant outliers. All the outliers were investigated, and all were from tests 3t or 4t. All but one of the outliers came from the walkman earpiece or the PC internal speakers (both of which gave audibly poor quality sound).

Finally, in order to determine whether equipment quality would make a difference to test results, i.e. to means of values, the four-factor analysis was adapted to calculate the effect of upper partial changes (the dominant effect) for each test run and equipment type. The resulting effects are given in Figure 6-3:

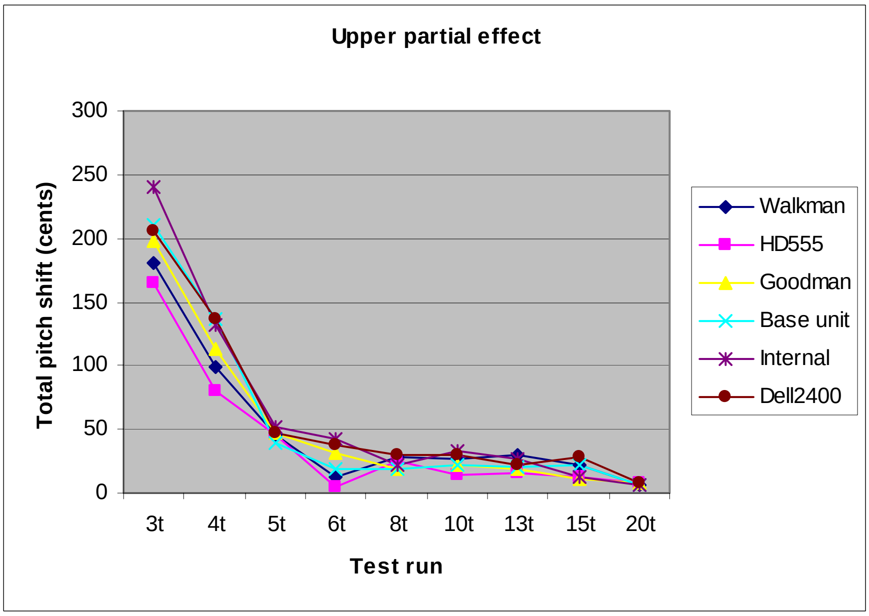


Figure 6-3 Virtual pitch shifts on different equipment

It will be seen that the results on all equipment are comparable, though the wide spread in results for test 6t is notable. Variation in the results due to experimenter error and fatigue is to be expected; each point in the plot above is derived from only 6 test results.

To get a measure of whether equipment difference is statistically significant, the quantity $\left(E_{t,e} - \overline{E}_t\right) / SD_t$ was calculated for each effect (i.e. each point on the plot above), where $E_{t,e}$ is the effect from an individual test run and equipment type, $\overline{E}_t$ is the mean effect for all equipment types for an individual test run, and SD_t is the standard deviation of all the residuals from an individual test run. Results are in Table 6-3.

Test id	Walkman	HD555	Goodman	Base unit	Internal	Dell2400
3t	-0.87	-1.57	-0.05	0.54	1.95	0.30
4t	-0.65	-1.58	0.05	1.21	0.97	1.21
5t	0.03	0.03	0.16	-0.75	0.54	0.16
6t	-0.85	-1.66	0.87	-0.31	1.95	1.50
8t	0.70	0.25	-0.43	-0.43	-0.09	0.92
10t	0.47	-1.26	-0.27	-0.27	1.33	0.72
13t	1.06	-0.96	-0.45	-0.20	0.56	0.05
15t	0.92	-0.61	-0.82	1.12	-0.61	2.14
20t	-0.19	0.29	0.29	-0.19	-0.19	0.17

Table 6-3 Scatter of virtual pitch shifts on different equipment

It will be seen that despite the outliers and mild lack of normality in the residuals identified above, all the effects lie within 2 standard deviations of the mean. Therefore, at a 95% confidence level there is no statistical significance between the results from the various equipment types.

From the analysis of these 864 tests on different equipment, I conclude that differences in equipment quality are not significant at any but the lowest frequencies, and possibly not at these. To allow for subsequent analysis using equipment quality as a factor if it proves necessary, the test subject is asked on each test run about the equipment used to run the tests.

6.5 Musical experience of the test subject

It is to be expected that the musical background (and in particular practice in judging pitches accurately) will influence the results. One potentially interesting outcome of the tests may be that subjects with differing musical background and experience may experience different virtual pitch effects. To investigate this, a trial of the tests was set up with seven subjects of differing musical experience. One subject failed to complete the tests due to lack of time. The other six subjects described their musical experience as follows:

Subject	Musical experience
WAH	The author. Experienced bellringer and choral singer and amateur pianist. Very familiar with the tests, having devised them and run many trials
DS	Experienced orchestral player
JW	Used to sing in a choir and play the trumpet at school, though it was 24 years ago
PS	Not musical
SB	Not musical
SL	Not musical

Each subject took the same test, a four-factor test varying hum intensity, and prime, tierce and upper partial frequencies, at a nominal frequency of about 990Hz. The test method was that of comparison against reference sounds. The audio equipment used was a pair of Goodman Pro CD 3100 headphones (i.e. mid-range consumer equipment) in a quiet room.

Figure 6-4 is a normal plot of residuals from all six test runs (96 tests in total) and shows that the residuals are normally distributed.

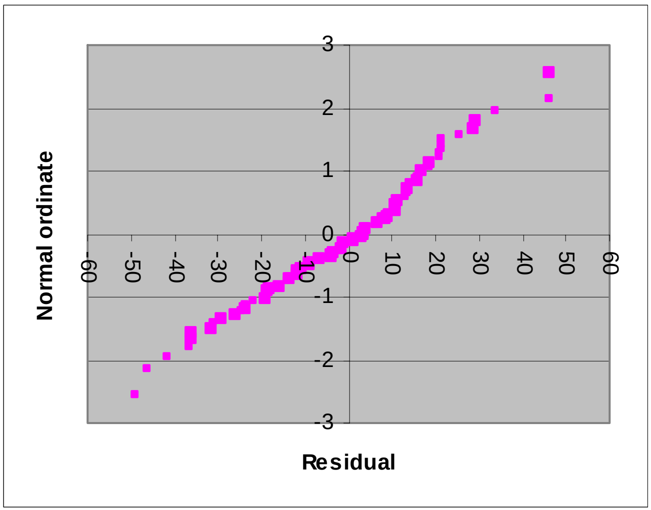


Figure 6-4 Normal probability plot of virtual pitch for different test subjects

Residuals for each individual test subject appear in Figure 6-5:

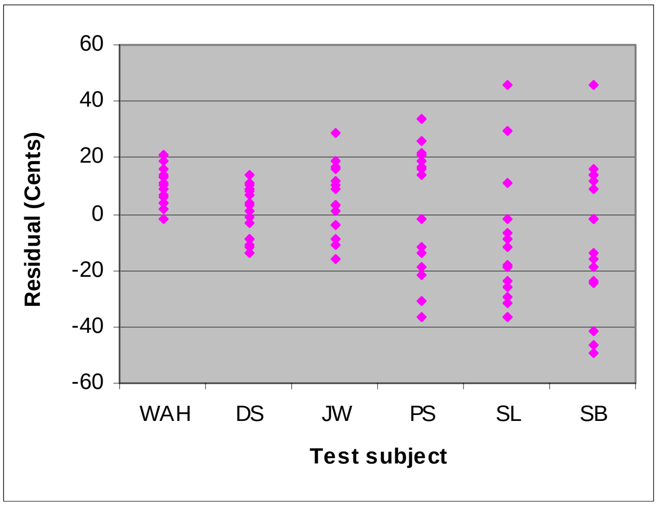


Figure 6-5 Residuals of virtual pitch for different test subjects

A clear pattern emerges, with the scatter in the residuals rather less for test subjects with more musical experience.

The number of test results for each individual subject (16 results) is not sufficient for the individual subject results to be statistically significant. However, the effect of each of the four factors on the test results for each subject, compared with the aggregate effect for all six subjects, is as follows (Table 6-4):

		Test subject					
Factor	**Aggregate**	**WAH**	**DS**	**JW**	**PS**	**SL**	**SB**
Hum intensity	-12.08	-1.25	-5.00	-2.50	-19.38	13.75	-45.63
Prime frequency	2.92	1.25	2.50	-3.83	-17.04	-5.08	20.62
Tierce frequency	4.79	8.75	-1.25	7.50	3.13	13.83	13.29
Upper partial spacing	27.50	25.00	40.00	46.25	39.38	1.17	-3.29

Table 6-4 Virtual pitch shifts for different test subjects

Again, there is a clear difference between the test subjects. It was recognised that it would be necessary to investigate the musical background of test subjects for the formal tests.

6.6 Which partials affect virtual pitch

To extend the tests to include more than four factors makes them lengthy and tedious to run (a full-five factor test would require 32 individual tests). Therefore, trials were carried out using various combinations of partials in order to focus on the most important factors. Most of the test trials were conducted by the author, though seven other people (including the five above) also performed some trial tests. The partials examined were chosen based on their prominence in the spectrum of bells. The partials considered were as follows:

- Hum frequency and amplitude
- Prime
- Tierce
- Quint
- The block of three partials between nominal and superquint
- Upper partial frequency and amplitude (as explained in the previous chapter, these partials do not need to be varied independently as this does not occur in practice).

Some validation of the trial results was carried out by comparing pitch shift predictions with actual peals of bells using the approach described in chapter 10. This was done to ensure that the results of the trials were plausible (i.e. that there was no procedural or analytical error). Results of the trials and comparisons are as follows.

Hum - varying the hum frequency was seen to have a much greater effect on virtual pitch than is experienced in real bells. The speculation is that a spectral pitch effect is affecting the results due to the artificiality of the test environment and because the hum frequency is much lower than any of the other partials. More investigation is required into this effect and the decision was taken to conduct all tests with true-harmonic hums to avoid the problem. Modern bells have true-harmonic hums in any case so the restriction does not reduce the usefulness of the test results. Preliminary experiments varying the hum amplitude with true-harmonic hums showed no significant effect of amplitude on pitch shift.

Upper partial spacing - varying upper partial spacing was found to have a dominant effect on virtual pitch. The values for pitch shift from the trials were found to correspond well with those observed in bells tuned with stretch tuning. Upper partial spacing was included as one of the four factors in the tests.

Upper partial amplitude – the experiment described in section 8.3 showed no significant effect on virtual pitch of changes in superquint and octave nominal amplitude and as a result the amplitudes of all partials were set to the values derived in section 3.3.

Prime and Tierce - varying these partials was found to have a minor effect on virtual pitch, but the direction of the effect varies for different test subjects. Given the prominence of these partials, they were included in the final test as two of the four factors.

Quint, and the three partials between nominal and superquint - trials did not show any particular effect on virtual pitch. Given the prominence of these partials in some bells, the decision was taken to use the last factor of the four available to test for their effect by varying the quint in some tests, and the other three partials in others.

In the trials, no significant effect was seen from any of the above factors acting in combination, though the analysis of variance done on the formal tests will identify the effect, if any, of factor combinations.

6.7 Ranges of the various test parameters

The values chosen for ranges of the test factors are based on the analysis of 2,066 bells documented in chapter 5, and the partial amplitudes investigated in chapter 3.

Table 6-5 gives the values chosen for the tests. The average amplitudes are in arbitrary units. The tuning figures are given, as always, in cents relative to the nominal. In the higher partials, tuning figures are only given for the octave nominal, as all the others can be calculated from the regression results.

Partial	Average amplitude	Low tuning	High tuning
hum	2.79	-2400	-2200
prime	5.09	-1400	-1200
tierce	8.35	-900	-800
quint	1.00	-500	-300
nominal	10.60		
superquint	8.79		
octave nominal	7.10	1140	1280
I-7	4.94		
I-8	4.03		
I-9	3.30		
I-10	2.05		
I-11	1.52		

Table 6-5 Parameters for partials for virtual pitch tests

Regression values based on $c_p = c_{on} \cdot m + c$ from which the tuning of the remaining upper partials can be calculated, taken from the results in chapter 5, are given in Table 6-6.

Partial	m	c
Superquint	0.557	-0.021
I-7	1.363	0.399
I-8	1.688	0.448
I-9	1.929	0.087
I-10	2.157	-1.995
I-11	2.364	-10.192

Table 6-6 Upper partial regression parameters

Results of the test trials with the various users were used to determine the spacing and centre-point of the reference sounds, so as to cover the range of results seen in the trials, with a little room for further variation at each end of the range.

The duration of the test and reference sounds was chosen as 250ms. This was done because:

- It is typical of the spacing between the sounds of adjacent bells in change-ringing, i.e. it is the time for which each bell is heard before the next strikes
- The short duration was expected to encourage virtual rather than spectral listening.

For some of the trials, the length of the sounds was increased to 1s to see if test results changed. No significant difference was seen with the longer sounds. This agrees with the tests reported on in (Terhardt & Seewann 1984), which showed that there was no significant difference in the pitch estimation ability of test subjects for test sounds 100ms and 3s long.

Values of the various parameters chosen for the nine final tests are as listed in Table 6-7.

Test ID	Test nominal (Hz)	Reference nominal spacing (cents)	Duration (s)	Low reference nominal (Hz)	Quint	Partials between nominal and s'quint
3	314.9803	20	0.25	292.1943	0	0
4	396.8503	12.5	0.25	378.6556	1	0
5	500	10	0.25	478.8016	0	1
6	629.9605	7.5	0.25	609.8213	1	0
8	793.7005	7.5	0.25	768.3266	0	1
10	1000	7.5	0.25	968.0309	1	0
13	1259.921	7.5	0.25	1219.643	0	1
15	1587.401	5.0	0.25	1553.386	1	0
20	2000	5.0	0.25	1957.144	0	1

Table 6-7 Parameters for virtual pitch tests

In this table, for the quint, and the partials between nominal and superquint, 0 indicates that a typical value is used, and 1 indicates that a high / low range is used as the fourth factor.

6.8 Documentation for the tests

The test web pages include instructions as to how to conduct the tests. These were changed as a result of user feedback during the trials to ensure that as far as possible subjects could conduct the tests without external help. The later trials proved that such unaided test execution was practical.

6.9 Software implementation of the tests

Software was written to automate the generation of test sounds, and to implement the actual tests. Outline details are given here sufficient to explain and justify the approach taken.

6.9.1 Test sound generation

From the test parameters given above, an Excel spreadsheet was produced which calculated all the test frequencies and other parameters for the 144 test sounds and the 144 reference sounds. The use of a spreadsheet in this way made it easy to make changes to test parameters and recalculate frequencies and parameters for all the tests. Full details of all parameter files for the virtual pitch test and reference sounds are given in Appendix 10.

These files of data were exported as comma-delimited .CSV files for the next stage of the process.

Software was written in C++ (using the class libraries developed by the author over a number of years for sound analysis) to generate sound files from the input data in the .CSV files. The sound files were generated by additive combination of cosine waves. As the partials in the sounds are inharmonic, there is no phase relationship between them and phase can be ignored when generating the sounds. Other experimenters have also found that phase does not affect virtual pitch perception. For an example see (Patterson and Wightman 1976). For speed, rather than use the inbuilt cosine function in C++, a recurrence relation due to Petr Vicherek (Cross 1998) was used to generate successive values of the cosine function. This recurrence relation relies on the following identity:

$$\cos(a + n \times b) = 2 \times \cos(a + (n-1) \times b) \times \cos(b) - \cos(a + (n-2) \times b)$$

where a is some initial phase angle, n is a progress parameter (e.g. time or step number), and b, a parameter related to frequency, is the phase angle change per unit step. The recurrence relation is very quick because the two cosine terms in n on the right-hand-side are the previous two values of the recurrence. Following a setup stage involving three calls to the inbuilt cosine function, each successive value is calculated using one floating point multiply and one floating point add. The author has extensive experience with the stability of this recurrence relation as a result of previous use, including a 24 hour test run which showed no change in frequency over the duration of the test.

Each partial was also given an individual amplitude envelope. The hum (a relatively long lasting partial) decayed to 60% of its initial value in 1s. The rest of the partials decayed to 60% of their initial values in 200mS. These values were chosen to represent partial envelope decays measured in a number of bell recordings. The discussion in (Plomp 1967a) and the experiment in section 8.3 of this thesis suggest that virtual pitch shifts are not significantly affected by partial amplitude, but the amplitude envelopes were included in these test sounds to make the experiments as realistic as possible.

The sounds are generated in floating point format and then normalised to a standard amplitude before being converted to 16-bit integer values. All sound files therefore have the same peak amplitude. As a compromise between size and quality, the sound files were saved as 22,050 samples per second, 16 bit, monophonic files in .wav format. In total, 16 test files and 16 reference files were produced for each of the 9 tests. The software generating the sound files for each test also produced a data file listing the test and reference filenames for the test.

Figure 6-6 is an example of the spectrum of one of the test sounds: compare this with the real bell spectrum in figure 3-1.

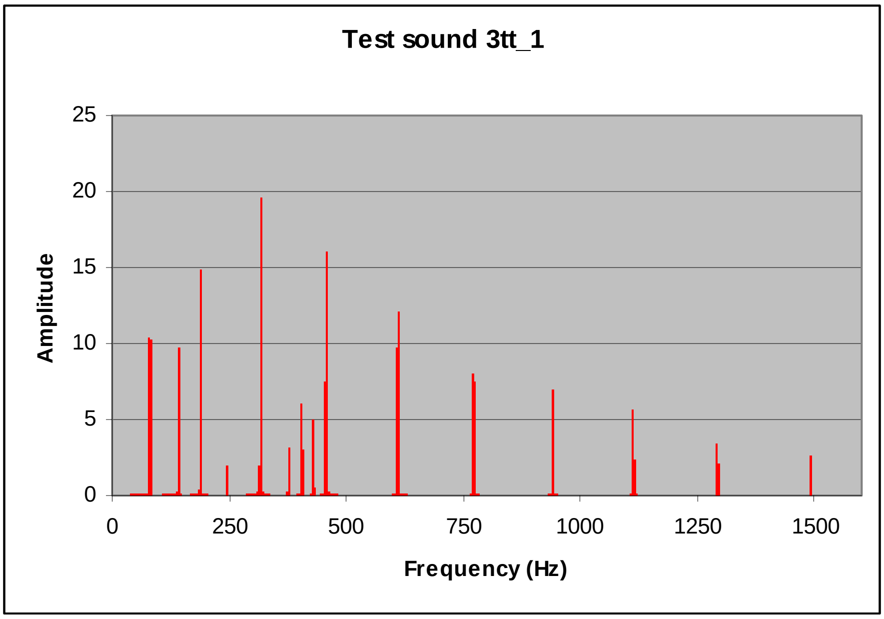


Figure 6-6 Sample spectrum of virtual pitch test sound

6.9.2 Test execution software

The test execution software was written in ASP VB-Script to run in a server environment. This software can run on any computer with Microsoft IIS installed, allowing the software to run on a test PC, on a departmental server, or within the author's website hosted by an external hosting company.

The ASP software consisted of two modules test_engine.asp and mailer.asp. test-engine is called from a webpage index.htm described in the next section. index.htm and the two asp programs interact as shown in Figure 6-7.

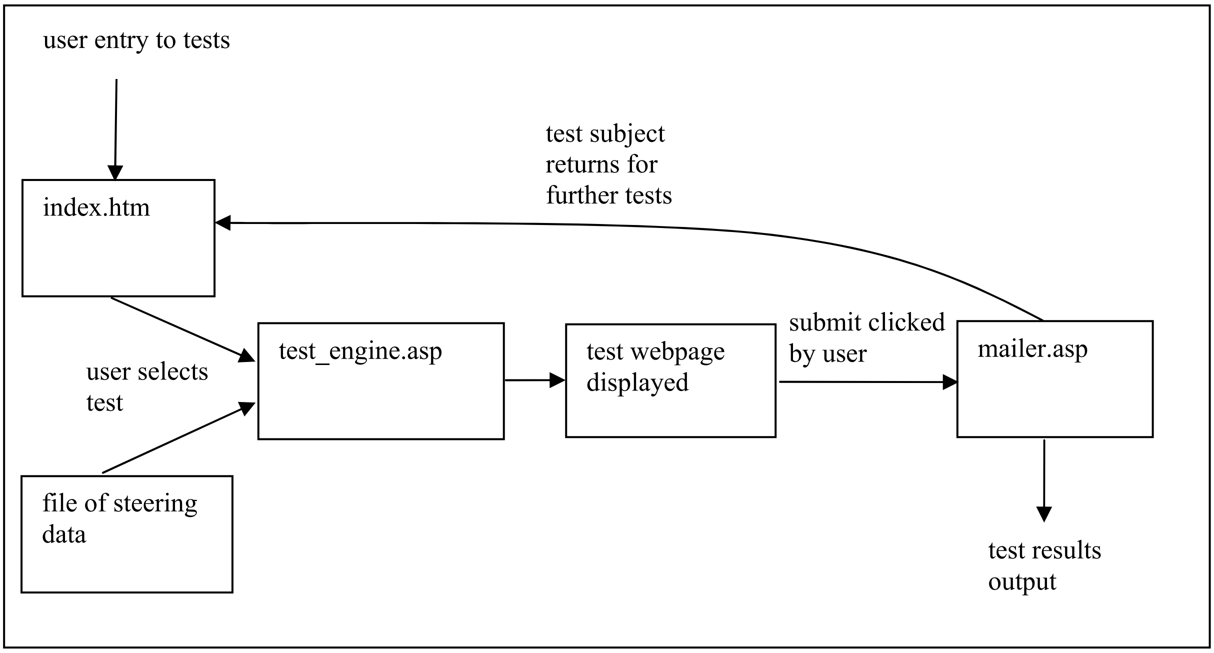


Figure 6-7 Overview of virtual pitch test software

Parameters supplied to the program test_engine.asp are:

- The name of a steering file containing lists of test and reference sound filenames
- A flag indicating whether the test results are to be emailed, displayed on the screen or appended to a file in a known location.

When called from an html page with appropriate parameters, test_engine.asp generates a random test order (for details see chapter 7) and then builds and displays a webpage structured as an html form from which the user will run the test. Test instructions appear at the head of the html page. A reference file is selected by the test subject for each test file by selecting an exclusive radio button. Tests within the page can be carried out in any order. At the foot of the page the test user is asked to provide an email address and indicate the quality of the equipment used for the test. The test run is completed when the test subject clicks the Submit button. On submit, the form contents are passed to the second ASP program, mailer.asp.

On receipt of the form contents from test_engine.asp, mailer.asp carries out a number of validation checks, including ensuring that all 16 tests have been responded to, and that the email address supplied appears to be valid. If any validation checks fail the user is returned to test_engine.asp (with the already completed form contents present) and asked to complete missing tests or correct the data supplied. If validation checks pass, mailer.asp creates a page of results or text for the results file, and then emails, displays or saves the results based on a parameter passed from test_engine.asp.

Once the results have been emailed, saved or displayed a “thank you” webpage is displayed and the test subject encouraged to return to carry out further tests.

This program structure allows the test subject to correct errors in a test block without losing previous work if they are detected by mailer.asp. However, the user cannot suspend and return to a block of tests.

6.9.3 Web environment for tests

The complete set of tests is called from a webpage (index.htm in the test directory) which contains basic instructions, and links to the 9 blocks of tests. Parameters such the name of the steering file for each test block and the parameter indicating how the results should be submitted are encoded in the URL for each test block, used to call test_engine.asp.

The sound files are played across http using whatever media player is defined as the default for .wav files. A webpage advice.htm provides the test subject with advice for setting up the test PC and for checking correct operation before testing starts.

Each sound file is 11 Kbytes in size. If the tests are being run from a server, the first time an individual sound file is called it passes across the network and is cached locally in the PC. Subsequent accesses to the sound file are served locally (and quickly). As a result, there is a slight delay when a sound file is called for the first time but after that the files are played virtually instantaneously. In particular, as the reference files are the same throughout a block of sixteen tests, once a reference file has been accessed it is cached for the remainder of the block of tests.

6.10 Details of test procedure

The detailed test procedure is best explained by providing copies of the web pages and instructions displayed as the user performs the tests.

6.10.1 Initial index page

The following web page is displayed when the test subject first enters the tests, and after test completion if he or she returns to take a further test:

Welcome to the Virtual Pitch tests. These tests are designed to test theories about the origin of the strike note in bells. There are a number of different tests. Each test is simple to do and should take no more than 15 minutes - they are better done quickly, initial impressions matter. You can do as few or as many of the tests as you like, and try them as often as you like. Unfortunately you can't break off and resume a test, but you can take one again if you didn't complete it the first time. The tests at 400Hz and 315Hz are quite difficult and you may want to leave them until last.

Links to individual test pages:	
1000 Hz nominal	630 Hz nominal
1260 Hz nominal	500 Hz nominal
1600 Hz nominal	400 Hz nominal
2000 Hz nominal	315 Hz nominal
800 Hz nominal	

Taking the tests involves listening to test tones and comparing them to see which sounds higher and which lower. The tests are designed to work in any standard web browser (both PC and Macintosh) but do require that your computer has sound output. You can use headphones or computer speakers as you prefer - you will be asked what equipment you used at the end of each test. For advice on how to check that your computer is set up correctly to run the tests, click here. I do ask for your name and email address, so that if your results are particularly interesting I can contact you for follow up. Your email address will only be used to contact you about the results of these tests.

In each test you are asked to choose tones which have the same pitch or note. In each test there is a test sound and sixteen reference sounds. The reference sounds are the same for each test, and are arranged in order from flattest (A) to sharpest (P). The test sounds are displayed in random sequence - if you repeat a test, they will appear in a different order. There are no right and wrong answers to the tests. People's ears respond differently, and these tests are designed to investigate that.

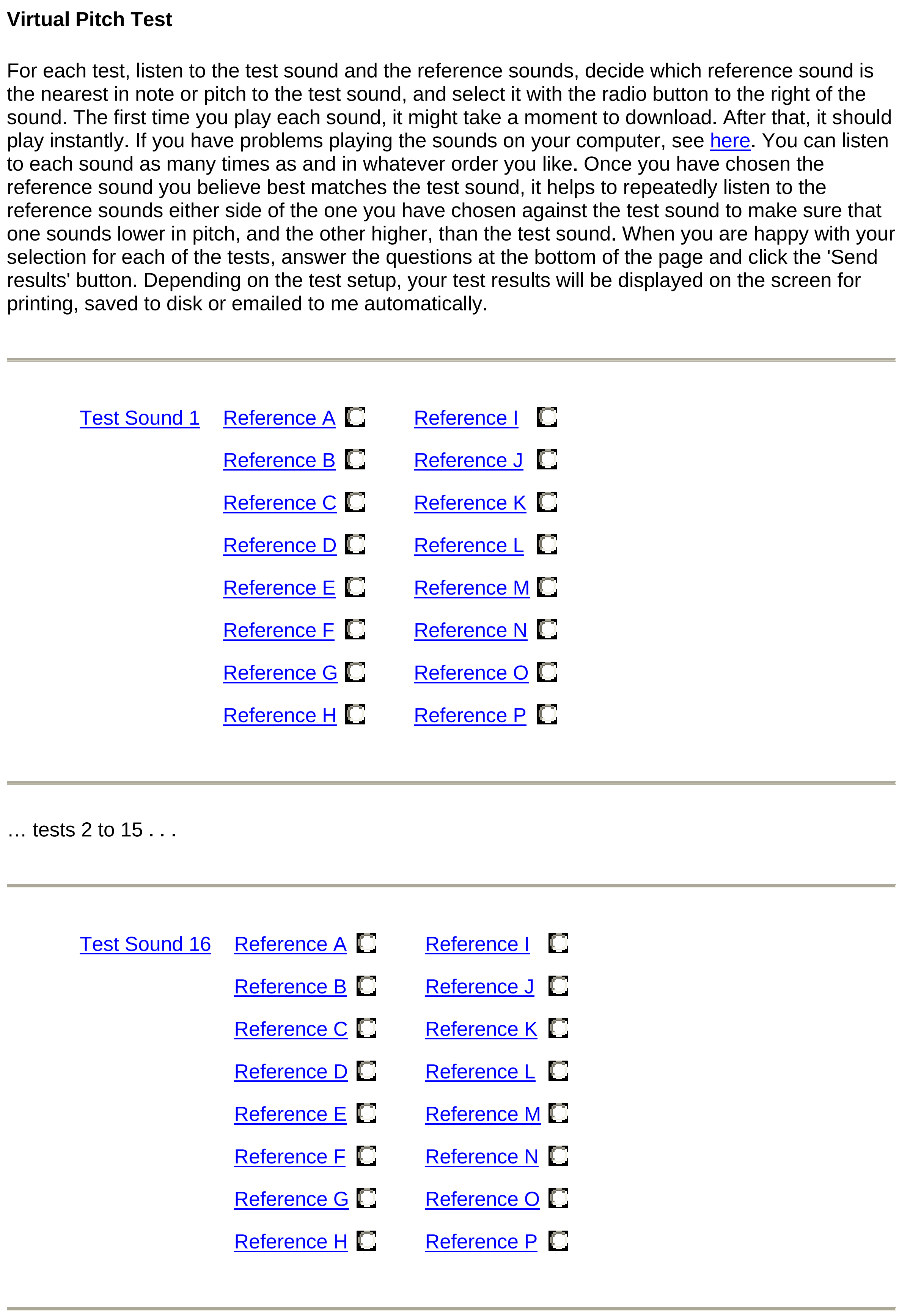
For each test, listen to the test sound and the reference sounds, decide which reference sound is the nearest in note or pitch to the test sound, and select it with the radio button to the right of the sound. The first time you play each sound, it might take a moment to download. After that, it should play instantly. If you have problems playing the sounds on your computer, see here. You can listen to each sound as many times as and in whatever order you like. Once you have chosen the reference sound you believe best matches the test sound, it helps to repeatedly listen to the reference sounds either side of the one you have chosen against the test sound to make sure that one sounds lower in pitch, and the other higher, than the test sound. When you are happy with your selection for each of the tests, answer the questions at the bottom of the page and click the 'Send results' button. Depending on the test setup, your test results will be displayed on the screen for printing, saved to disk or emailed to me automatically.

These instructions are repeated at the start of each test page for your convenience. If you have any comments or questions on these tests, send me an email.

Page last updated on 1st February 2007. Page created by Bill Hibbert, Great Bookham, Surrey.

6.10.2 Test page

The following web page is displayed when the test subject enters a specific test:



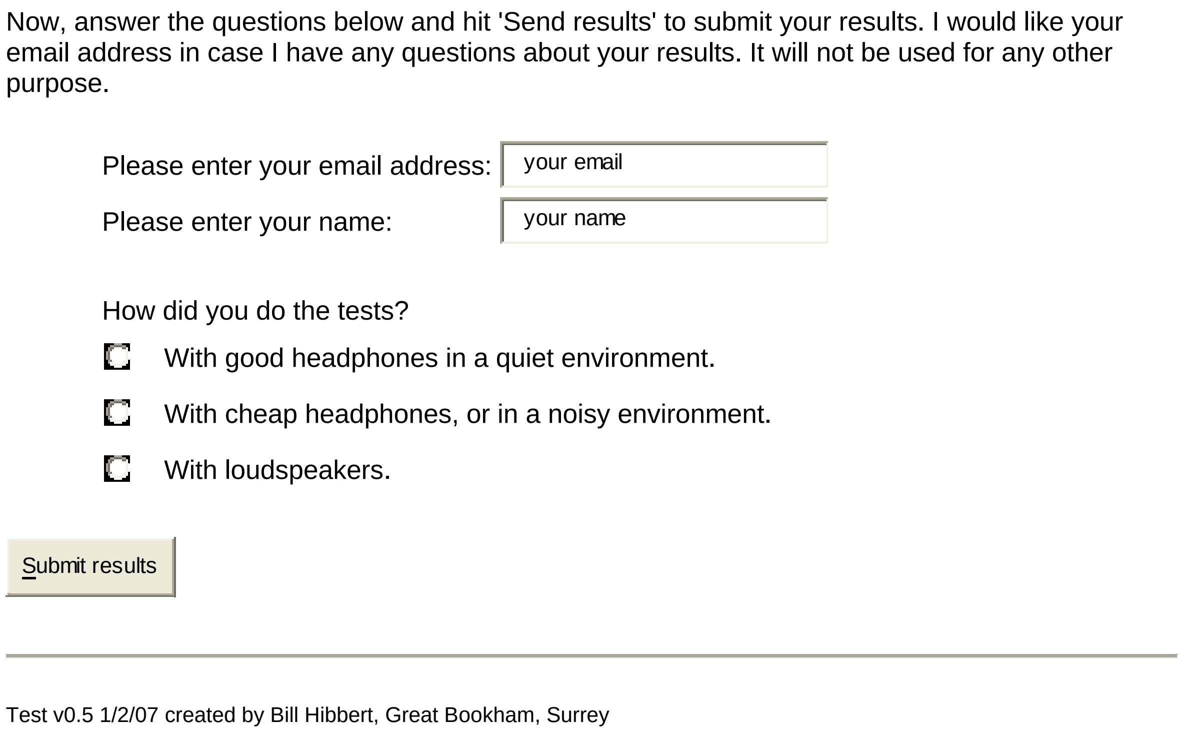
Virtual Pitch Test

For each test, listen to the test sound and the reference sounds, decide which reference sound is the nearest in note or pitch to the test sound, and select it with the radio button to the right of the sound. The first time you play each sound, it might take a moment to download. After that, it should play instantly. If you have problems playing the sounds on your computer, see here. You can listen to each sound as many times as and in whatever order you like. Once you have chosen the reference sound you believe best matches the test sound, it helps to repeatedly listen to the reference sounds either side of the one you have chosen against the test sound to make sure that one sounds lower in pitch, and the other higher, than the test sound. When you are happy with your selection for each of the tests, answer the questions at the bottom of the page and click the 'Send results' button. Depending on the test setup, your test results will be displayed on the screen for printing, saved to disk or emailed to me automatically.

Test Sound 1	Reference A	○	Reference I	○
	Reference B	○	Reference J	○
	Reference C	○	Reference K	○
	Reference D	○	Reference L	○
	Reference E	○	Reference M	○
	Reference F	○	Reference N	○
	Reference G	○	Reference O	○
	Reference H	○	Reference P	○

… tests 2 to 15 . . .

Test Sound 16	Reference A	○	Reference I	○
	Reference B	○	Reference J	○
	Reference C	○	Reference K	○
	Reference D	○	Reference L	○
	Reference E	○	Reference M	○
	Reference F	○	Reference N	○
	Reference G	○	Reference O	○
	Reference H	○	Reference P	○



Now, answer the questions below and hit 'Send results' to submit your results. I would like your email address in case I have any questions about your results. It will not be used for any other purpose.

Please enter your email address: your email

Please enter your name: your name

How did you do the tests?

- ○ With good headphones in a quiet environment.
- ○ With cheap headphones, or in a noisy environment.
- ○ With loudspeakers.

Test v0.5 1/2/07 created by Bill Hibbert, Great Bookham, Surrey

7 STATISTICAL DESIGN OF VIRTUAL PITCH TESTS

7.1 Introduction

The background to the experiments and their overall design is covered in previous chapters, especially in chapter 6. The purpose of this chapter is to explain and justify the statistical design and analysis of the four-factor analysis of variance employed.

Each set of tests performed by the subject comprised 16 tests. The sixteen tests had combinations of high and low levels of four factors, i.e. sixteen combinations in all. The tests in each set of 16 were compiled ‘on the fly’ from pre-built sets of test sounds as described in chapter 6 and were presented in random order to eliminate any effect from test ordering.

A large number of exploratory experiments were carried out by and on the author to validate the experimental procedure, establish the range of various effects so that tests were appropriately calibrated, and to provide sample data to validate the analysis techniques. The results of 240 of these tests are used in this chapter to illustrate and validate the test design.

7.2 Randomisation of test order

The webpage from which the tests are conducted is built by a server-side script (ASP VBScript) each time the page is called. Browser caching is disabled in the webpage so that each time it is recalled, the generating script runs again. The random ordering is achieved using a Fisher-Kerr random shuffle in the script. The VBScript code fragment that does this is as follows:

```
Randomize
For i=0 To 15
    testindex(i) = i + 1
Next
For i=15 To 0 Step -1
    j = Int(Rnd() * (i + 1))
```

```
        temp = testindex(i)

        testindex(i) = testindex(j)

        testindex(j) = temp

    Next
```

The array testindex[0..15] holds the integers 1 to 16 and determines the order in which the tests are displayed. The Randomize command seeds the VB random number generator. Rnd() returns a floating point value ≥ 0 and < 1.0. The values of j are therefore integers ≥ 0 and $\leq$ i. The shuffle steps along the array from end to start swapping the current element with a random element between the current element and the start of the array.

This random shuffle was validated by noting that its purpose is to ensure that any test is equally likely to succeed any other test. Therefore, a statistic of interest is the difference between successive values in testindex. The actual code above was inserted in a test harness which repeated the code 5,000 times and, for each run, collected counts of differences between successive values. The ideal frequencies of differences were obtained simply by counting the gaps in a list of 16 numbers and scaling up for the actual number of trials. Table 7-1 gives the results:

Difference	Actual count	Ideal count	Difference	Actual count	Ideal count
-15	308	312.5	**15**	337	312.5
-14	613	625	**14**	636	625
-13	909	937.5	**13**	932	937.5
-12	1212	1250	**12**	1327	1250
-11	1518	1562.5	**11**	1526	1562.5
-10	1860	1875	**10**	1879	1875
-9	2140	2187.5	**9**	2232	2187.5
-8	2544	2500	**8**	2463	2500
-7	2893	2812.5	**7**	2727	2812.5
-6	3011	3125	**6**	3162	3125
-5	3339	3437.5	**5**	3449	3437.5
-4	3874	3750	**4**	3689	3750
-3	4132	4062.5	**3**	4085	4062.5
-2	4410	4375	**2**	4348	4375
-1	4763	4687.5	**1**	4682	4687.5

Table 7-1 Results from random shuffle test

The chi-squared value for the above results calculated as $\chi^2 = \sum \frac{(actual - ideal)^2}{ideal}$ was 35.15 with 29 degrees of freedom. The probability that the actual results are drawn from a population with the ideal distribution is 0.19965 or a fraction less than 20%. This means that the hypothesis that the observed distribution matches the theoretical distribution, i.e. that the shuffle is random, should be accepted - to reject it at a 95% confidence level we would need the probability to be below 5%.

7.3 Analysis and interpretation of results

The experiment design and the approach to analysis of the results are based on standard approaches described by Montgomery (2001). All analysis and charting of results was done in Microsoft Excel, but due to restrictions in Excel's statistical functions, the analysis approaches are explicitly derived below and were entered as formulas in the spreadsheets, apart from values for the t, F, χ^2 and normal distributions for which the built-in functions were used.

To explain and validate the approaches, several hundred test results (obtained by the author while validating the software used to deliver the tests) are used as examples below.

The test results consist of, for each test tone, the reference tone filename which was matched against it. These test tone filenames are translated into the nominal frequency of the test tone using an Excel lookup. The nominal frequencies are then translated into the pitch deviation in 1/100ths of a semitone, or cents, from the test tone nominal frequency using the standard formula (Equation 3-1). It is possible to analyse the tests using raw frequencies rather than cents. The results are virtually identical, because the log function is virtually linear over the range of pitch shifts of interest. However, as cents are the more natural way to present the final results of the analysis (for instance, it allows meaningful comparison of effects at different nominal frequencies), it seems appropriate to use them throughout.

7.4 Four-factor test

7.4.1 Overview of experimental analysis

Four factors, each at two levels, have $2^4 = 16$ combinations or treatments. A set of tests is put together comprising all 16 treatments. This set of tests is then conducted multiple times to average out experimental error and uncontrolled factors. Each set of tests is called a replicate. In the trials, most tests were conducted on the author. In the tests reported on in Chapter 9, each of the replicates was conducted by a different test subject. The means of all the replicates of each treatment are taken.

A set of contrasts is defined. These contrasts are linear combinations of the 16 treatment means, chosen in such a way as to isolate out the effects of each factor, singly and in combination. The contrasts are chosen to be linearly independent.

Because of this linear independence, the sum of squares of the difference of each individual test result from the overall mean of the test results, can be partitioned into a sum of squares for each treatment combination, and a sum of squares for the residual differences in the test results due to random effects and errors. This allows the effect of each factor on the overall test results, both individually and in combination, to be estimated.

Tests of statistical validity can be applied to these factor effects to determine which effects are actually meaningful at a particular confidence level, and which may just be due to random effects. Based on this analysis, a model of the dependence of the test results on various factors is hypothesised. Tests on the residuals arising from this model are used to establish the validity of the model and that the residuals appear to be normally distributed.

This overall approach is called analysis of variance or ANOVA and is a standard approach to the design and analysis of experiments of this type. The particular design used here, because it has the same number of test results for all treatment combinations, is termed a balanced design and is straightforward to implement and analyse.

To validate the test and analysis approach, the results of 15 replicates (i.e. 240 tests in total) performed by the author are presented and analysed below.

7.4.2 Factors, Treatments and Models

The four factors are denoted by A, B, C and D. The convention adopted through the analysis is that a factor at high level is indicated by the presence of the appropriate letter, a factor at low level by the absence of the letter. The treatment with all factors at a low level is denoted by I.

Table 7-2 below shows the way the 16 treatments appear in the test data - in the actual tests, the treatments are presented in random order.

Coding	**Factor A**	**Factor B**	**Factor C**	**Factor D**
I	low	low	low	low
A	high	low	low	low
B	low	high	low	low
AB	high	high	low	low
C	low	low	high	low
AC	high	low	high	low
BC	low	high	high	low
ABC	high	high	high	low
D	low	low	low	high
AD	high	low	low	high
BD	low	high	low	high
ABD	high	high	low	high
CD	low	low	high	high
ACD	high	low	high	high
BCD	low	high	high	high
ABCD	high	high	high	high

Table 7-2 Factors and treatments in the four-factor analysis of variance

In the sample tests presented below, the factors used are:

A: hum intensity

B: prime frequency

C: tierce frequency

D: octave nominal frequency

but of course the test design and analysis is valid for any four factors.

The hypothesis assumed in the four factor test is that all 15 factors and factor combinations affect the test result to some degree. The purpose of the test is to identify the actual extent to which the 15 factors and factor combinations influence the test results, so that the model can if possible be refined and simplified.

If we define a set of variables a, b, c, d related to the factors A, B, C and D in that, when factor A is at its low level, $a = -1$, and when A is at its high level, $a = +1$, then we can propose a model for the observed measurements of y as:

$$
\begin{aligned}
y &= \tau_I \\
&+ \frac{1}{2}\left(\tau_A \cdot a + \tau_B \cdot b + \tau_C \cdot c + \tau_D \cdot d\right) \\
&+ \frac{1}{2}\left(\tau_{AB} \cdot ab + \tau_{AC} \cdot ac + \tau_{AD} \cdot ad + \tau_{BC} \cdot bc + \tau_{BD} \cdot bd + \tau_{CD} \cdot cd\right) \\
&+ \frac{1}{2}\left(\tau_{ABC} \cdot abc + \tau_{ABD} \cdot abd + \tau_{ACD} \cdot acd + \tau_{BCD} \cdot bcd + \tau_{ABCD} \cdot abcd\right) \\
&+ \varepsilon
\end{aligned}
$$

where ε represents the effect of experimental error and random factors, and the multipliers of $\frac{1}{2}$ are needed because a, b, c, d have a range of 2 (-1 to +1). This model might seem to presume a linear relationship between y and intermediate levels of the various factors, but this is not the case, as we are only going to use this model with factors at either the low or the high level. The τ_i are called effects because they quantify the effect of each factor and factor combination.

If we can establish values and confidence limits on the various τ we can begin to understand how the observed values of y depend on the various factors, in isolation and combination.

7.4.3 Calculation of contrasts and effects

The results below are taken from Montgomery (2001 pp. 218 et seq.). They will not be explicitly derived, though sufficient explanation will be given to allow constraints and conditions on their usage to be made clear. The notation has been slightly modified from that used by Montgomery to suit the details of these experiments.

A particular set of experiments involves a number of replicates of a set of tests. Each set of tests involves a number of individual treatments with factors set at different levels. If r is the number of replicates of the tests, a is the number of treatments, and N is the total number of tests done, then

$$N = r \times a$$

If there are k factors, each at two levels, then $a = 2^k$. In the four factor tests, $k = 4$ and $a = 16$.

The individual test results are y_{ij} where i ranges over the treatments (i.e. $1 \le i \le a$), and j ranges over the replicates (i.e. $1 \le j \le r$).

The mean of all the measurements (the grand mean) is:

$$\overline{y} = \frac{1}{N}\sum_{i=1}^{a}\sum_{j=1}^{r} y_{ij} \tag{7-1}$$

The mean of the measurements for the i^{th} treatment is:

$$\overline{y}_i = \frac{1}{r}\sum_{j=1}^{r} y_{ij} \tag{7-2}$$

In this experiment, there are 16 of these means, which we can denote $\overline{y}_I$, $\overline{y}_A$, $\overline{y}_B$, $\overline{y}_{AB}$,..., $\overline{y}_{BCD}$, $\overline{y}_{ABCD}$ to indicate the factor levels in the tests in each treatment. The order of the treatments is essentially arbitrary, but for consistency, the order used in Table 7-2 above (known as Yates' order) will be used.

To establish the effect on the test results of an individual factor, take all the treatment means with that factor present at the high level, and deduct all those treatment means with that factor present at the low level, to calculate a quantity called the contrast for that factor. For example, to estimate the contrast Γ_A of factor A independent of all the other factors and factor combinations, we calculate:

$$\begin{aligned}\Gamma_A = {} & \left(\overline{y}_A + \overline{y}_{AB} + \overline{y}_{AC} + \overline{y}_{ABC} + \overline{y}_{AD} + \overline{y}_{ABD} + \overline{y}_{ACD} + \overline{y}_{ABCD}\right) \\ & - \left(\overline{y}_I + \overline{y}_B + \overline{y}_C + \overline{y}_{BC} + \overline{y}_D + \overline{y}_{BD} + \overline{y}_{CD} + \overline{y}_{BCD}\right)\end{aligned}$$

As there are 16 contrasts (the 15 factors and factor combinations, together with the case with all factors at the low level) which are all linear combinations of the 16 treatment means, a matrix formulation is helpful. If we define:

$$\begin{aligned}\mathbf{G}^T &= \left(\Gamma_I \quad \Gamma_A \quad \cdots \quad \Gamma_{BCD} \quad \Gamma_{ABCD}\right) \\ \mathbf{M}^T &= \left(\overline{y}_I \quad \overline{y}_A \quad \cdots \quad \overline{y}_{BCD} \quad \overline{y}_{ABCD}\right)\end{aligned}$$

then we can define a 16×16 matrix $\mathbf{C}$ of linear combinations or contrast coefficients such that:

$$\mathbf{G} = \mathbf{C} \cdot \mathbf{M} \tag{7-3}$$

The individual elements of $\mathbf{C}$ are c_{ik} where i (the columns) ranges across the treatment means, and k (the rows) ranges across the effects. The particular contrast coefficient matrix used for this analysis is:

$$\mathbf{C} = \begin{pmatrix}
+1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 \\
-1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 \\
-1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 \\
-1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 \\
+1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 \\
+1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 \\
+1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 \\
+1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 \\
+1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 \\
+1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 \\
-1 & +1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 & -1 & +1 \\
-1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 \\
-1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 \\
-1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 \\
+1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & +1 & -1 & -1 & +1
\end{pmatrix}$$

For convenience in displaying the results, the treatment means, and the columns of this matrix, are in the order:

I, A, B, AB, C, AC, BC, ABC, D, AD, BD, CD, ACD, BCD, ABCD

(i.e. Yates' order), whereas the contrasts, and the rows of this matrix, are in the order:

I, A, B, C, D, AB, AC, AD, BC, BD, CD, ABC, ABD, ACD, BCD, ABCD

(i.e. the single factor contrasts, then the two factor contrasts, then the three factor contrasts, etc.).

This matrix is populated by geometric analogy. It should be clear that the second row (for effect Γ_A) implements the linear combination shown above for Γ_A. The rows for $\Gamma_B, \Gamma_C, \Gamma_D$ are populated in a similar fashion. The rows for the effect of combinations of factors are produced by multiplying, element by element, the rows for the appropriate individual factors.

The matrix $\mathbf{C}$, which is a Hadamard matrix, has some interesting and important properties:

- All rows apart from the first sum to zero

- All rows are orthogonal, i.e. $\mathbf{C}^T \cdot \mathbf{C} = \mathbf{C} \cdot \mathbf{C}^T = 16 \cdot \mathbf{I}$ where $\mathbf{I}$ is the 16×16 unit matrix
- The matrix inverse is closely related to its transpose: $\mathbf{C}^{-1} = \frac{1}{16}\mathbf{C}^T$, which follows directly from the previous property.

To calculate the effects $\tau_I, \tau_A, \tau_B, ..., \tau_{ABCD}$ needed to populate the model from the contrasts $\Gamma_I, \Gamma_A, \Gamma_B, ..., \Gamma_{ABCD}$, since each contrast is the average of $a/2$ differences in means,

$$\tau_k = \frac{2 \cdot \Gamma_k}{a} \tag{7-4}$$

7.4.4 Analysis of variance

We now have to find a way to decide if the calculated effects are significant given the influence of random factors and experimental error. To do this, we need to analyse the variances, or rather (since analysis of variance is a bit of a misnomer), the squared deviations of various quantities from means.

The sum of squares of deviations of all measurements from the global mean is:

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{r}\left(y_{ij} - \bar{y}\right)^2 \tag{7-5}$$

The sum of squares of deviations from the mean for treatment k is:

$$SS_k = \frac{r \cdot \left(\sum_{i=1}^{a} c_{ik} \cdot \bar{y}_i\right)^2}{\sum_{i=1}^{a} c_{ik}^2}$$

Since $\sum_{i=1}^{a} c_{ik} \cdot \bar{y}_i = \Gamma_k$ (taking the k^{th} row of $\mathbf{G} = \mathbf{C} \cdot \mathbf{M}$) and since $c_{ik} = \pm 1$, this result simplifies to

$$SS_k = \frac{r \cdot \Gamma_k^2}{a} \tag{7-6}$$

Finally, the sum of squares of deviations of results from their corresponding treatment mean (resulting from errors and unrecognised effects) is:

$$SS_E = \sum_{i=1}^{a}\sum_{j=1}^{r}\left(y_{ij} - \bar{y}_i\right)^2 \tag{7-7}$$

Then, provided the contrasts Γ_k are linearly independent, which they are because **C** is a Hadamard matrix, it emerges after a long algebraic manipulation that:

$$SS_T = \sum_{k=1}^{a} SS_k + SS_E \tag{7-8}$$

since the cross terms sum to zero because of the way the means are defined. This remarkable result lies at the heart of analysis of variance. What it means is that the total sum of squares for all the test results can be decomposed into independent sums of squares for each factor and combination of factors, and a residual sum of squares covering errors and random effects.

Note that we have two ways to calculate SS_E, either directly from the defining formula summing squares of the deviations of results from their treatment mean, or indirectly from $SS_E = SS_T - \sum_{k=1}^{a} SS_k$. This allows us to check that the various calculations have been implemented correctly.

We can use the independence of the various sums of squares to provide us with the first insight into which factors and factor combinations are important, simply by looking at the percentage contribution of each, and of the error sum of squares, to the overall sum of squares.

Provided the residuals are normally distributed with mean 0 and standard deviation σ, application of Cochran's theorem leads to the conclusions that:

- SS_T/σ^2, the SS_k/σ^2 and SS_E/σ^2 are independent chi-squared random variables

- The degrees of freedom in the SS_k and SS_E sum to the total degrees of freedom in SS_T.

The test for normality of residuals is an important part of the validation of this test design and will be further explored when the test data is examined below.

Provided the residuals are normal, we then have the basis for tests of statistical significance of the effects. The actual degrees of freedom for this test are:

- 1 for each of the $(a-1)$ contrasts Γ_k
- $N-a$ for the error sum of squares
- $N-a+1\cdot(a-1)=N-1$ for the total sum of squares.

Quantities such as ${}^{SS_k}\!/_{SS_E}$ follow the F distribution with one degree of freedom in the numerator and $N-a$ degrees of freedom in the denominator and we can use this directly to establish the significance of each effect, provided we assume the standard deviation for all treatments and the residual errors is the same.

We now have a second way to investigate the significance of the various factors and factor combinations, by using the F distribution to calculate a P-value for each contrast or effect. The Excel formula used to calculate the P-value is FDIST(F value, treatment degrees of freedom, error degrees of freedom). The P-value gives the probability of an effect as big as that observed arising by chance if the hypothesis of no effect were true. A P-value of 0.05 or less means that the effect is statistically significant at the 95% level.

7.4.5 Confidence intervals

For practical application it also helps to establish confidence intervals on the effects. First, we need to establish the variance in the effects. For the four-factor test, the variance of the measurements for the i^{th} treatment is $\frac{1}{(r-1)}\sum_{j=1}^{r}\left(y_{ij}-\bar{y}_i\right)^2$. The average variance s^2 for all measurements is

$$\begin{aligned} s^2 &= \frac{1}{a}\frac{1}{(r-1)}\sum_{i=1}^{a}\sum_{j=1}^{r}\left(y_{ij}-\bar{y}_i\right)^2 \\ &= \frac{1}{a(r-1)}SS_E \end{aligned} \tag{7-9}$$

This quantity is also known as the mean square error MSE. We are going to use s as an estimator for the standard deviation of the measurements and assume that the standard deviation for all treatments and replicates is the same. Note that this assumption may not be justified especially for tests at a different nominal frequency; a separate analysis will be done at each frequency.

Since each treatment mean is the average of r measurements, the variance of each mean is $\frac{s^2}{r}$. Since each contrast is the sum or difference of a treatment means, the variance of each contrast is $\frac{a \cdot s^2}{r}$. Finally, since $effect = \frac{2}{a} \cdot contrast$, and $Var(cy) = c^2 \cdot Var(y)$, the variance of the effects is $\frac{4}{a^2} \cdot \frac{a}{r} \cdot s^2 = \frac{4}{a^2 r(r-1)} \cdot SS_E$.

The standard error se (i.e. our estimate of the standard deviation) of the effects is:

$$se = \frac{2}{a} \cdot \sqrt{\frac{SS_E}{r(r-1)}} \tag{7-10}$$

With this basic result in place, we can look at how to establish confidence limits on the effects. If $P(p)$ is the probability of some proposition p, v is some value for which we want to define a confidence interval, v_M is an actual measured value of v, and α is the confidence level we want to achieve (e.g. a 5% chance that v lies outside the confidence interval, in which case $\alpha = 0.05$), then

$$P(v_M - \lambda \le v \le v_M + \lambda) = 1 - \alpha$$

defines the low and high limits $v_M - \lambda, v_M + \lambda$ within which we can be $1-\alpha$ (e.g. 95%) confident that v will be found if we conduct multiple runs of the experiment. Note that this is a two-sided limit. If we want to achieve an α chance (e.g. 5%) that v lies outside the range $v_M \pm \lambda$, then

$$P(v_M - \lambda \le v) = P(v \le v_M + \lambda) = 1 - \alpha/2$$

e.g. there must only be a 2.5% chance that v is lower than the lower limit, and a 2.5% chance that it is higher than the higher limit. If $v_M \pm \lambda$ spans zero at the chosen level of probability, then we should regard the measured v_M as not significant. An equivalent way to look at this issue is to say that that v_M is not significant if $|v_M| \le \lambda$.

There are a number of different ways in which the critical value λ can be established. The first, and very crude method, is to assume (wrongly!) that the effects are normally distributed with standard deviation se, so that 5% probability implies $\lambda \approx 2 \cdot se$.

A slightly more realistic approach is to recognise that effects, because they are derived from differences in means, follow the t distribution with $N-a$ degrees of freedom. On this assumption, the correct value, from Montgomery (2001 p. 74) for λ is:

$$\lambda = t_{\alpha/2, N-a} \cdot se \tag{7-11}$$

However, because we will be examining multiple effects in the tests, the value of α to be used needs to be chosen with care. If we are looking for significance in a number n of effects out of the total of 16, then use of a confidence level of α on the individual effects means that overall there is a chance $n \cdot \alpha$ of a wrong conclusion. If $n = 16$ and $\alpha = 0.05$ there is a 0.8 or 80% chance of a wrong conclusion about one of the effects. Put another way, at a confidence level of 5% there is a 1 in 20 chance of a wrong conclusion, so there is a strong chance that one of the 16 effects will be judged significant when it is not. A resolution for this, due to Bonferroni, is to drop the chance of each effect being outside the range to α/n, so that the correct value to use for the confidence intervals, from Montgomery (2001 p. 75) is:

$$\lambda = t_{\alpha/2n, N-a} \cdot se \tag{7-12}$$

As will be seen, based on the trials most of the effects from the virtual pitch test are not significant, and the formal tests reported on in chapter 9 use $n = 4$, i.e. we are looking for significance only in 4 of the 16 effects. To give an idea of the difference between critical values calculated by the various methods, here are values of λ/se for each approach, for 3 and 30 replicates, with $\alpha = 0.05$ and four of the sixteen effects judged significant:

Critical value	**r = 3**	**r = 30**	**Excel formula**
Crude normal assumption	2.00	2.00	
t distribution	2.35	2.25	TINV(0.025,degs of freedom)
Bonferroni	2.93	2.75	TINV(0.025/4,degs of freedom)

Table 7-3 Comparison of confidence interval calculations

The Bonferroni approach widens the confidence interval, i.e. makes it less likely that an effect will be judged significant.

7.4.6 Summary of calculations required to process test results

The following processing steps are implemented in Excel spreadsheets to carry out the analysis of the test results:

- Lookup tables translate the filenames of the reference file selected by the user for each test to a nominal frequency, and then convert it to cents based on the nominal of the test waveform
- The treatment means and grand mean are calculated by applying Equations (7-1) and (7-2)
- The contrasts and effects are calculated by applying Equations (7-3) and (7-4)
- The various sums of squares are calculated by applying Equations (7-5), (7-6) and (7-7)
- The calculations are verified by comparing the results from Equations (7-7) and (7-8)
- The F distribution is used to estimate the significance of each effect through the calculation of P-values
- The standard error in the effects is calculated using 7-9

- The Bonferroni confidence interval is calculated for each effect using Equations (7-10) and (7-12).

Additional tests are also carried out to verify the assumption that residuals are normally distributed. These calculations are explained in the next section.

7.4.7 Sample results

240 test results (15 replicates of 16 tests) are analysed in detail in this section to demonstrate and validate the analysis and processing. These results were produced by the author while verifying various aspects of the tests. As a result, they are not being presented as part of the research results.

Table 7-4 shows the results of the 240 tests. The figures in this table are the pitch shifts in cents observed for each test, calculated from the nominal frequencies of the reference files matched in each test, compared with the nominal frequency of the test sounds (which was the same in every case). Table 7-5 shows the analysis as already described.

The four factors used in these tests are:

Factor	**Parameter**	**Low level**	**High level**
A	Hum intensity	0.5 x notional level	2.0 x notional level
B	Prime frequency	1400 cents below nominal	1200 cents below nominal
C	Tierce frequency	900 cents below nominal	800 cents below nominal
D	Upper partials (represented by Oct. Nom. frequency)	1140 cents above nominal	1280 cents above nominal

The tests were done on a range of equipment from very poor to quite good quality (from an audio reproduction perspective). No particular care was taken in test execution, as in most cases the focus was software testing rather than accuracy of results. Therefore, as will be seen there are a number of outliers in the results. This is all to the good, because it helps test the robustness of the analysis and the ability to detect outliers.

	Goodman headphones			Cheap headset			Goodman headphones			Compaq inbuilt speaker			Comp.	Dell speakers	
Factors	repl. 1	repl. 2	repl. 3	repl. 1	repl. 2	repl. 3	repl. 1	repl. 2	repl. 3	repl. 1	repl. 2	repl. 3	laptop	repl. 1	repl. 2
I	-15.3	-15.3	-25.3	-5.3	-15.3	-15.3	-25.3	-20.3	-20.3	-25.3	-20.3	-25.3	-15.3	-10.3	-15.3
A	-15.3	-25.3	-15.3	-10.3	-15.3	-10.3	-15.3	-15.3	-20.3	-20.3	-32.8	-25.3	-20.3	-10.3	-25.3
B	-10.3	-10.3	-10.3	-5.3	-5.3	-10.3	-10.3	-0.3	-10.3	-15.3	-10.3	-15.3	-10.3	-20.3	-5.3
AB	-15.3	-15.3	-5.3	-10.3	-15.3	-5.3	-15.3	-10.3	-10.3	-10.3	-10.3	-20.3	-20.3	-10.3	-20.3
C	-5.3	-5.3	-15.3	-5.3	-5.3	-10.3	-5.3	-10.3	-10.3	-10.3	-0.3	-15.3	-5.3	-0.3	-5.3
AC	-15.3	-5.3	4.7	-5.3	-10.3	-10.3	-5.3	-5.3	-5.3	-15.3	-15.3	-25.3	-10.3	-5.3	-10.3
BC	-5.3	-0.3	-5.3	4.7	4.7	14.7	-5.3	-5.3	-0.3	-5.3	-10.3	-10.3	4.7	4.7	-10.3
ABC	-10.3	-5.3	4.7	-0.3	-0.3	4.7	-0.3	4.7	9.7	-10.3	-5.3	-10.3	4.7	-10.3	-5.3
D	4.7	-0.3	-0.3	4.7	-5.3	4.7	4.7	4.7	-0.3	9.7	24.7	14.7	4.7	4.7	-0.3
AD	-5.3	-0.3	-0.3	4.7	4.7	4.7	9.7	-5.3	9.7	4.7	4.7	-0.3	-0.3	4.7	-0.3
BD	4.7	4.7	-0.3	4.7	4.7	4.7	4.7	-5.3	4.7	9.7	9.7	4.7	-0.3	-0.3	4.7
ABD	4.7	4.7	4.7	-0.3	-0.3	4.7	9.7	4.7	4.7	9.7	4.7	9.7	9.7	-0.3	4.7
CD	9.7	4.7	14.7	9.7	-0.3	4.7	14.7	9.7	14.7	14.7	9.7	14.7	4.7	14.7	4.7
ACD	9.7	4.7	9.7	9.7	9.7	14.7	14.7	9.7	14.7	9.7	14.7	9.7	9.7	14.7	9.7
BCD	9.7	9.7	14.7	19.7	19.7	9.7	19.7	9.7	14.7	14.7	9.7	19.7	14.7	19.7	14.7
ABCD	9.7	9.7	9.7	9.7	14.7	14.7	9.7	14.7	14.7	9.7	9.7	9.7	9.7	14.7	9.7

Table 7-4 Pitch shifts (in cents) from sample virtual pitch test runs

	Contrast	Effect	SS	Percent	Deg free	F value	P-value	Bonferroni conf. int.	
Total	-17.18		31647.09					low	high
A	-9.16	-1.15	78.74	0.25%	1	2.81	0.0951	-3.03	0.74
B	33.16	4.15	1031.13	3.26%	1	36.79	0.0000	2.26	6.03
C	69.83	8.73	4571.73	14.45%	1	163.12	0.0000	6.84	10.62
D	142.50	17.81	19035.79	60.15%	1	679.19	0.0000	15.93	19.70
AB	-1.50	-0.19	2.12	0.01%	1	0.08	0.7836	-2.07	1.70
AC	0.50	0.06	0.23	0.00%	1	0.01	0.9276	-1.82	1.95
AD	2.50	0.31	5.85	0.02%	1	0.21	0.6483	-1.57	2.20
BC	3.50	0.44	11.50	0.04%	1	0.41	0.5225	-1.45	2.32
BD	-19.83	-2.48	368.68	1.16%	1	13.15	0.0004	-4.37	-0.59
CD	-8.50	-1.06	67.69	0.21%	1	2.42	0.1216	-2.95	0.82
ABC	-4.50	-0.56	18.96	0.06%	1	0.68	0.4116	-2.45	1.32
ABD	0.17	0.02	0.03	0.00%	1	0.00	0.9753	-1.87	1.91
ACD	-1.83	-0.23	3.14	0.01%	1	0.11	0.7381	-2.12	1.66
BCD	4.50	0.56	18.97	0.06%	1	0.68	0.4115	-1.32	2.45
ABCD	-12.84	-1.60	154.47	0.49%	1	5.51	0.0198	-3.49	0.28
Error SS			6278.06	19.84%	224				
Residual SS			0.00						

Standard error of effects:	0.683
Bonferroni conf. interval:	1.887

Table 7-5 Analysis of sample virtual pitch test runs

The presentation of the results should be easy to follow after the discussion above. Contrasts, effects and sums of squares (SS) are given for each factor and factor combination. The figure for residual SS is the difference between the two methods of calculating the error sum of squares and confirms that the calculations have been set up correctly.

The column 'Percent' shows the relative contribution of each factor combination and the within-treatment variance to the total sum of squares. It is clear that B, C and D acting in isolation explain most of the frequency changes observed, with some contribution from BD (prime and upper partials acting together). About 20% of the sum of squares is down to error.

The next two columns present the results of the comparison against the F distribution and confirm that only factors B, C, D and BD are significant at the 95% level.

The last two columns present the confidence intervals calculated using the Bonferroni correction. The cells with a grey background are those where the confidence interval does not span zero. Broadly, these follow the results of the comparison against the F distribution.

Finally, the figures below the main table show the standard error in the effects, and the critical value used to calculate the confidence intervals.

The assumption has been made throughout the analysis that the measurements and their errors are normally distributed. To investigate this, the following procedure was followed:

- A model was used to predict the result for each treatment. This model included only the factors for effects B, C and D (i.e. the biggest effects predicted by the analysis). This also provides a check that the analysis is correctly identifying the major effects
- Residuals were calculated for each measurement taken

- The residuals were tested for normality as described below
- The residuals were plotted against replicate number (to compare variances for a replicate and look for outliers)
- The residuals were plotted against predicted values (to look at adequacy of the simple model adopted).

The residuals were checked for normality by:

- Finding the rank or ordinal position of each residual in the total set of residuals
- Obtaining the value of the normal distribution (with mean 0 and standard deviation 1) pertaining to this ordinal position
- Plotting this quantity against the residual.

The Excel formula used to get the normal value was:

NORMSINV((RANK(value, set of all values, 1) – 0.5) / N)

where N is the total number of values.

Figure 7-1 below is the resulting normality plot:

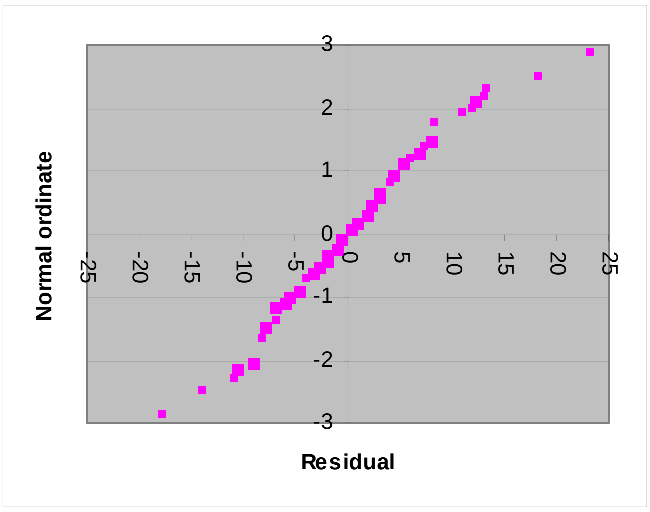


Figure 7-1 Normal probability plot of sample test runs

Apart from four outliers (two at each end of the range) the remaining 236 measurements lie virtually on a straight line. There is little evidence that the residuals are not normally distributed.

The four outliers identified from this plot are:

Replicate	Treatment
Compaq PC, inbuilt speaker, run 2	D
Compaq PC, inbuilt speaker, run 2	A
Compaq PC, inbuilt speaker, run 3	AC
Cheap headset, run 3	BC

The sound quality from the inbuilt speaker in the Compaq PC was dreadful and the tests were difficult to do on this equipment. The cheap headset used was a pair of (free) Walkman earpieces. It is no surprise that this equipment gave rise to outliers.

The plot of residuals against replicate is shown in Figure 7-2:

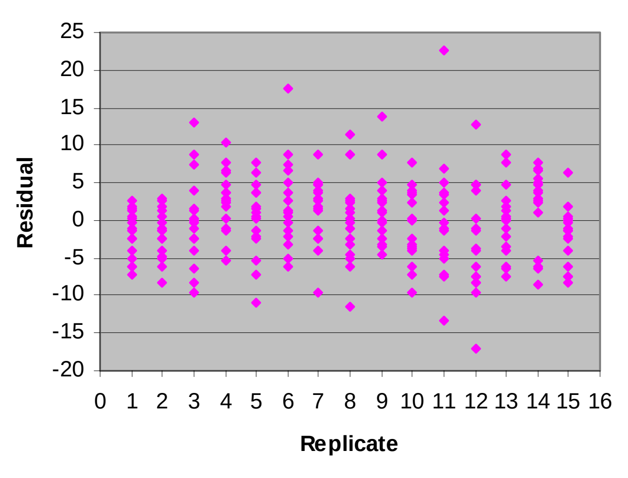


Figure 7-2 Plot of residuals against replicate for sample test runs

Again, the four outliers appear (in replicates 6, 11 and 12). Other than that, there is little to remark on in this plot. There appears to be a difference in variance and also in mean value for tests on different equipment. Tests to establish the differences due to equipment are described in chapter 6.

The plot of residuals against model value is shown in Figure 7-3:

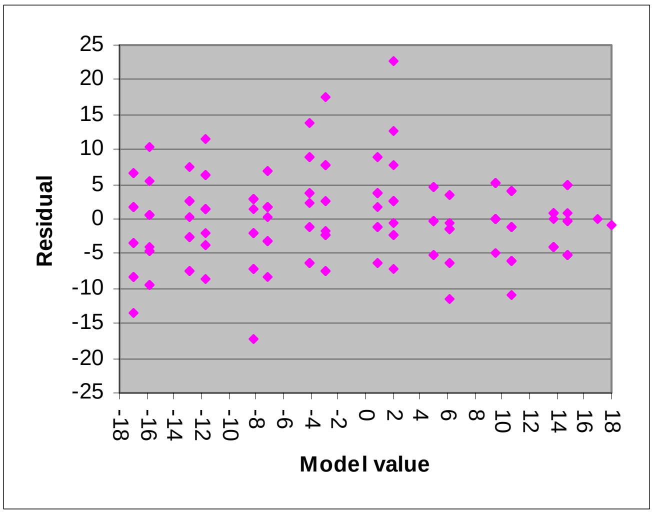


Figure 7-3 Plot of residuals against model value for sample test runs

The effect of the 5-cent spacing in the reference tones can be clearly seen in this plot (the units of the model value are pitch shift in cents from the grand mean). The usual outliers are present. There is also a slight reduction in variance for higher pitch shifts, which may be due to the simple model employed.

The conclusions from this analysis of residuals are that:

- The assumption that residual errors are normally justified is valid
- The simple model provides a good explanation for the results seen.

7.4.8 Number of replicates to be employed

Clearly, having more replicates is going to provide more certainty in the results of the testing. Using the 15 replicates analysed above, we can investigate the improvement in the standard error of the effects (which directly determines the confidence limits for the results) as more replicates are introduced. Figure 7-4 plots the standard error in the effects calculated using Equation (7-9) against number of replicates:

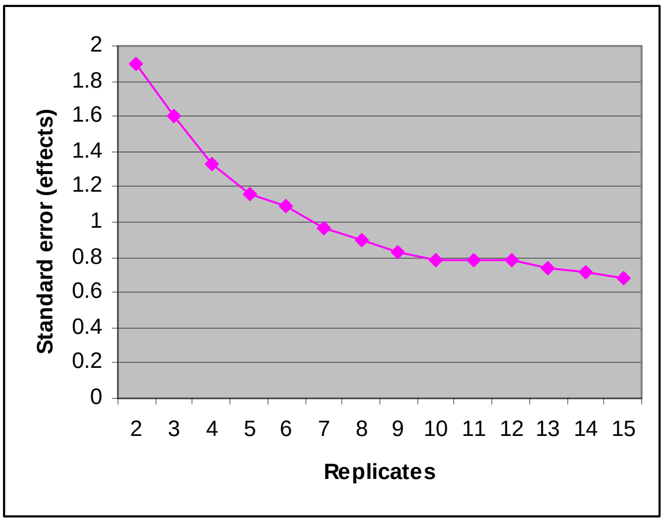


Figure 7-4 Plot of standard error against number of replicates for sample test runs

The replicates are introduced in the same order as they are presented in the analysis above. It is interesting to note that the introduction of replicates 11 and 12 (those using the Compaq internal speaker giving rise to three out of the four outliers) mean the standard error doesn’t drop, due to the poor results on this equipment.

The plot suggests that using up to ten testers for the formal testing will bring about a useful improvement in precision. Using more than about a dozen or fifteen is probably not justified.

8 FURTHER VERIFICATION OF VIRTUAL PITCH TESTS

8.1 Introduction

To further validate the design of the virtual pitch tests, two other experiments with modifications of the standard design were run by the author to validate different aspects of the test design. Both experiments use the approach of comparison against reference tones.

The first experiment, documented in section 8.2, explores in more detail the relationship between pitch shift and upper partial spacing. The linear relationship demonstrated in these tests validates the range of values for octave nominal tuning and upper partial spacing used in the tests. The tests produced numerical results in good agreement with those from the standard design, providing confirmation of the analysis approach.

The second experiment documented in section 8.3 explores the effect of the amplitude of superquint and octave nominal on virtual pitch. The test results show that the effect is not very significant, so that the work done to establish typical partial amplitudes for bells, although improving the validity of the test results for bells, is not critical to the test results. This experiment also shows that virtual pitch effects even survive complete absence of one or both partials, showing that it is the whole set of equally-spaced partials that gives rise to the virtual pitch effect, not any one individual partial.

8.2 Regression tests for the upper partial effect

To provide further validation of the analysis approach from the four-factor analysis of variance, the effect of a single factor on pitch shift was investigated in more detail via tests conducted by the author. The hypothesis to be tested in this experiment was that of a linear relationship between the pitch shift and the octave nominal values. A set of 16 test waveforms was created with the upper partials changing in steps between the low and high values used in the four-factor tests. The test nominal was 1000Hz. The parameters chosen for the test sounds are given in Table 8-1. All partials except the upper partials took the relative frequencies given them in the reference files in the virtual pitch tests. The upper partial frequencies were calculated from the octave nominal using the model established in chapter 5 (Table 5-4). All amplitudes and decay times were as for the virtual pitch tests.

Partial	**Frequency (Hz)**	**Cents to nominal**
Hum	250	-2400
Prime	500	-1200
Tierce	601.5125	-880
Quint	771.1054	-450
Nominal	1000	
Middle 1	1189.207	300
Middle 2	1278.247	425
Middle 3	1357.388	529
Octave Nominal	1931.873 - 2094.588	1140 - 1280 in 15 equal steps

Table 8-1 Partial frequencies for regression test sounds

The perceived pitches of these test waveforms were matched against a set of reference tones as for the four-factor test, using the same test software and procedure. A linear regression was done between the pitch shifts calculated from the nominals of the reference sounds, and the octave nominals of the test waveforms, to establish the relationship between them.

8.2.1 Experimental results

Eight replicates of 16 tests were run, i.e. 128 tests in all, on four different types of equipment; a Dell 2400 PC with external speakers, Goodman Pro CD 3100 headphones, a Dell E521 PC with external speakers, and an HP nc6400 laptop with internal speakers. Table 8-2 below gives the pitch shifts in cents observed on each test run.

Octave Nominal (cents)	**Dell 2400 1**	**Dell 2400 2**	**Good-man 1**	**Good-man 2**	**Dell E521 1**	**Dell E521 2**	**HP laptop 1**	**HP laptop 2**
1140	-10.5	-10.5	-10.5	-10.5	-13.5	-13.5	-4.5	-10.5
1149.333	-10.5	-13.5	-7.5	-7.5	-10.5	-10.5	-7.5	-7.5
1158.667	-7.5	-10.5	-7.5	-4.5	-16.5	-10.5	-7.5	-10.5
1168	-10.5	-10.5	-7.5	-7.5	-10.5	-7.5	-4.5	-10.5
1177.333	-7.5	-7.5	-7.5	-4.5	-10.5	-10.5	-7.5	-7.5
1186.667	-10.5	-10.5	-1.5	-4.5	-1.5	-7.5	-4.5	1.5
1196	-7.5	-7.5	-4.5	-1.5	1.5	-4.5	-7.5	-1.5
1205.333	4.5	1.5	-1.5	1.5	4.5	4.5	1.5	-1.5
1214.667	7.5	-1.5	4.5	-1.5	7.5	4.5	-1.5	-4.5
1224	1.5	-4.5	-1.5	1.5	4.5	4.5	4.5	7.5
1233.333	7.5	4.5	4.5	4.5	4.5	7.5	1.5	4.5
1242.667	7.5	10.5	7.5	7.5	1.5	4.5	4.5	4.5
1252	10.5	10.5	4.5	4.5	10.5	10.5	10.5	1.5
1261.333	10.5	10.5	4.5	7.5	4.5	10.5	4.5	7.5
1270.667	13.5	1.5	10.5	10.5	10.5	13.5	7.5	7.5
1280	13.5	13.5	7.5	7.5	13.5	13.5	1.5	4.5

Table 8-2 Test results for regression test

These results are shown graphically in Figure 8-1, together with the regression fit explained below. The spacing between the nominals in the reference files was 3 cents and this granularity can be seen in the plot:

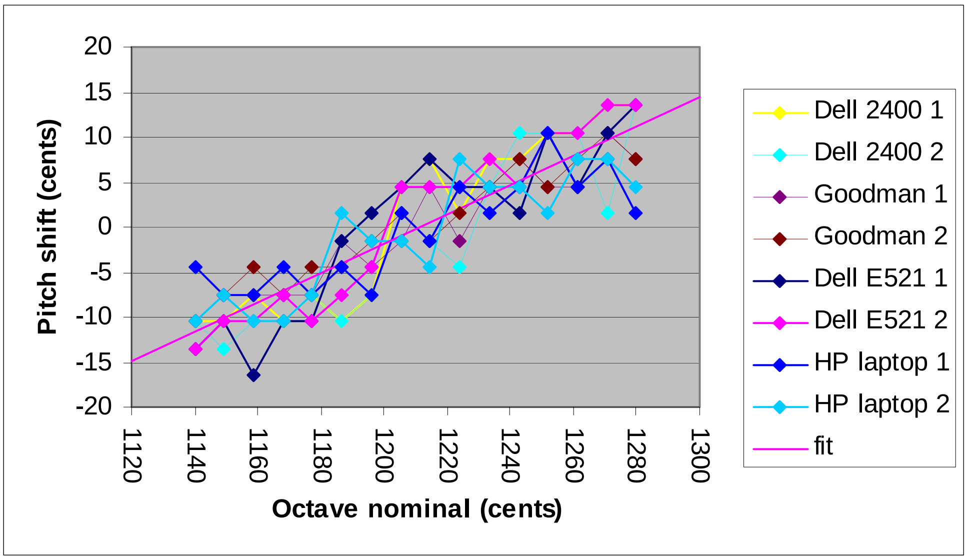


Figure 8-1 Pitch shift vs. Octave Nominal for the regression experiment

The above plot suggests that there is no reason not to suppose a linear relationship between octave nominal and pitch shift. A suitable model for a regression analysis in this case is:

$$y_i = \beta_0 + \beta_1 \cdot x_i + \varepsilon_i$$

where y_i is the measured value, β_0 (the intercept) and β_1 (the slope) are model parameters to be established, x_i is the octave nominal value (which ranges over the set of sixteen values in Table 8-2) and ε_i represents the effect of experimental error and random factors. The results of the regression analysis (with 95% confidence intervals) are:

$$\beta_0 = -198.7 \pm 19.6$$
$$\beta_1 = 0.164 \pm 0.016$$

A normal probability plot of the residuals against this model appears as Figure 8-2:

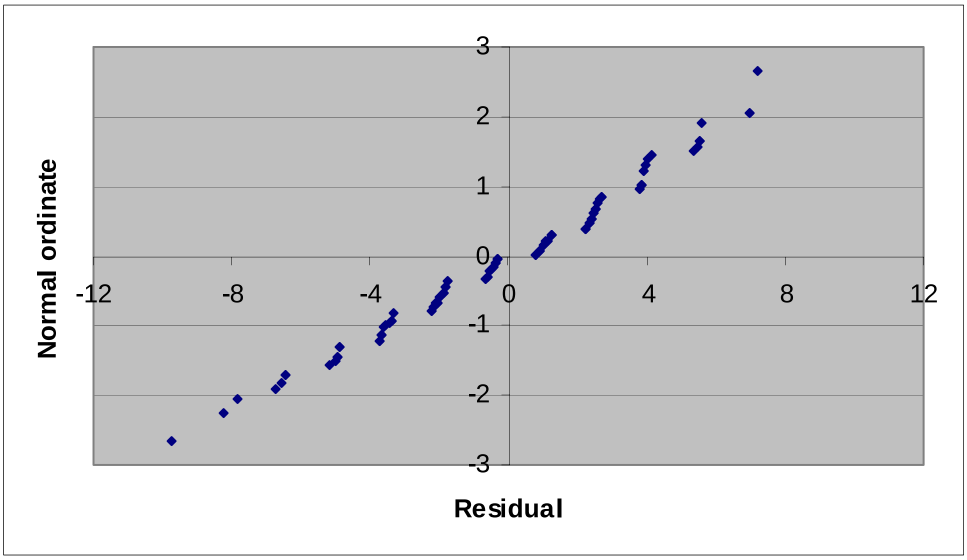


Figure 8-2 Normal plot of regression test results

Based on this plot, there is no reason not to suppose that the residuals are normally distributed, and that therefore the linear relationship is an adequate model for the data.

8.2.2 Validation of models and analysis techniques

The actual results of the regression above give good confirmation of the analysis and models being used. The dependence of pitch shift on the octave nominal is broadly linear over this range of values. Therefore, this range is appropriate for use in the testing.

The total pitch shift in cents for an octave nominal change from 1140 cents to 1280 cents according to this experiment is 23.0 cents, at a nominal frequency of 1000Hz. In section 6.4, another set of virtual pitch tests by the author was reported on. These were done to investigate the effect of equipment quality. The tests in section 6.4 used the same two extreme octave nominal values, but were analysed using the four-factor approach. The average upper partial effect (as shown in Figure 6-3) for six test runs at 1000Hz was 23.6 cents. This is in excellent agreement with the result obtained in the regression experiment, and is confirmation that the models with their associated calculations have been set up correctly.

The test sounds created for this experiment are used again in section 10.2.3 below.

8.3 Effect of partial amplitude on virtual pitch

To ascertain the effect on pitch shift of changes in partial intensity, a further experiment was designed and conducted by the author. The hypothesis to be tested was whether changes in the amplitude of the superquint or octave nominal affected virtual pitch shifts. The experiment was an adaptation of the design of the experiment in the previous section.

Salient features of the design were:

- Test sounds with a nominal frequency of 1000Hz were used, as in the previous section
- The upper partials were assigned two sets of values, corresponding to an octave nominal of 1140 cents and 1280 cents
- The amplitude of the superquint and octave nominal were each independently presented at 0%, 33%, 67% and 100% of the amplitude used in the virtual pitch tests
- All other partial frequencies and amplitudes were as in section 8.2
- The experiment used the approach of comparison against reference tones.

The experiment design comprised 16 test waveforms constructed as follows in Table 8-3:

Test	Superquint amplitude	Octave Nom. amplitude	Octave nom. cents
t1	0%	0%	1280
t2	0%	33%	1140
t3	0%	67%	1140
t4	0%	100%	1280
t5	33%	0%	1140
t6	33%	33%	1280
t7	33%	67%	1280
t8	33%	100%	1140
t9	67%	0%	1140
t10	67%	33%	1280
t11	67%	67%	1280
t12	67%	100%	1140
t13	100%	0%	1280
t14	100%	33%	1140
t15	100%	67%	1140
t16	100%	100%	1280

Table 8-3 Test parameters for amplitude experiment

This design is one half fraction of a full experiment design for two variables at four levels and one variable at two levels. The half fraction is orthogonal, so that statistical analysis of the results is straightforward. Examples of the spectra of the test waveforms follow as Figures 8-3 to 8-6:

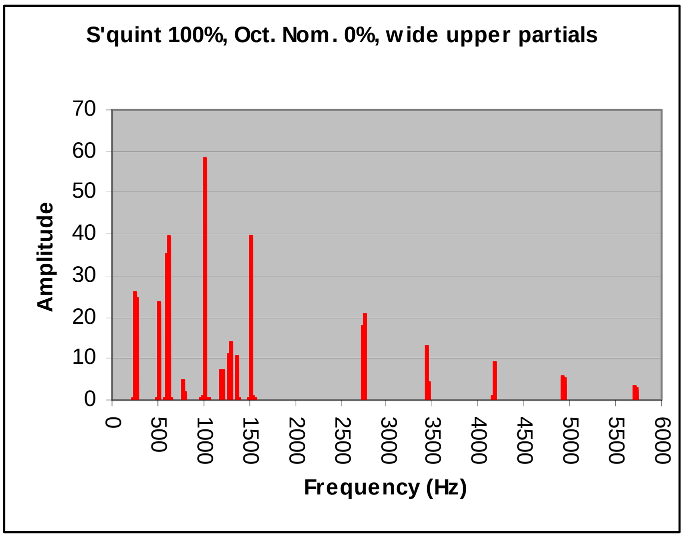


Figure 8-3 Spectrum of amplitude test waveform 13

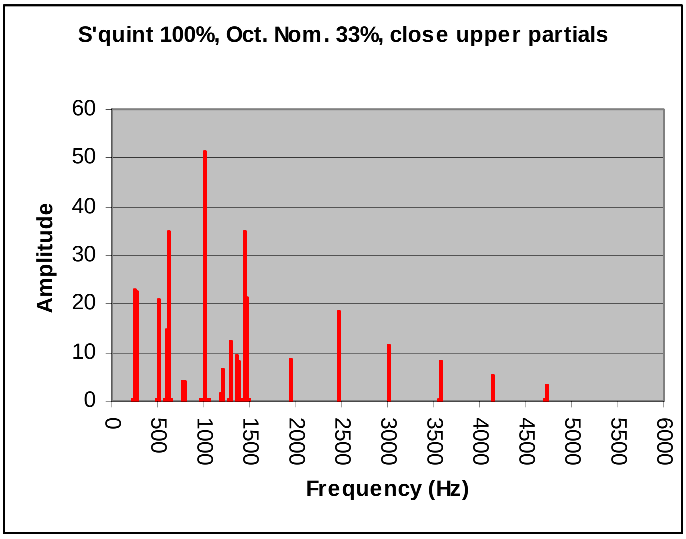


Figure 8-4 Spectrum of amplitude test waveform 14

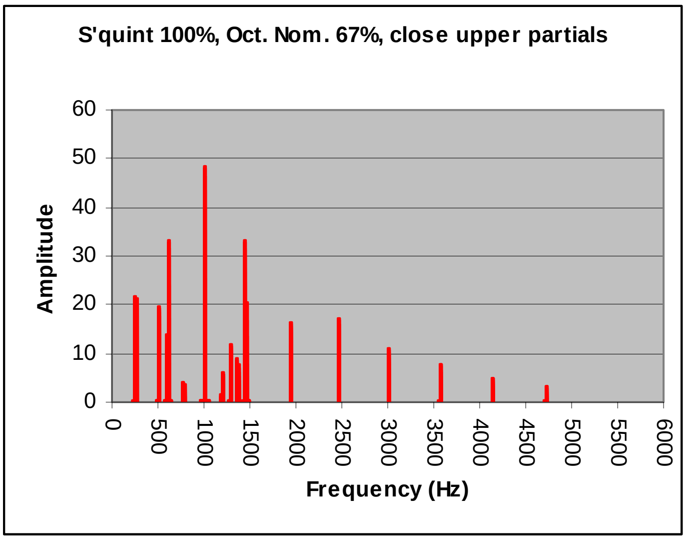


Figure 8-5 Spectrum of amplitude test waveform 15

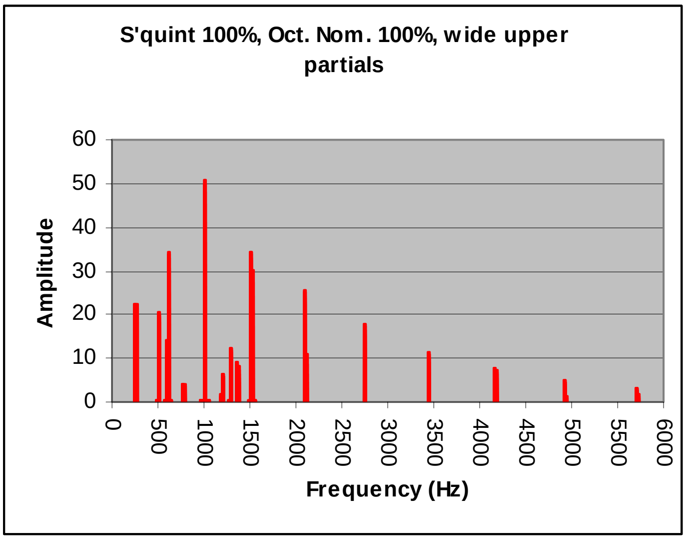


Figure 8-6 Spectrum of amplitude test waveform 16

As well as the growing amplitude of the octave nominal at around 2000Hz, the stretched upper partials in Figures 8-3 and 8-6 can be seen compared with the closer upper partials in Figures 8-4 and 8-5.

Sixteen replicates of 16 tests each were performed by the author, four on each of four equipment types, as follows:

- Dell 2400 PC external speakers
- HP nc6400 laptop internal speakers
- Sennheiser HD555 headphones
- Dell E521 PC external speakers.

The test runs produced a total of 256 individual test results.

The results were analysed by defining contrasts in such a way as to separate out the effect of superquint and octave nominal amplitude and octave nominal frequency on the pitch shift. Each contrast was chosen to isolate the effect of interest while averaging over the other effects. If $\bar{y}_1$, $\bar{y}_2$, … , $\bar{y}_{16}$ are the average pitch shifts for each of the treatments t1, t2, … , t16 and sq_0, sq_{33}, sq_{67}, sq_{100}, on_0, on_{33}, on_{67}, on_{100} are the eight contrasts of interest, then:

$$
\begin{aligned}
sq_0 &= \frac{1}{2}(\bar{y}_1 + \bar{y}_4 - \bar{y}_2 - \bar{y}_3) & on_0 &= \frac{1}{2}(\bar{y}_1 + \bar{y}_{13} - \bar{y}_5 - \bar{y}_9) \\
sq_{33} &= \frac{1}{2}(\bar{y}_6 + \bar{y}_7 - \bar{y}_5 - \bar{y}_8) & on_{33} &= \frac{1}{2}(\bar{y}_6 + \bar{y}_{10} - \bar{y}_2 - \bar{y}_{14}) \\
sq_{67} &= \frac{1}{2}(\bar{y}_{10} + \bar{y}_{11} - \bar{y}_9 - \bar{y}_{12}) & on_{67} &= \frac{1}{2}(\bar{y}_7 + \bar{y}_{11} - \bar{y}_3 - \bar{y}_{15}) \\
sq_{100} &= \frac{1}{2}(\bar{y}_{13} + \bar{y}_{16} - \bar{y}_{14} - \bar{y}_{15}) & on_{100} &= \frac{1}{2}(\bar{y}_4 + \bar{y}_{16} - \bar{y}_8 - \bar{y}_{12})
\end{aligned}
$$

It should be clear from the experiment design how each of these contrasts isolates the effect of interest. The average pitch shift s_{av} across all equipment types and partial amplitudes is:

$$s_{av} = \frac{1}{8}(\bar{y}_1 + \bar{y}_4 + \bar{y}_6 + \bar{y}_7 + \bar{y}_{10} + \bar{y}_{11} + \bar{y}_{13} + \bar{y}_{16} - \bar{y}_2 - \bar{y}_3 - \bar{y}_5 - \bar{y}_8 - \bar{y}_9 - \bar{y}_{12} - \bar{y}_{14} - \bar{y}_{15})$$

95% confidence intervals on the above quantities were calculated in a similar fashion to the calculations in section 7.4.5, modified because each contrast only involves four means, not all 16. Values of each quantity calculated from the test results were as follows (Table 8-4):

Quantity	Interpretation	Value
sq_0	Pitch shift due to upper partial change with 0% superquint	15.47 ± 2.52
sq_{33}	Pitch shift due to upper partial change with 33% superquint	16.87 ± 2.52
sq_{67}	Pitch shift due to upper partial change with 67% superquint	17.53 ± 2.52
sq_{100}	Pitch shift due to upper partial change with 100% superquint	17.44 ± 2.52
on_0	Pitch shift due to upper partial change with 0% oct. nom.	14.81 ± 2.52
on_{33}	Pitch shift due to upper partial change with 33% oct. nom.	16.31 ± 2.52
on_{67}	Pitch shift due to upper partial change with 67% oct. nom.	17.81 ± 2.52
on_{100}	Pitch shift due to upper partial change with 100% oct. nom.	18.37 ± 2.52
s_{av}	Pitch shift due to upper partial change for all treatments	16.83 ± 2.52

Table 8-4 Pitch shifts for partials at different amplitudes

Displayed graphically, the shifts at different partial amplitudes appear as shown in Figure 8-7. In this figure, the error bars are the 95% confidence intervals listed above:

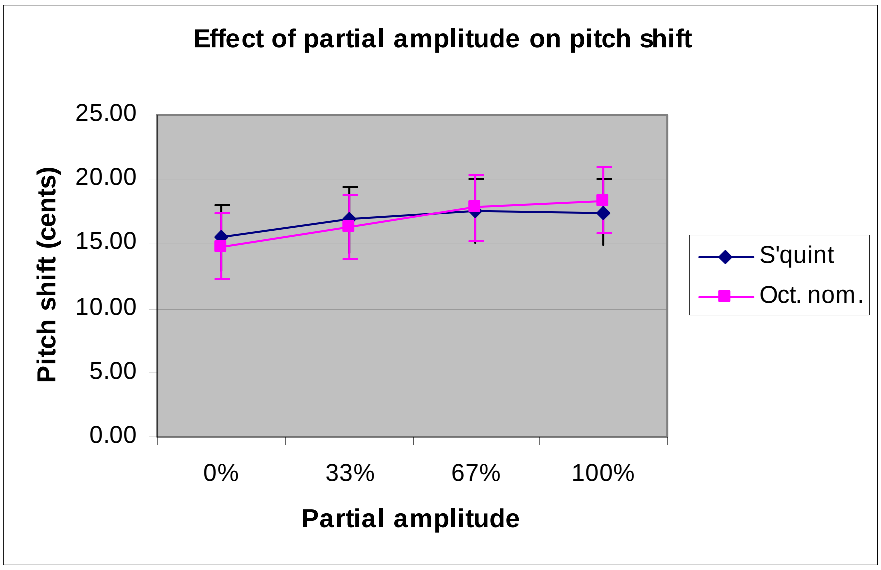


Figure 8-7 Pitch shifts for partials at different amplitudes

Though this chart suggests there is a slight increase in pitch shift due to increases in partial amplitude from 0% to 100%, the change is within the confidence interval and therefore is not statistically significant.

The fact that the virtual pitch shifts still exist even when both superquint and octave nominal have zero amplitude is remarkable and interesting. It is predictable from other work in the literature as discussed in section 2.4, and gives confirmation that the whole series of equally spaced rim partials contribute to the virtual pitch effect, not just the lowest three in the series. Inspection of the average shifts for individual pieces of equipment plotted in Figure 8-8 lends weight to this interpretation. These shifts were calculated by averaging out the partial amplitude effects leaving only the upper partial effect for each of the four equipment types:

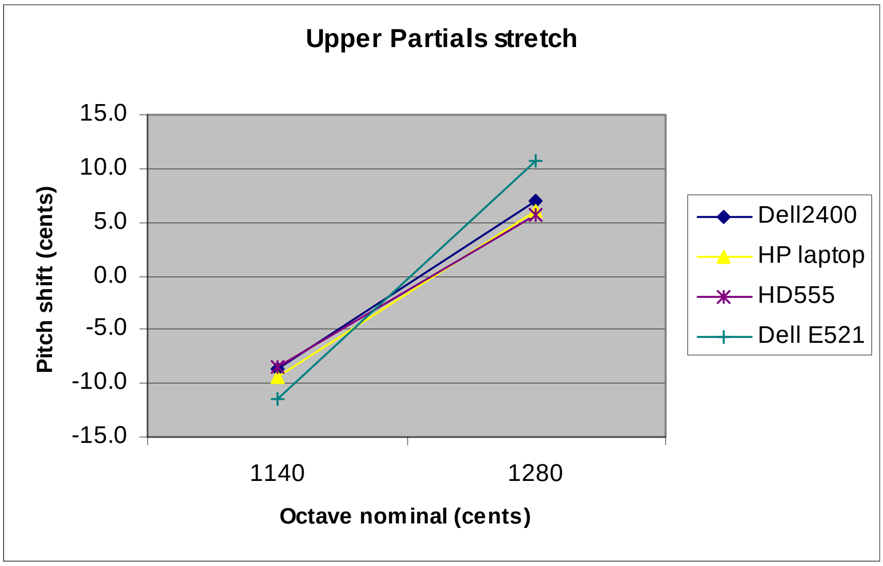


Figure 8-8 Shift due to upper partials on four types of equipment

The shifts measured on the Dell E521 external speakers are noticeably greater than on the other three equipment types. These external speakers were audibly better quality than the other equipment used. It is suggested that the better high frequency response of these speakers offsets the reduced amplitude of the superquint and octave nominal, allowing the partials higher in frequency than the octave nominal to have more influence on the perceived pitch.

8.3.1 Summary of effect of partial amplitude

The experiment documented in this section shows that, although partial amplitude has a minor effect on virtual pitch shifts, changes in amplitude between 33% and 100% of the typical value for superquint and octave nominal do not have a statistically significant effect on virtual pitch shifts. Therefore, although typical partial amplitudes and decay envelopes were used to construct the test and reference waveforms used in the virtual pitch tests, the results of these tests will be relatively insensitive to partial amplitude.

The other discovery from this experiment, that virtual pitch in bell-like sounds survives the removal of superquint and octave nominal, was unexpected, although trials leading to this experiment design involving the removal of superquint or octave nominal from real bell sounds using notch filters also showed that the strike pitch survived the modification to the sound. The inference is that all the rim partials contribute to strike pitch formation in bells, not just the lowest two or three in the series.

The author’s practical experience of bell tuning and strike pitch also suggests that partial amplitude is not a significant factor in the pitch of bells.

9 VIRTUAL PITCH TEST RESULTS

9.1 Introduction

The experiment in the second half of chapter 4 showed that, for bell sounds with nominal frequencies between 500 Hz and 2000 Hz, the majority of listeners perceived pitches primarily determined by the nominal. Outside this range, there is ambiguity, with pitch perception possible based on other partials.

The various trials and preliminary tests documented in the previous two chapters have suggested that:

- Shifts in pitch away from the half nominal are seen due to virtual pitch effects
- These shifts are determined mainly by changes in the tuning of the upper partials, with some shift also due to prime and tierce
- Interactions between prime, tierce and upper partial tuning do not affect pitch shift, i.e. a simple model will suffice.

The purpose of the experiments documented in this chapter is to establish whether the above results are true for a greater number of test subjects, over the range of nominal frequencies. The tests comprised 9 sets, at nominal frequencies from 315 Hz to 2000 Hz. Each test set consisted of 16 individual tests. The detailed design of the tests is described in the previous two chapters. This chapter documents the results.

9.2 Experimental environment and procedure

Three test subjects, including the author, carried out the tests on an HP nc6400 laptop using high quality headphones (Sennheiser HD555). Further test subjects were invited to volunteer via requests on three internet newsgroups subscribed to by bellringers, and via a letter to the bellringer’s weekly journal *The Ringing World*. The requests asked for volunteers with ‘A musical ear, a PC or Mac with sound, and a broadband connection’. 27 additional people carried out tests in response to these requests, for a total of 30 subjects. Some completed tests at all nine frequencies, others a subset of all tests. The tests were presented in such a way as to encourage all subjects to carry out the test at 1000Hz first. This was done deliberately to encourage a high number of responses to this test, to allow an assessment to be made of the power of the tests with the greatest number of users. The frequency of 1000Hz was chosen because it was near the middle of the range where pitch is based on nominal only, i.e. ambiguity over which partial(s) determine pitch is unlikely.

The 27 volunteers took the tests over a six-week period from mid October to end November 2007. Following the initial announcements, further requests were posted two weeks into the test period to encourage further volunteers to participate. Periodic chasing emails were sent out to all subjects who had completed at least one test to encourage them to attempt more. These emails and a progress chart on the test website encouraged subjects to try tests at nominal frequencies for which the lowest number of results had been submitted. Test results were analysed as they were received (to ensure that the tests were working as planned). No discussion took place with any test subject about their individual results until the tests were complete.

The total number of test sets completed at each frequency is shown in Figure 9-1.

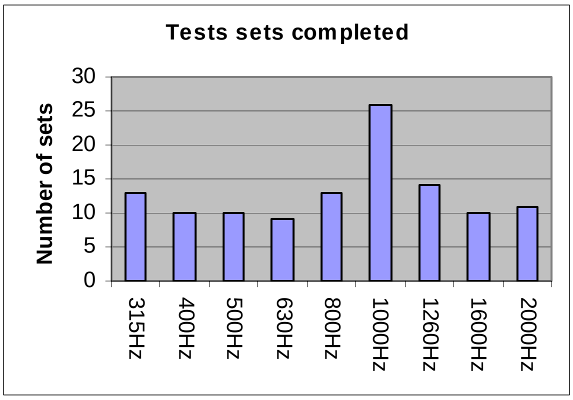


Figure 9-1 Virtual pitch test sets completed

Sufficient test sets were received at every frequency to ensure statistically significant results. In all, 116 sets of test results were received, giving a total of 1,856 individual tests. All results submitted have been included in the analysis.

The equipment used by the test subjects, as declared in their response to the tests, was as follows:

Equipment	**No. of test sets**
Good headphones in a quiet environment	36
Cheap headphones, or in a noisy environment	7
Loudspeakers	73
Total	**116**

9.3 Detailed analysis of test results at 1000Hz

As mentioned in the introduction, the high number of test results for the test at 1000Hz allows an examination of the power of the tests. The results for the tests at this frequency are examined in detail here to illustrate the approach to be used at all frequencies.

The test results were compared against a simple model, with a linear relationship for each factor and no interactions between factors. Figure 9-2 below is a normal probability plot for the residuals against this model:

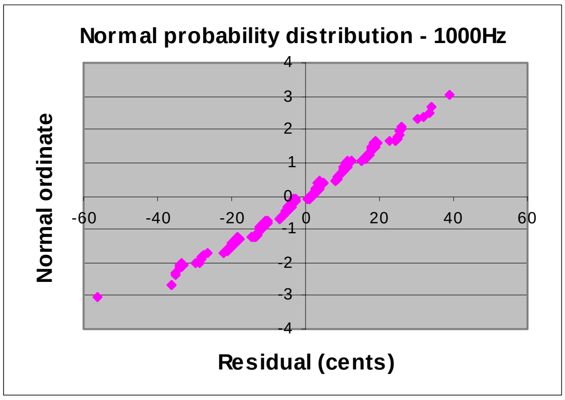


Figure 9-2 Normal plot – 1000Hz virtual pitch test

The overall shape of the plot is linear, with one outlier at -56 cents. There is a distinct repeating structure with a periodicity of about 7.5Hz. The reference sounds for this test were spaced 7.5Hz apart which explains the granularity in the results. Based on this plot, there is no reason to suppose that the test residuals are not normally distributed.

Figure 9-3 below is a plot of cumulative mean square error in the measurements against the number of result sets included. Mean square error is calculated using Equation 7-9 from section 7.4.5. The results are plotted in the order in which they were received, i.e. the ordering in this plot is essentially arbitrary.

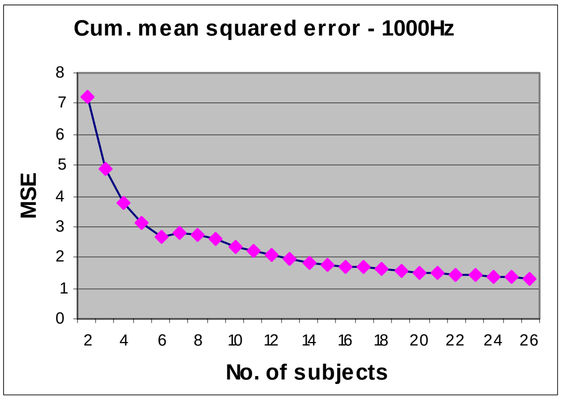


Figure 9-3 Mean square error – 1000Hz virtual pitch test

The line does not fall monotonically because different test subjects have a wider or narrower spread of results, determined by their ability to judge pitch accurately in the test conditions, or differences in the effects they experience. The asymptotic value for MSE suggests a practical limit to the ability of test subjects, on average, to judge pitch. For these test subjects, at this frequency, the plot suggests that increasing the number of test subjects above about 10 or 12 will not improve the usefulness of the results.

The plot in Figure 9-4 shows the residuals for each test subject against the simple model. Test subjects are charted in the same order as in the previous plot and there is a clear alignment between spread of residuals in Figure 9-4 and a rise or fall in the MSE in Figure 9-3 – most pronounced for subject 7. The outlier from subject 8 is the outlier at the lower left of the normal probability plot in Figure 9-2.

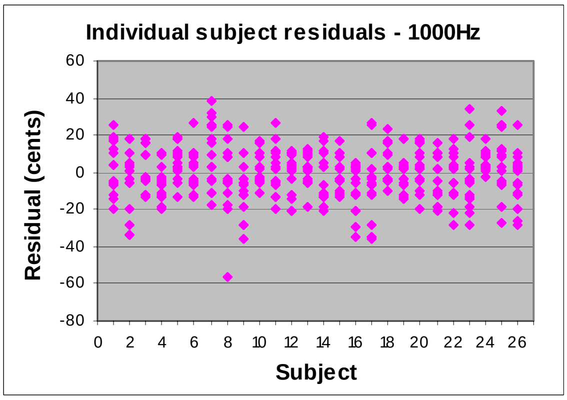


Figure 9-4 Individual residuals – 1000Hz virtual pitch test

Numerical results from the analysis of variance are presented in Table 9-1. Shaded rows are those for which the pitch shift is statistically significant as explained below.

Factors	Effect	Sum of Squares	Percent	Deg free	F value	P-value	Bonferroni conf. interval	
Total		222157.064					Low	High
A (quint)	-0.9014	84.510	0.04%	1	0.46	0.4989	-4.56	2.76
B (prime)	6.2380	4046.890	1.82%	1	21.94	0.0000	2.58	9.90
C (tierce)	8.3293	7215.279	3.25%	1	39.11	0.0000	4.67	11.99
D (upper partials)	36.0216	134946.041	60.74%	1	731.48	0.0000	32.36	39.68
AB	0.6130	39.078	0.02%	1	0.21	0.6456	-3.05	4.27
AC	0.1082	1.217	0.00%	1	0.01	0.9353	-3.55	3.77
AD	0.2524	6.626	0.00%	1	0.04	0.8498	-3.41	3.91
BC	0.4687	22.852	0.01%	1	0.12	0.7251	-3.19	4.13
BD	-2.9928	931.505	0.42%	1	5.05	0.0252	-6.65	0.67
CD	-1.3341	185.111	0.08%	1	1.00	0.3171	-5.00	2.33
ABC	-0.9736	98.573	0.04%	1	0.53	0.4652	-4.63	2.69
ABD	-2.2716	536.674	0.24%	1	2.91	0.0889	-5.93	1.39
ACD	-1.1899	147.251	0.07%	1	0.80	0.3722	-4.85	2.47
BCD	-0.5409	30.424	0.01%	1	0.16	0.6849	-4.20	3.12
ABCD	-0.8293	71.529	0.03%	1	0.39	0.5339	-4.49	2.83
Error SS		73793.505	33.22%	400				

Table 9-1 Virtual pitch test results at 1000Hz

The detailed theory behind this analysis is presented in chapter 7. In summary, the meaning of the columns is as follows:

- The factors column shows, for each of the middle 16 rows of the table, which of the four factors (quint, prime, tierce and upper partial tuning) act alone or in combination. Each row is a different treatment or combination of factors. In this notation, absence of a factor means that quint, prime, tierce and upper partials are at the lower limit of their range. Presence of the factor means that the relevant partials are at the higher limit
- The first and last rows of the table show the total sum of squares and the error sum of squares
- The effect column shows the actual pitch shift in cents for each of the factors and factor combinations. The shift of 36 cents due to change in upper partials is over 1/3 of a semitone and is musically very significant
- The sum of squares column shows the squared deviation from the treatment mean summed across the results of all test subjects

- The percentage column shows the contribution of each treatment to the total sum of squared deviations. The change in upper partials acting alone accounts for 60% of the total and is therefore the dominant effect
- The degrees of freedom for each treatment are calculated as in chapter 7 for use in calculating the F distribution
- The column headed F value is the ratio of the sum of squared deviations for each treatment to the error sum of squares. As the variances are normally distributed, this statistic will follow the F distribution with appropriate degrees of freedom
- The P-value column uses the F distribution to calculate the probability of seeing a result at least as extreme as the effect observed in this particular set of experiments, if the hypothesis of no effect were true. If the resulting value is 0.05 or less, then the effect observed is significant at the 95% level.
- The columns headed Bonferroni conf. interval give an alternate view of the significance of each treatment effect. The low and high values are the 95% confidence interval for the effect calculated as in chapter 7 using the t-distribution with Bonferroni correction. If the low and high values do not span zero, the treatment effect is significant at the 95% level.

It will immediately be seen that the two different ways of establishing significance are compatible, with the confidence interval approach a little more conservative. Treatments B, C, D and BD have P-values below 0.05, and it is only for treatments B, C and D that the 95% confidence limits do not span zero, with treatment BD as a marginal case. Use of confidence limits to establish significance has the advantage that it also provides a likely range of values for the results. The assumption of a simple model ignoring factors in combination is justified by these results.

Finally, it is useful to look at the pitch shift experienced by each individual subject for the upper partial factor, to see if all subjects experience the pitch shift - the analysis of variance reported on above looks only at the average effect experienced by all test subjects. Figure 9-5 below shows the effect due to upper partials experienced by each subject individually:

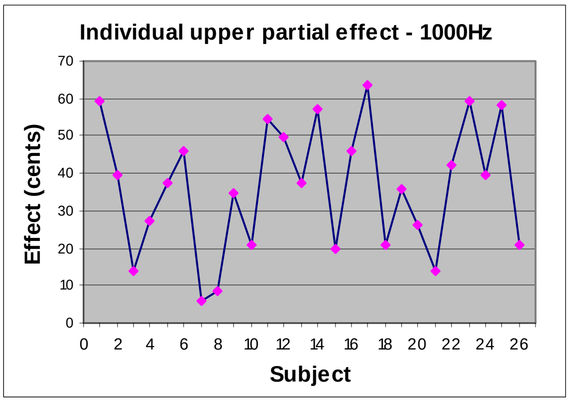


Figure 9-5 Individual upper partial effects – 1000Hz

Yet again, the order of test subjects is the same as in previous plots. Clearly, there is considerable variance in the effect experienced by the different test subjects in this test environment. It is not possible to determine from this analysis how much of the difference is due to the innate capability of each test subject to experience pitch shift or distinguish pitch accurately, and how much is due to random effects. It is notable however that the effect for all subjects is positive, leading to the conclusion that some influence on pitch due to change in upper partials is experienced by all subjects though its magnitude may vary.

The conclusions to be drawn from this detailed analysis of the results of the 1000Hz test are as follows:

- All 26 test subjects experience the pitch shift effect to a greater or lesser degree, and in the same direction
- Averaged across all test subjects, the pitch shift due to change in upper partials is highly significant both statistically and from a musical perspective
- Statistically significant but musically less significant shifts also arise from changes in the prime and tierce

- No significant shifts occur as a result of changes in the quint partial or from any factors acting in combination.

9.4 Test results at all frequencies – overview of results

The charts on the following pages (Figures 9-6 to 9-14) show the normal probability plot, mean square error, residuals and individual upper-partial effects for the tests at all nine frequencies. The plots for the 1000Hz tests, already given above, are repeated here to allow comparison.

Looking at the normal probability plots, and the individual subject residuals, there are 10 test results that could be considered to be outliers, either because they do not lie on a smooth normal probability curve, or because they are well separated from the rest of the test results. As these 10 outliers represent only 0.54% of the total number of test results, it was felt that their influence on the results of the experiment was minimal. To avoid having to define criteria for exclusion, the outliers were included in the analysis.

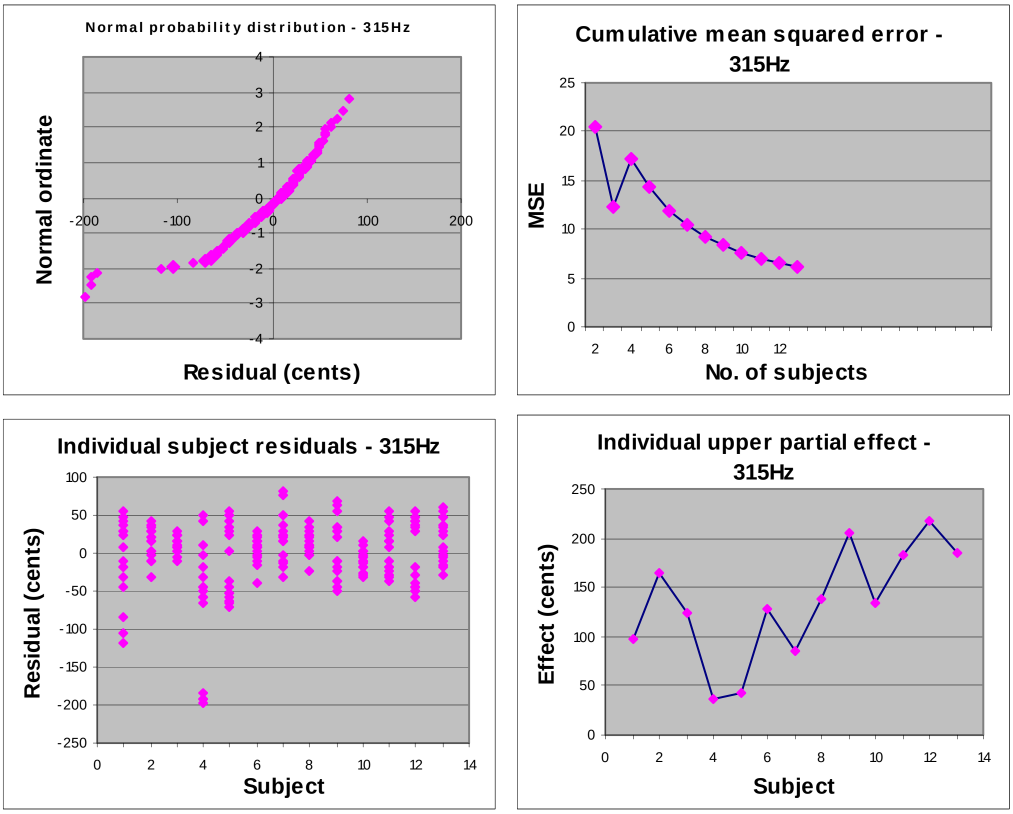


Figure 9-6 Virtual pitch test results at 315Hz

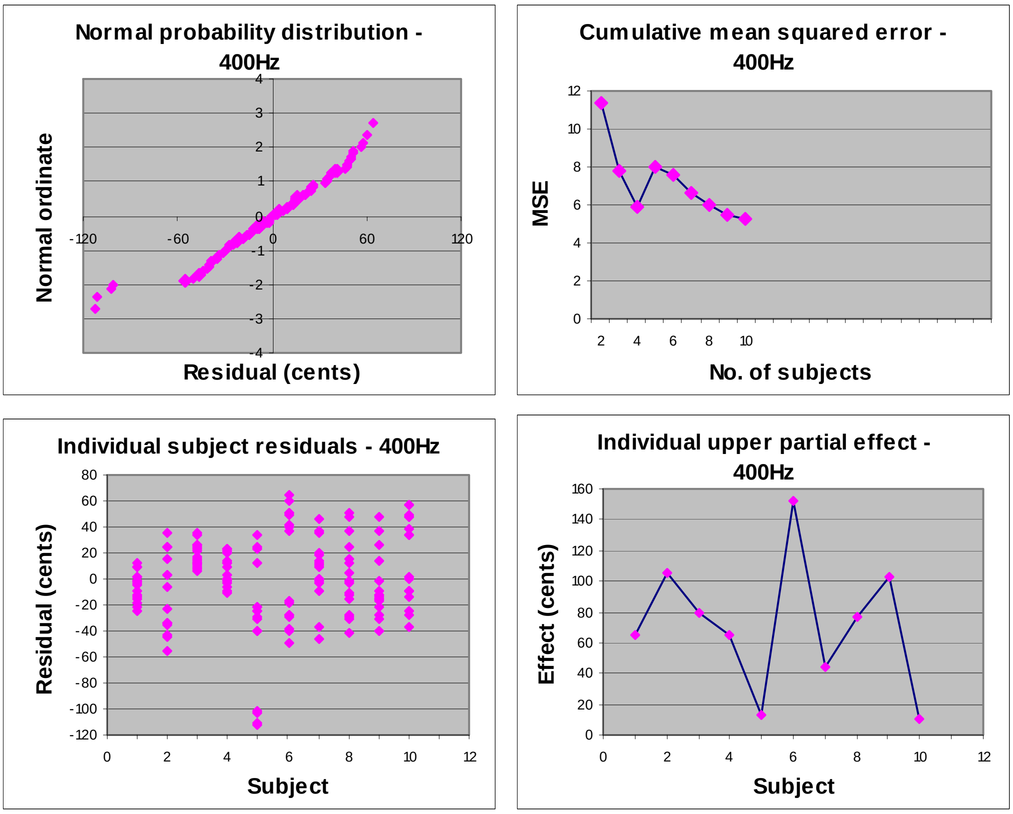


Figure 9-7 Virtual pitch test results at 400Hz

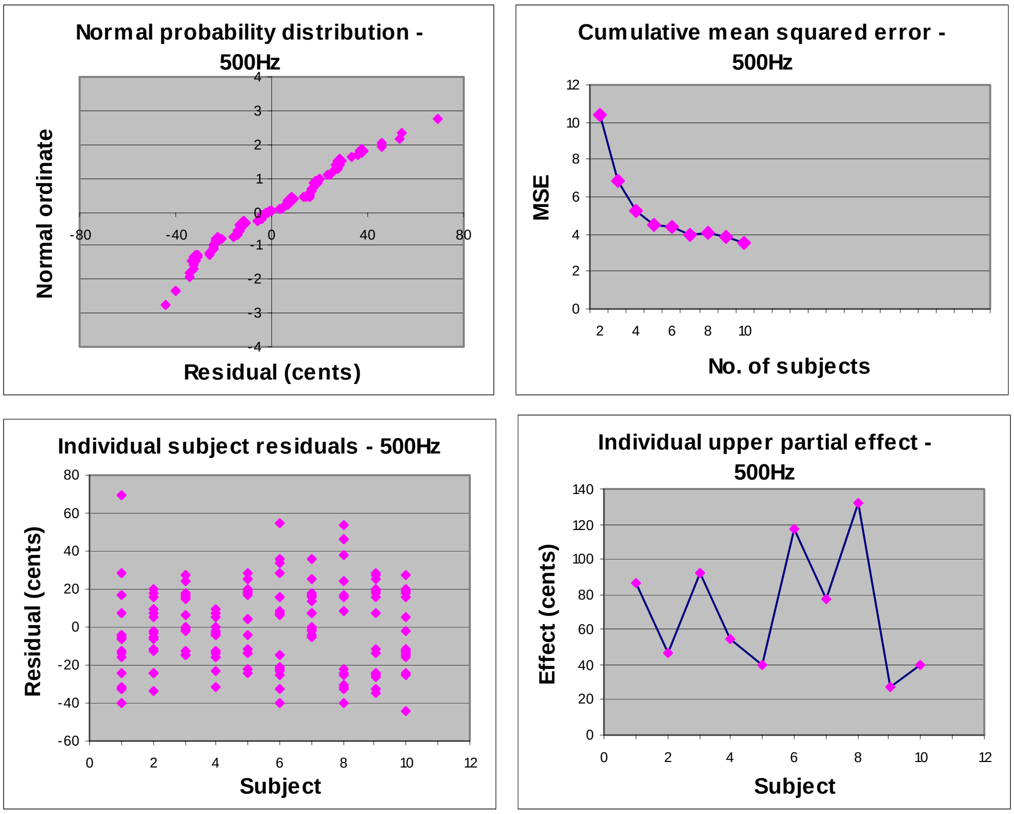


Figure 9-8 Virtual pitch test results at 500Hz

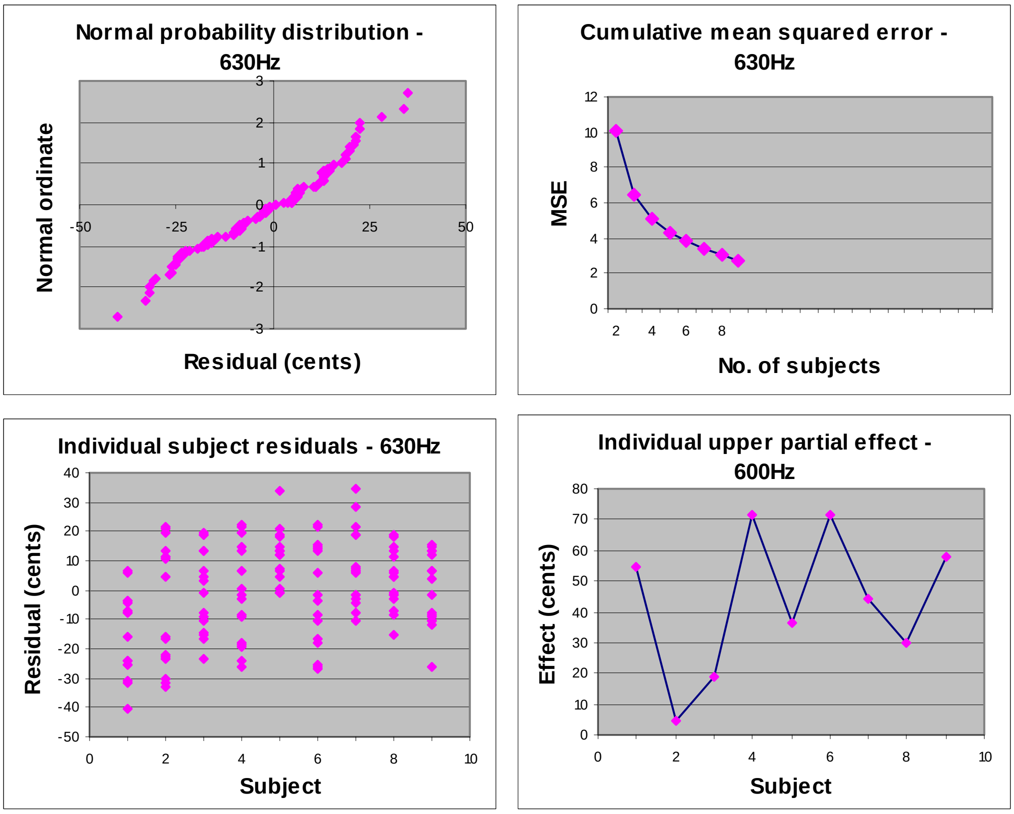


Figure 9-9 Virtual pitch test results at 630Hz

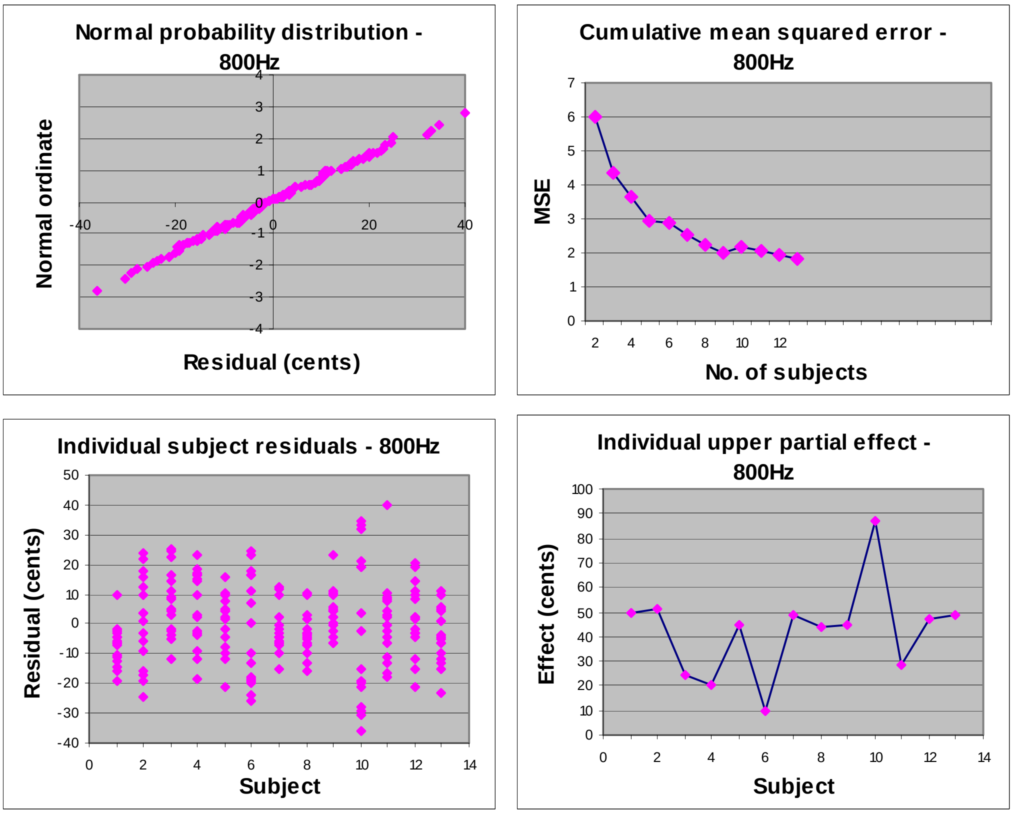


Figure 9-10 Virtual pitch test results at 800Hz

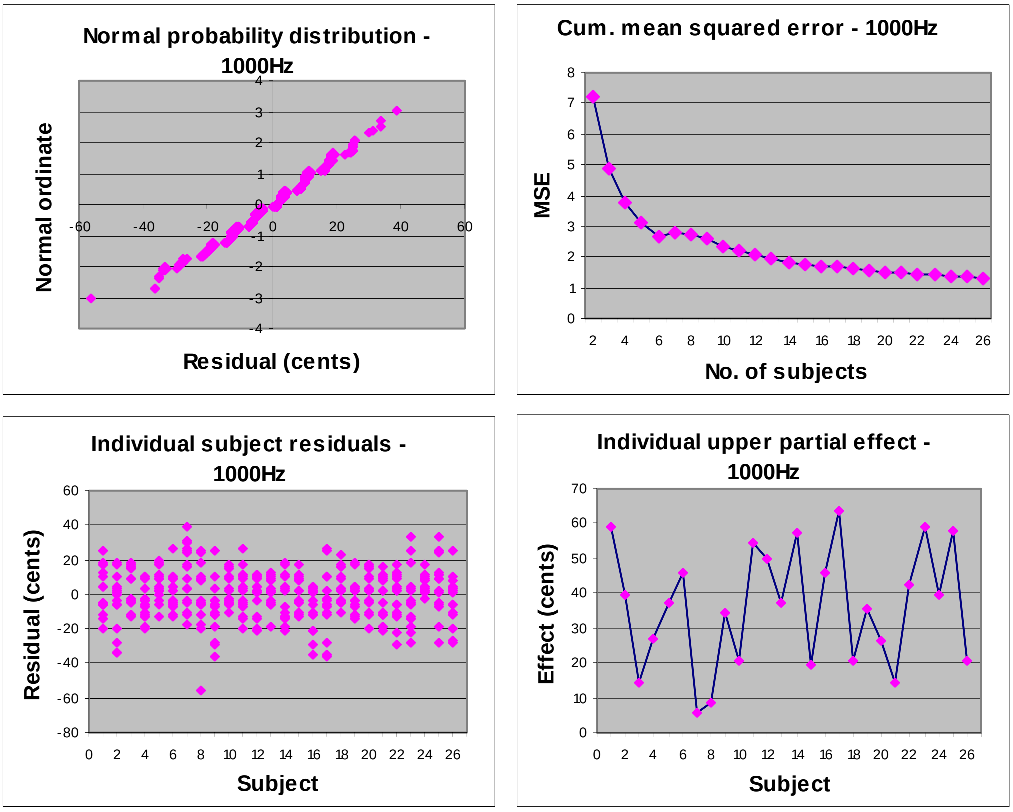


Figure 9-11 Virtual pitch test results at 1000Hz

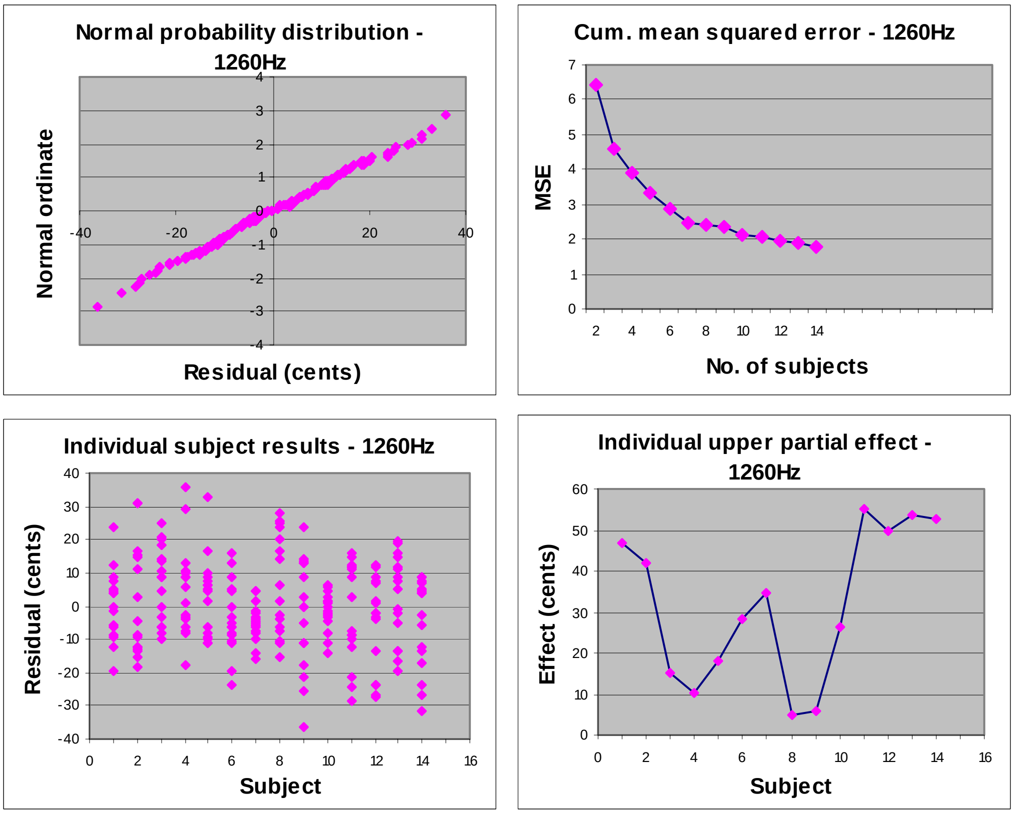


Figure 9-12 Virtual pitch test results at 1260Hz

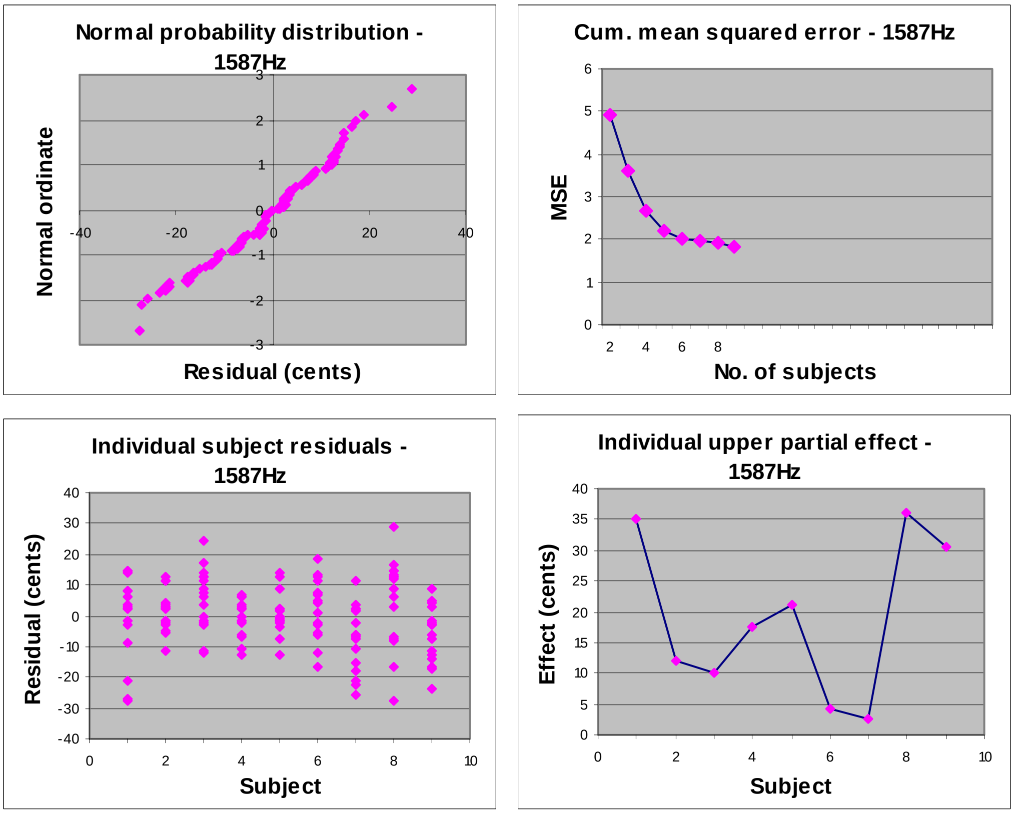


Figure 9-13 Virtual pitch test results at 1587Hz

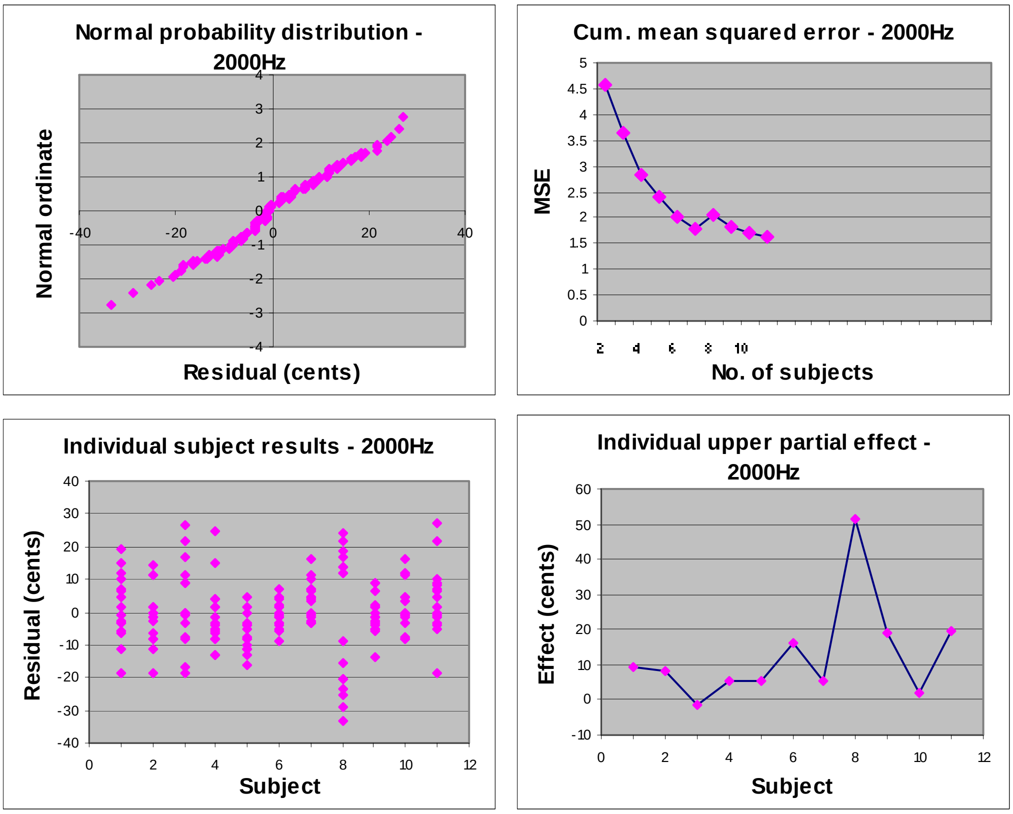


Figure 9-14 Virtual pitch test results at 2000Hz

Comparison of the normal probability plots suggests that there are departures from normality of residuals at the lowest and highest frequencies. The mean square error also fluctuates considerably with increasing numbers of test subjects at these frequencies. At frequencies in the middle of the range, residuals appear normally distributed and the mean square error decreases almost monotonically with increasing number of test subjects. An overview follows as Table 9-2:

Test set	Normal distribution plot	Mean square error plot	Individual user results
315Hz	Non-normal, significant outliers	Fluctuations	Significant outliers
400Hz	Normal, but significant outliers	Fluctuations	Significant outliers
500Hz	Normal	Steady decrease	
630Hz	Normal	Steady decrease	
800Hz	Normal	Steady decrease	
1000Hz	Normal	Steady decrease	
1260Hz	Normal	Steady decrease	
1587Hz	Normal	Steady decrease	
2000Hz	Near-normal	Fluctuations	Results for subject 8 have an atypical distribution

Table 9-2 Comparison of virtual pitch test results at different frequencies

Interpretation of the mean square error plots is of course a little informal because the shape of the plot depends on the ordering of test subjects.

The characteristics of the test results at different frequencies shown in Table 9-2 are directly explicable from the second experiment documented in chapter 4, showing the different pitch perception mechanisms at different frequencies. A summary of the results of this experiment appears in Figure 4-10. At nominal frequencies between 500Hz and 1600Hz, pitch perception by test subjects is dominated by the nominal and is relatively unambiguous. At lower frequencies, pitch perception is ambiguous between the nominal and I-7 partials. At frequencies from 2000Hz upwards, pitch perception is ambiguous between the nominal, prime, tierce and hum partials. The ambiguity at lower and higher frequencies leads to the results in Table 9-2.

9.5 Test results at all frequencies – statistical significance

Table 9-3 below shows the P-values for the test results for all treatments and frequencies. To recap, this quantity shows the probability of the effects observed in the test results being due to chance if the hypothesis of no effect is true. A value of 0.05 or less means that the effects observed are statistically significant at the 95% level. Significant values are highlighted in the table.

Treatment	315Hz	400Hz	500Hz	630Hz	800Hz	1000Hz	1260Hz	1587Hz	2000Hz
A (quint)		0.841		0.793		0.499		0.849	
A (middle partials)			0.889		0.330		0.853		0.972
B (prime)	0.047	0.057	0.022	0.019	0.001	0.000	0.007	0.001	0.806
C (tierce)	0.256	0.070	0.032	0.001	0.000	0.000	0.000	0.001	0.000
D (upper partials)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
AB	0.927	0.524	0.625	0.312	0.990	0.646	0.595	0.567	0.551
AC	0.559	0.981	0.944	0.680	0.990	0.935	0.665	0.238	0.648
AD	0.976	0.934	1.000	0.735	0.399	0.850	0.451	0.849	0.752
BC	0.601	0.612	0.329	0.910	0.085	0.725	0.814	0.304	0.505
BD	0.878	0.494	0.443	0.349	0.038	0.025	0.091	0.037	0.461
CD	0.689	0.759	0.834	0.388	0.199	0.317	0.422	0.731	0.806
ABC	0.976	0.628	0.329	0.349	0.726	0.465	0.970	0.238	0.861
ABD	0.878	0.688	0.296	0.735	0.706	0.089	0.720	0.849	0.277
ACD	0.830	0.981	0.530	0.574	0.631	0.372	0.720	0.675	0.752
BCD	0.976	0.531	0.889	0.735	0.117	0.685	0.738	0.909	0.916
ABCD	0.735	0.714	0.576	0.970	0.846	0.534	0.853	0.970	0.420

Table 9-3 Statistical significance of virtual pitch test results at various frequencies

Notwithstanding the significant effects for treatment BD (prime and upper partials acting in combination) at 800Hz, 1000Hz and 1587Hz, the assumption will be made that factors in combination do not have a significant effect at any frequency. None of the BD treatments are significant at the 98% level.

The effect of the upper partials is highly significant at all frequencies, and prime and tierce highly significant at frequencies in the middle range. Apart from the BD interaction, no other factors or treatments come close to having a significant effect at any frequency, a powerful argument for a simple model. As a result, only effects due to prime, tierce and uppers will be further analysed.

9.6 Test results at all frequencies – magnitude of effects

The actual size of the pitch shift effect for prime, tierce and upper partials observed in these experiments is given in Table 9-4, together with the 95% confidence interval for each value.

Frequency	**Prime Effect (cents)**	**Tierce Effect (cents)**	**Upper partial Effect (cents)**	**Confidence Interval (cents)**
315Hz	12.500	7.115	134.039	17.259
400Hz	10.125	9.625	71.437	14.656
500Hz	8.250	7.750	71.500	9.915
630Hz	6.562	9.271	43.229	7.698
800Hz	6.274	8.245	42.236	5.109
1000Hz	6.238	8.329	36.022	3.661
1260Hz	4.888	10.960	31.585	4.978
1587Hz	6.319	5.903	18.819	5.049
2000Hz	0.398	8.125	12.784	4.477

Table 9-4 Virtual pitch effects in cents at various frequencies

Where the confidence interval exceeds the effect (as for Tierce at 315Hz, 400Hz and 500Hz, and Prime at all frequencies except 800Hz, 1000Hz and 1587Hz) the effect is not statistically significant at the 95% level, but is still included for the sake of completeness. To provide some context for the above figures, listeners with musical ability can judge pitch to within 5 or 10 cents. Therefore, all of the shifts due to upper partials are musically significant. Most of the shifts due to prime and tierce are not musically significant.

The effects arise as a result of changes in the relevant partials (prime, tierce or upper partials). The most useful way for practical application to present the effects is as a shift of the strike pitch in cents for a 1 cent shift in the relevant partial. The low and high values for each partial in the experiment expressed (as always) as cents relative to the nominal are:

Partial	**Low**	**High**
Prime	-1400	-1200
Tierce	-900	-800
Octave Nominal	1140	1280

Dividing the figures in Table 9-4 by these values gives the results in Table 9-5 for pitch shift in cents per 1 cent change in the relevant partial:

Frequency	**Prime**	**Tierce**	**Octave Nominal**
315Hz	0.062 ± 0.086	0.071 ± 0.173	0.957 ± 0.123
400Hz	0.051 ± 0.073	0.096 ± 0.147	0.510 ± 0.105
500Hz	0.041 ± 0.050	0.077 ± 0.099	0.511 ± 0.071
630Hz	0.033 ± 0.038	0.093 ± 0.077	0.309 ± 0.055
800Hz	0.031 ± 0.026	0.082 ± 0.051	0.302 ± 0.036
1000Hz	0.031 ± 0.018	0.083 ± 0.037	0.257 ± 0.026
1260Hz	0.024 ± 0.025	0.110 ± 0.050	0.226 ± 0.036
1587Hz	0.032 ± 0.025	0.059 ± 0.050	0.134 ± 0.036
2000Hz	0.002 ± 0.022	0.081 ± 0.045	0.091 ± 0.032

Table 9-5 Pitch shift per cent of partial change

Figure 9-15 shows these results graphically, plotted not as absolute shifts, but as pitch shift in cents per cent change in the relevant partial.

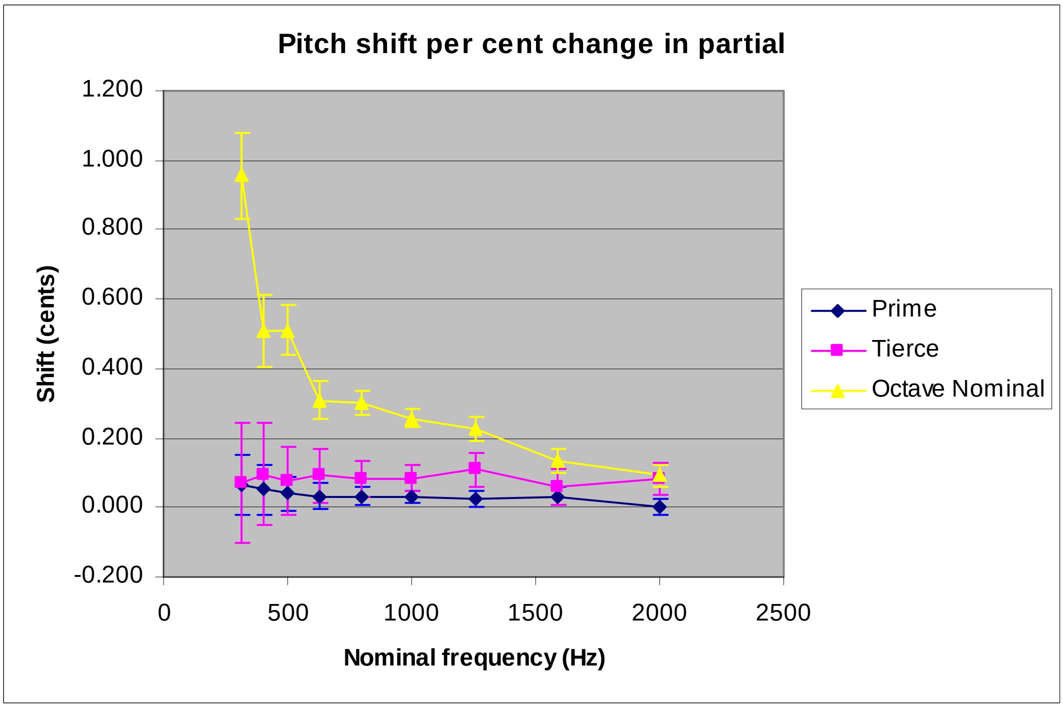


Figure 9-15 Pitch shift per cent change in partials

At the lowest nominal frequencies (315Hz and possibly 400Hz) there is, as has been shown previously, ambiguity as to whether the nominal or I-7 partial is the main determinant of pitch. If a particular subject hears a pitch based on partial I-7, then changes in octave nominal have a direct, first-order effect on pitch (because a change in octave nominal changes partial I-7, as shown in chapter 5). If the subject hears a pitch based on the nominal partial, then changes in octave nominal have an indirect, second order effect due to virtual pitch shifts. The effects observed for pitch shift against octave nominal at 315Hz and possibly 400Hz are an aggregate of the two pitch mechanisms. It is not possible to estimate the relative contribution of the two effects from the results of this experiment. However, the importance of these results is that they quantify the average effect across a number of test subjects, which is the information needed in practice.

9.7 Approximate model of pitch shifts

The model of pitch shift based on prime, tierce and octave nominal is already simple. However, for routine calculations an even simpler model suggests itself:

- The shifts due to prime and tierce are approximated by their average value over all frequencies. The effects for these two partials are sufficiently small for this approximation to be valid
- A curve can be fitted to the effects for the octave nominal. From the shape of the curve in Figure 9-15, a log fit suggests itself.

A regression fit of $\log_e$(octave nominal effect) against $\log_e$(frequency) gives a slope of -1.10012 and an intercept of 6.12396. The octave nominal effect at frequency f is therefore approximated by:

$$\begin{aligned}&\exp(6.12396 - 1.10012 \times \log_e(f))\\&= 456.66839 \times f^{-1.10012}\end{aligned}$$

The average value of the prime effect is 0.036. The average value of the tierce effect is 0.085.

Figure 9-16 shows the resulting approximations overlaid on Figure 9-15:

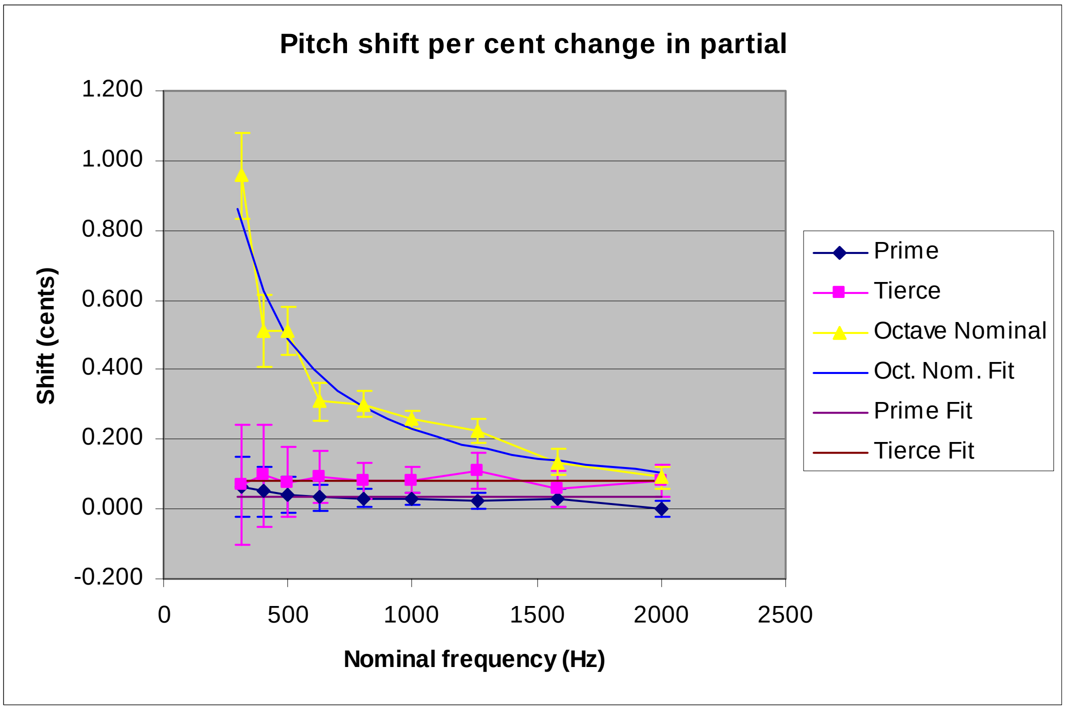


Figure 9-16 Approximate model for pitch shifts

Almost all points lie close to or within the confidence interval in the measurements. Note that no theoretical justification is being claimed for these approximations – they are just convenient shorthand used to estimate pitch shifts at an arbitrary frequency.

The baseline for the analysis of variance used to estimate these pitch shift effects had all factors at their low level. The values of these low level factors in cents relative to the nominal were:

Partial	Reference
Prime	-1400
Tierce	-900
Octave Nominal	1140

Then, for a particular bell, if p is the cents of the prime relative to the nominal, t is the cents for the tierce, o is the cents for the octave nominal and f is the nominal frequency (in the range 400Hz to 2000Hz), then the pitch shift s in cents due to virtual pitch effects is:

$$\begin{aligned} s &= (p+1400)\times 0.036 \\ &+ (t+900)\times 0.085 \\ &+ (o-1140)\times 456.668\times f^{-1.10012} \end{aligned}$$

This formula will be put to the test against actual examples in the next chapter.

9.8 Precision of test results

In chapter 6, a number of candidate designs for pitch measurement experiments were compared. Two different techniques were used in this research; the comparison of sounds with similar timbre used for the virtual pitch tests reported on in this chapter, and the method of post-vocalisation used for the experiments in chapter 4. Sufficient test results have been accumulated to make it realistic to compare the precision of the two techniques, by looking at the standard error in residuals from the two sets of experiments. Table 9-6 compares the standard errors from the two experiments, both as cents (i.e. relative values) and in absolute frequencies.

	Post vocalisation (Chapter 4)			Comparison of sounds with similar timbre		
Nominal frequency	Number of subjects	Standard error (cents)	Standard error (Hz)	Number of subjects	Standard error (cents)	Standard error (Hz)
315	17	45.6	8.4	13	45.0	8.3
396.8	15	40.1	9.3	10	33.4	7.7
500	16	55.7	16.3	10	22.6	6.6
630	15	39.4	14.5	9	16.6	6.1
793.6	16	40.4	18.8	13	13.3	6.1
1000	16	51.9	30.4	26	13.6	7.9
1259.8	15	51.8	38.3	14	13.5	9.8
1587.4	14	41.8	38.8	10	10.9	10.0
2000	17	45.9	53.7	11	10.7	12.4
2519.5	15	40.0	58.9			
3174.5	16	19.3	35.6			
4000	17	40.1	93.9			

Table 9-6 Precision of test techniques

The precision of the results is comparable at low frequencies (where, as we have seen, there is considerable ambiguity over pitch). At higher frequencies the comparison of sounds with similar timbre as used in the virtual pitch tests gives much more precise results. The comparison is shown graphically in Figures 9-17 and 9-18 below, comparing the standard errors in first cents, and then Hertz.

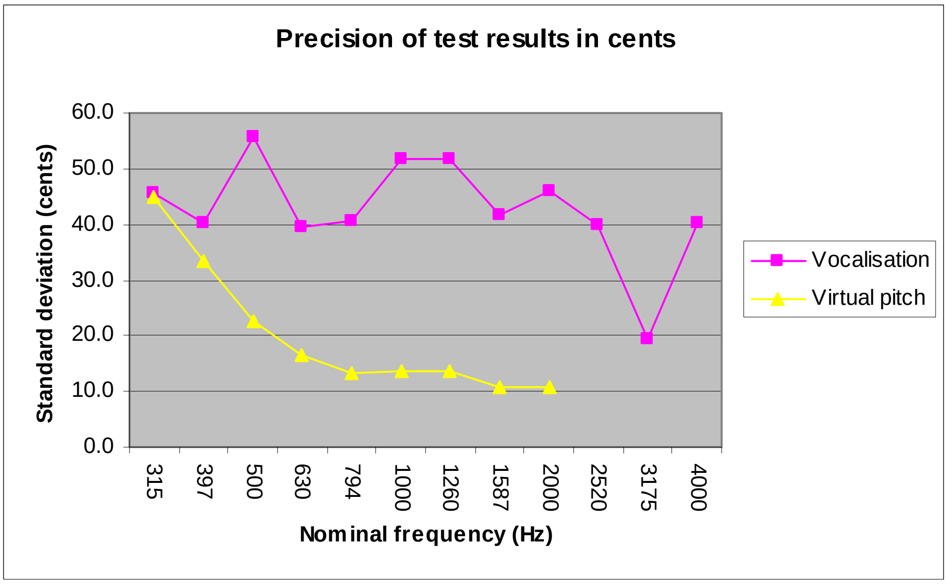


Figure 9-17 Comparison of experiment precision in cents

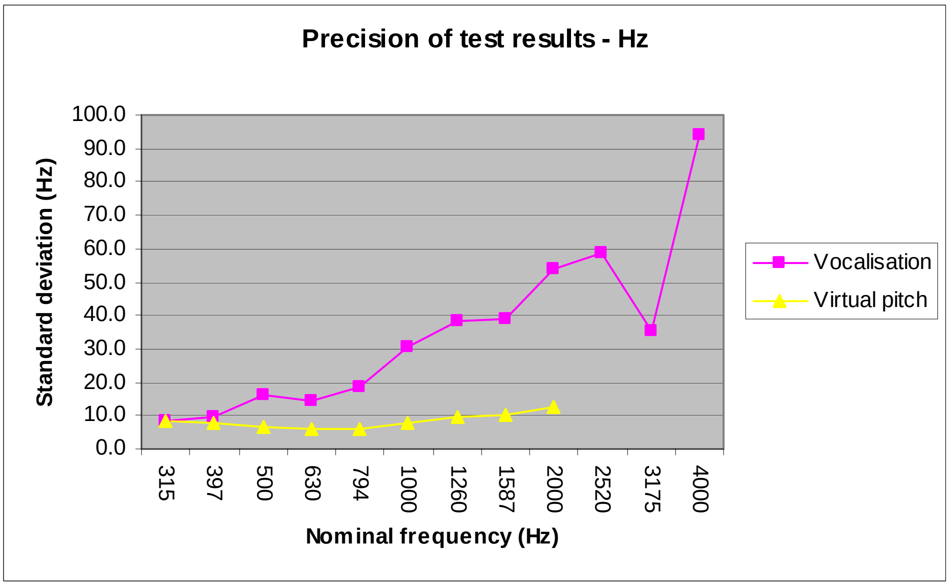


Figure 9-18 Comparison of experiment precision in Hertz

This comparison provides useful insight into the test techniques to be used for further experiments in this area. It also provides a cross check on the working practices of bell founders. Some founders claim that it is un-necessary to tune bell partials to closer than 10 cents because this is the practical limit of listeners to judge pitch. The results of the virtual pitch tests, where in the dominance region for virtual pitch the standard error in pitch ranges from 11 to 17 cents, support this practice.

9.9 Musical experience of test subjects

The 30 test subjects who carried out the formal virtual pitch tests were asked about their musical knowledge and experience. 28 of the test subjects responded to this request.

To investigate the results from each test subject, two quantities were calculated for each replicate of 16 tests at an individual nominal frequency for an individual test subject. The first calculation involved dividing each test subject's upper partial effect at a nominal frequency, by the average upper partial effect for all test subjects at that nominal frequency, to give the relative upper partial effect for that individual at that nominal frequency. This quantity measures, in relative terms, the extent to which each test subject experienced pitch shifts at that nominal frequency.

The second calculation involved dividing the standard deviation of the residuals of each test subject's 16 test results at a nominal frequency, by the standard deviation of residuals for all test subjects at that nominal frequency, to give the relative standard deviation of residuals for that subject at that nominal frequency. This quantity measures, in relative terms, the precision with which the test subject was able to judge pitch at that nominal frequency.

In addition, for each test subject the average of the relative standard deviations at all nominal frequencies was calculated and the subjects were ranked in increasing order of this quantity. It was assumed that this ranking gave an overall indication of the ability of each test subject to judge pitch accurately under the experimental conditions.

Figure 9-19 below shows the relative standard deviations for all tests and test subjects, with the test subjects ranked in order of their ability to judge pitch, with the most precise on the left. Note that the identification of the test subjects in this plot is different from that in figures 9-6 to 9-14.

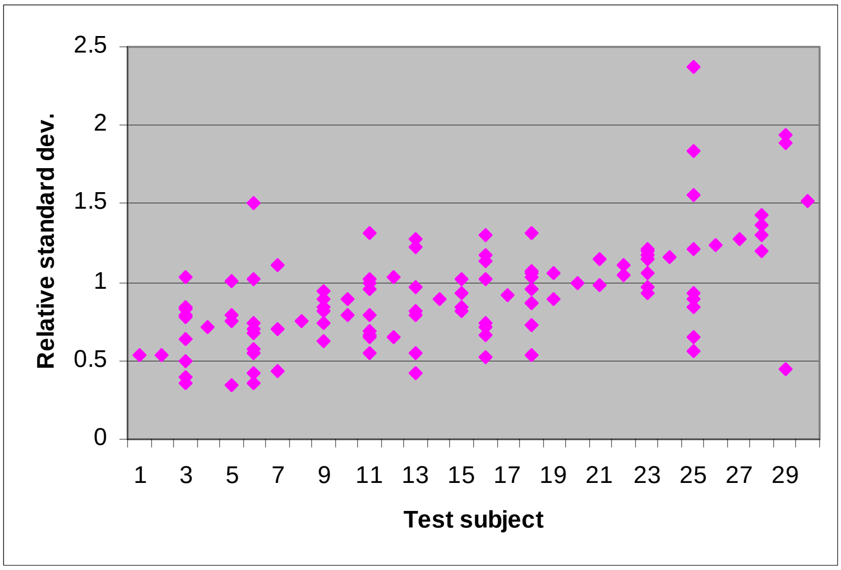


Figure 9-19 Relative standard deviation of individual test results

The clustering of the values in this chart for each individual suggests that the ability of each individual to judge pitch was broadly the same at all frequencies. Subjects 6, 25 and 29 are the exception to this, producing precise and imprecise results in different test runs.

Figure 9-20 below is a scatter plot of the relative upper partial effect against the relative standard deviation for each block of tests:

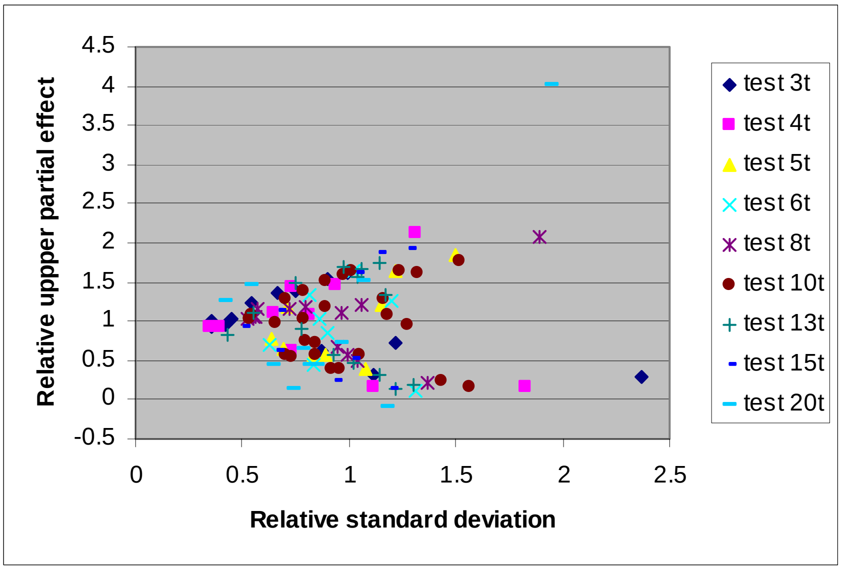


Figure 9-20 Upper partial effect related to accuracy of pitch judgment

It is clear from this plot that the more precise the individual's pitch judgments (i.e. lower relative standard deviations), the more the test results tend towards the average value for the upper partial effect. Apart from the one outlier for the test at 2000Hz, the relative upper partial effects at higher relative standard deviations are symmetrically distributed about the average value. The correlation coefficient for the values plotted in the above chart is 0.11, i.e. there is very little correlation between relative standard deviation of residuals and relative upper partial effect. Test subjects able to judge pitch less accurately experience the same pitch shifts, on average, as those able to judge pitch more accurately.

Table 9-7 below gives a summary of the response of each subject to the questions about their musical experience. The subjects are listed in increasing order of average relative standard deviation, i.e. subjects at the start of the list judged pitch more precisely. This is the same order as in figure 9-19.

Subject	Average rel. SD	Average upp. eff.	Tests conducted	Musical experience
1	0.54	1.04	1	Used to play clarinet, piano and guitar to grade 6 (~ 14 years ago)
2	0.54	1.09	1	Grade 8 flautist, plays regularly in a number of orchestras and bands
3	0.69	0.73	9	Sings in a choir (and has done so since age 9). Previously played piano and cello
4	0.72	1.15	1	Sings in a large choral society, medium sized church choir and madrigal group. Plays the piano and used to play the violin
5	0.72	1.30	4	Piano player. Sang in choir up to the age of 14 (~ 15 years ago)
6	0.73	1.32	9	Played piano and clarinet to grade 5, sang in school choir (~ 30 years ago)
7	0.75	0.57	3	Played piano and cello to a very basic level (~ 30 years ago)
8	0.75	1.38	1	Played classical guitar for 6 years to grade 5 (~ 10 years ago)
9	0.81	0.62	6	Regularly sings in a choir and plays a musical instrument
10	0.84	1.45	2	No musical experience
11	0.85	0.65	9	Occasionally sings in a choir and plays the piano. Previously sang extensively in several choirs (~ 18 years ago)
12	0.85	1.28	2	Sang in school choir, played recorder and cornet in brass band (~ 22 years ago)
13	0.86	1.38	7	Used to be a solo folk club singer (~ 20 years ago)
14	0.89	1.17	1	Plays church organ, various recorders, and sings in church choir
15	0.90	0.57	4	Sings occasionally in choir, at one time sang regularly and played violin (~ 6 years ago)
16	0.91	1.02	8	Sang in choir as child / teenager (~ 50 years ago)
17	0.92	0.39	1	Has sung in various choirs, choral societies and operetta companies for ~ 35 years
18	0.94	1.13	9	Sang in choir (~ 40 years ago). Currently learning the guitar
19	0.98	1.51	2	Musical director of a choir, plays piano and guitar
20	0.99	1.62	1	Has played classical guitar for ~ 50 years
21	1.06	1.67	2	
22	1.08	0.36	2	Regularly sings in a choir
23	1.11	1.21	9	Regularly sings in a choir, plays trombone and piano to grade 4
24	1.16	1.28	1	Has sung in several choirs, regularly plays clarinet to grade 8, and piano to grade 6
25	1.21	0.43	9	Has sung in choirs for 50 years, currently in a large choral society. Plays piano, flute, organ and carillon, but now only occasionally
26	1.24	1.64	1	Sang in church choir for several years (~ 15 years ago). Plays musical instruments by ear (including tuning them)
27	1.28	0.96	1	
28	1.33	0.19	4	Occasionally sings in a choir. Sang extensively in the past (~ 30 years ago)
29	1.43	2.37	3	Played trumpet for two years at school (~ 18 years ago)
30	1.52	1.77	1	No musical experience

Table 9-7 Musical experience of test subjects, sorted by pitch judgment

Table 9-8 lists the same information, but sorted by the average upper partial effect experienced by the test subjects, with the greatest pitch shift at the start of the list:

Subject	Average rel. SD	Average upp. eff.	Tests Conducted	Musical experience
29	1.43	2.37	3	Played trumpet for two years at school (~ 18 years ago)
30	1.52	1.77	1	No musical experience
21	1.06	1.67	2	
26	1.24	1.64	1	Sang in church choir for several years (~ 15 years ago). Plays musical instruments by ear (including tuning them)
20	0.99	1.62	1	Has played classical guitar for ~ 50 years
19	0.98	1.51	2	Musical director of a choir, plays piano and guitar
10	0.84	1.45	2	No musical experience
8	0.75	1.38	1	Played classical guitar for 6 years to grade 5 (~ 10 years ago)
13	0.86	1.38	7	Used to be a solo folk club singer (~ 20 years ago)
6	0.73	1.32	9	Played piano and clarinet to grade 5, sang in school choir (~ 30 years ago)
5	0.72	1.30	4	Piano player. Sang in choir up to the age of 14 (~ 15 years ago)
12	0.85	1.28	2	Sang in school choir, played recorder and cornet in brass band (~ 22 years ago)
24	1.16	1.28	1	Has sung in several choirs, regularly plays clarinet to grade 8, and piano to grade 6
23	1.11	1.21	9	Regularly sings in a choir, plays trombone and piano to grade 4
14	0.89	1.17	1	Plays church organ, various recorders, and sings in church choir
4	0.72	1.15	1	Sings in a large choral society, medium sized church choir and madrigal group. Plays the piano and used to play the violin
18	0.94	1.13	9	Sang in choir (~ 40 years ago). Currently learning the guitar
2	0.54	1.09	1	Grade 8 flautist, plays regularly in a number of orchestras and bands
1	0.54	1.04	1	Used to play clarinet, piano and guitar to grade 6 (~ 14 years ago)
16	0.91	1.02	8	Sang in choir as child / teenager (~ 50 years ago)
27	1.28	0.96	1	
3	0.69	0.73	9	Sings in a choir (and has done so since age 9). Previously played piano and cello
11	0.85	0.65	9	Occasionally sings in a choir and plays the piano. Previously sang extensively in several choirs (~ 18 years ago)
9	0.81	0.62	6	Regularly sings in a choir and plays a musical instrument
7	0.75	0.57	3	Played piano and cello to a very basic level (~ 30 years ago)
15	0.90	0.57	4	Sings occasionally in choir, at one time sang regularly and played violin (~ 6 years ago)
25	1.21	0.43	9	Has sung in choirs for 50 years, currently in a large choral society. Plays piano, flute, organ and carillon, but now only occasionally
17	0.92	0.39	1	Has sung in various choirs, choral societies and operetta companies for ~ 35 years
22	1.08	0.36	2	Regularly sings in a choir
28	1.33	0.19	4	Occasionally sings in a choir. Sang extensively in the past (~ 30 years ago)

Table 9-8 Musical experience of test subjects, sorted by pitch shift

The conclusion to be drawn from these tables is that there is no obvious link between musical experience as expressed by the test subjects and their ability to judge pitch accurately under the experimental conditions, or the pitch shift experienced in the tests. It is wrong to draw a general conclusion from this, as the test subjects listed were self-selected: they all succeeded in completing at least one test, and were both motivated and able to perform experiments of this type.

9.10 Summary and conclusions

These virtual pitch experiments have been successful. The tests have shown to a high level of statistical significance that a broad range of volunteers, selected only on their willingness to participate in the tests, of differing musical experience, and using unknown and possibly poor quality equipment, all experience pitch shifts when the conditions (as in these tests, and in change ringing in the English style) favour virtual as opposed to spectral pitch perception. The pitch shifts, of 1/3 of a semitone or more, are musically as well as statistically significant. This finding in turn vindicates and justifies the virtual pitch theory of the strike pitch of bells.

The tests have also provided further evidence of different pitch perception mechanisms in bells with different nominal frequencies. For nominals in the range 500Hz to 1600Hz, the nominal primarily determines pitch. Above and below this range, there is ambiguity as to which partials determine pitch, in line with the dominance region for virtual pitch.

In the range of frequencies where pitch is determined by the nominal, the experiment has shown that pitch shifts are dominated by changes in the upper partials, with a minor contribution from the prime and tierce. Other partials tested do not give rise to pitch shifts. A simple model has been established relating pitch shift to changes in upper partial, prime and tierce tuning.

Finally, this experiment has shown the viability of conducting such tests across the internet, allowing the easy participation of a large number of test subjects.

10 VALIDATION OF PITCH SHIFT MODEL

10.1 Introduction

The experiment in the second half of chapter 4 showed that, for bell sounds with nominals between 500Hz and 1600Hz, pitch is determined primarily by the nominal partial. The experiment documented in the previous chapter proved beyond all reasonable doubt that pitches based on the nominal are shifted significantly by changes in the upper partials, and slightly by changes in the prime and tierce frequency.

Two pieces of work are documented in this chapter. The first compares the observed pitch shifts with the results of Terhardt's model of virtual pitch. The shifts observed are not predicted by the Terhardt model, showing that the effect measured in the virtual pitch tests was not anticipated by him from his research.

The second piece of work involves the analysis of the tuning of a number of peals of bells. Some of these are tuned with stretch tuning, as explained below. The model of pitch shift derived in the previous chapter provides a good quantitative prediction of the degree of stretch employed by the bell tuners. This gives elegant and convincing confirmation of the existence of the virtual pitch effect and the validity of the derived model.

10.2 The Terhardt Virtual Pitch Model

This model is explained in (Terhardt, Stoll & Seewann 1982b) which gives an algorithm for establishing perceived pitches, both spectral and virtual, from a set of partial frequencies and amplitudes. This algorithm was used in the research on bell pitches documented in (Terhardt & Seewann 1984). In this work, the experimenters compared the results of the algorithm with pitch determination experiments on a number of bells. The predictions of the algorithm were a reasonable match for the experimental results as regards prediction of the note names of the pitches (i.e. to within half a semitone in either direction), but investigation of the detailed correspondence between measured and predicted pitches showed average deviations of between 15 and 40 cents. No attempt was made to explain the deviations as pitch shifts.

The Terhardt algorithm was implemented in software (C program ptp2svp) by Ernst Terhardt and Richard Parncutt. The software is available for download at (Terhardt & Parncutt 2004a). Documentation for the software is available at (Terhardt & Parncutt 2004b). An online implementation in Java is available at (Jensen 2005).

10.2.1 Terhardt algorithm trials - procedure

The ptp2svp software was incorporated by the author into a program that could import information on partial amplitudes and frequencies from the author's Wavanal software. The only modifications made to ptp2svp were to re-implement the user interface, on the input side to read files of partials data, and on the output side to display the resulting pitches in a window. The ptp2svp program takes information on each partial as a frequency (in kHz) and an intensity in decibels. The absolute intensity of the partials is unknown in the circumstances in which the software was used here. Therefore the following approach was adopted in the interface:

- The user was prompted for an intensity i_{user} in decibels for the loudest partial
- The n partial amplitudes a_n were converted into logarithmic intensities $i_n = 20 \times \log_{10}(a_n)$
- The maximum value $i_{\max}$ of these quantities across all partials was found
- The partial with the highest amplitude was assigned the intensity i_{user}
- All other partials were assigned an intensity $i_{user} - i_{\max} + i_n$.

The trials were run at two levels of i_{user}, 60dB and 85db (ptp2svp will not accept intensities above 90dB). The results at the two intensities were very similar.

In the first trial, all 144 test sounds from the virtual pitch tests were analysed with Wavanal, the resulting partials were run through ptp2svp, and the dominant pitches predicted by the algorithm were noted. It would have been possible to run the algorithm on the files of data from which the test sounds were generated, but it was deemed more realistic to analyse the test sounds actually used in the virtual pitch tests. The procedure to analyse all 144 sounds (16 sounds at each of 9 nominal frequencies) was laborious but straightforward.

Once this work had been done, the effect on the virtual pitch of each factor in the tests was established by adding up all eight pitches given by the algorithm with the factor at a high level, subtracting the eight pitches with that factor at a low level, and dividing by eight. This calculation gives a result which is directly comparable with the effects measured in the virtual pitch tests. Given the results of the virtual pitch experiments, effects due to factors acting in concert were not calculated.

Successful implementation of the algorithm was confirmed by the fact that the approximate frequencies of the pitches predicted were those to be expected for bell sounds with these partials. As will be seen, the algorithm also successfully predicts secondary strikes for the test sounds with low frequency nominals.

10.2.2 Terhardt trial results

For the test sounds with nominals of 500Hz and above, a single dominant pitch was predicted by the algorithm. Multiple pitches of roughly equal prominence were predicted by the algorithm for test sounds with nominals of 315Hz and 400Hz. This corresponds with the pitch ambiguity already noted for bell sounds with these nominals.

The effects predicted by the algorithm for nominals of 500Hz and above are as shown in Table 10-1. The effects given are for 85dB, those for 60dB were very similar:

Frequency	**Quint Effect**	**Middle Effect**	**Prime Effect**	**Tierce Effect**	**Upper partial Effect**
500Hz		0.000	0.455	0.065	0.000
630Hz	0.000		0.450	0.040	0.000
800Hz		0.000	0.395	0.015	0.000
1000Hz	0.000		0.300	0.010	0.000
1260Hz		0.000	0.195	0.005	0.000
1587Hz	0.000		0.110	0.000	0.000
2000Hz		0.000	0.050	0.000	0.000

Table 10-1 Pitch shift effects predicted by Terhardt algorithm

'Middle effect' refers to the effect of shifting the three partials between nominal and superquint. Comparison with the experimental results from the virtual pitch tests given in Table 10-2 below (extracted from Table 9-4 and the detailed test results) shows significant differences.

Frequency	**Quint effect**	**Middle Effect**	**Prime Effect**	**Tierce Effect**	**Upper partial Effect**
500Hz		0.500	8.250	7.750	71.500
630Hz	0.729		6.562	9.271	43.229
800Hz		1.803	6.274	8.245	42.236
1000Hz	-.0901		6.238	8.329	36.022
1260Hz		0.335	4.888	10.960	31.585
1587Hz	0.347		6.319	5.903	18.819
2000Hz		0.057	0.398	8.125	12.784

Table 10-2 Extract from virtual pitch test results

The algorithm as implemented in ptp2svp predicts no pitch shift as a result of upper partial changes, whereas we know from the experiments that the shift due to this cause is the dominant and significant effect. The prime and tierce effects predicted by the algorithm are at least an order of magnitude less that those seen in the experiments, though they are in the same direction. The Terhardt algorithm therefore does not predict the pitch shifts seen in practice at these nominal frequencies.

For the test sounds with 315Hz nominals, four pitches with roughly equal weight predominate in the results from ptp2svp. These four pitches have frequencies which correspond with the hum or two octaves below the nominal, the prime or one octave below the nominal, two octaves below partial I-7, and the nominal. Effects for the four observed pitches, due to the three factors, are shown in Table 10-3:

	Factor		
Predicted pitch	prime	tierce	upper partials
nominal / 4	0.356	0.097	0.001
nominal / 2	0.123	0.203	0.048
nominal	0.245	0.410	0.095
I-7 / 4	0.330	5.225	23.685

Table 10-3 Terhardt algorithm results for 315Hz

The algorithm is clearly predicting a secondary strike pitch, shown by the presence of a pitch whose frequency is determined by partial I-7 and is shifted by changes in the upper partials.

A similar set of results follows in Table 10-4 for the test sounds at a 400Hz nominal. For some, but not all, of the sounds, the algorithm predicted a pitch two octaves below I-7. Calculating the shift effect for the pitches based on I-7 is difficult due to the missing values, and in consequence they are omitted from Table 10-4.

	Factor		
Predicted pitch	prime	tierce	upper partials
nominal / 4	0.424	0.085	0.000
nominal / 2	0.131	0.251	0.029
nominal	0.264	0.504	0.056

Table 10-4 Terhardt algorithm results for 400Hz

The corresponding results for the virtual pitch experiments at 315Hz and 400Hz as follows (extracted from Table 9-4) are as follows:

Frequency	**Prime Effect**	**Tierce Effect**	**Upper partial Effect**
315Hz	12.500	7.115	134.039
400Hz	10.125	9.625	71.437

Table 10-5 Extract from virtual pitch test results for low frequencies

The virtual pitch test results are an amalgam of pitch shifts experienced for multiple virtual pitches as predicted by the Terhardt algorithm. However, for both 315Hz and 400Hz test sounds, the large upper partial effect seen in the experiment results is not predicted by the algorithm; nor are the significant prime and tierce effects.

Therefore, on the basis of these trials one can confidently say that the pitch shifts seen in practice are not predicted by the Terhardt algorithm.

10.2.3 Pitch shifts for smaller changes in the upper partials

Though the result of the trial described in the previous section appears conclusive, examination of the documented algorithm and of the source code of ptp2svp shows that the algorithm does look for multiple partials in a near-harmonic series and establishes the virtual pitch by searching for near-common divisors. It was thought that the range in upper partials between the low and high values in the test sounds might be too great for the algorithm to work properly. The change in the octave nominal, of 140 cents or 1.4 semitones, though not atypical of partials seen in real bells, is large. Therefore, the set of sixteen test waveforms created for the regression test in section 8.2 was used for a second Terhardt algorithm trial

The partial frequencies and amplitudes of the sixteen test sounds were extracted using Wavanal and passed through the ptp2svp program, at both 60dB and 85dB levels. Two dominant virtual pitches were predicted by the algorithm, at about 250Hz and 500Hz. The results were identical for all sixteen test waveforms, in which the octave nominal changed from 1140 cents to 1280 cents, and the other upper partials changed similarly in line with the derived model.

No shifts in the perceived pitches were predicted by ptp2svp, to five significant figures. This is despite the fact that the experiment in section 8.2 on these sounds showed a pitch difference of 23 cents between the sounds with the highest and lowest octave nominals. The sounds with the highest and lowest octave nominals sound clearly different in pitch when sounded in sequence.

Based on these two trials, one can say with confidence that the pitch shifts observed in practice are never predicted by the Terhardt algorithm, at least in the software implementation trialled here. The effect experienced by the virtual pitch test subjects was not anticipated or discovered by Terhardt.

10.3 Comparison of experimentally predicted shifts with real peals of bells

An important objective of this research work is to establish results that are of practical significance. The virtual pitch test sounds are abstracted from those of real bells, and as a result we cannot be certain that the effects measured in the tests will apply in practice. It is desirable to seek practical circumstances in which the model of pitch shift developed at the end of chapter 9 can be verified against real bells. Fortunately, such circumstances are not difficult to find. The rest of this chapter explains them and tests the extent to which the virtual pitch test results are valid in practice.

10.3.1 Stretch tuning of peals of bells

Some peals of bells are tuned with stretched trebles. By this term is meant the practice of tuning the nominals of the small bells in a peal (especially in peals of ten, twelve or more bells) significantly sharp - one quarter or one half of a semitone. This stretch tuning was probably the practice rather than the exception in peals of ten and twelve up to the late 19th century (when true harmonic tuning with tuning forks was introduced). From the start of the 20th century, treble bells were tuned to temperament rather than sharp. The practice of stretch tuning was revived by the Taylor bellfoundry from the early 1950s until the late 1970s or early 1980s, and was also employed on rare occasions in this time period by the Whitechapel bellfoundry.

The reason for the original practice, and the re-adoption for a time by Taylors, has not until now been satisfactorily explained. People say that stretched trebles sound brighter, but other people dislike the sound, because the tuning extension is quite noticeable if two bells, notionally an octave apart but with the smaller tuned with stretch, are rung together. The author's experience, as explained in section 3.5 above, is that treble bells with closely spaced upper partials sound flat because of the pitch shifts, and that stretch tuning helps to compensate for this. The decision to stretch the tuning of a peal of bells and by what degree is a compromise between these two competing influences. Two different tuning strategies are in play; in a stretched peal, the target is to get the bells to sound in tune with each other by tuning the perceived pitches of the bells; in an unstretched peal, the target is to get the nominal frequencies as close to the temperament value as possible. The description in section 2.5 of tuning of strike pitches rather than nominals by the Whitechapel Bellfoundry (a practice they continued into the 1920s) provides a good illustration of the first approach.

The hypothesis to be tested is that stretch tuning is done to compensate for pitch shifts due to differences in the partial structure between the trebles (highest pitched bells) and tenors (lowest pitched bells). These differences in partial structure arise primarily from differences in mechanical profile between small and large bells, as explained in sections 3.5.1 and 3.6. Small bells are cast thick and heavy so that their mechanical dynamics when rung full-circle better match those of the bigger bells. In carillons, mechanical dynamics of bells are not of significance, but the small bells are cast to a thicker profile than the bigger bells to ensure they produce a sufficient volume of sound.

Peals of twelve were chosen to test the hypothesis for two reasons:

- Due to the tuning discrepancies generally found in peals of bells (tuning of nominals to within 10 cents of temperament is typically judged 'in tune'), as many bells as possible tuned together are needed to average out the variations
- The significantly different weights and sizes across a peal of twelve mean that significant differences in upper partials result, accentuating the resulting pitch shifts.

10.3.2 Analysis of peals of bells with stretch tuning

The author's collection of bell recordings was searched for peals of 12, tuned with stretch, cast or retuned in the 1950s or 1960s. Four such peals were found and all are analysed in this part of the thesis. Two additional peals were analysed. One is a classic 19th century stretched peal. The other is a peal of 20th century Mears & Stainbank bells, eight of which were cast and tuned at a time when Mears were still tuning strike pitches, not nominals, and the remaining four added recently with tuning sympathetic to the old approach. Due to the more primitive tuning methods used in these two peals, less precise results are to be expected. Finally, to show the difference between stretched and unstretched tuning, a modern unstretched peal of twelve is analysed.

The nominals of the bells in all seven peals fell into the range 450 Hz to 2000 Hz where the experiments in chapter 4 have shown that the nominal is the primary determinant of pitch. The complete list of peals appears as Table 10-6:

Peal	Provenance	Notes
Melbourne, Australia	Cast Mears 1889, retuned Taylor 1962	Stretched peal using modern tuning methods
Tewkesbury Abbey, Gloucestershire	Cast Taylor 1962	Stretched peal using modern tuning methods
Cambridge, University Church	Cast Phelps 1722/3, some bells replaced, retuned Taylor 1952	Stretched peal using modern tuning methods
Cornhill, London	Mixed set of bells, some replaced and all retuned Mears 1960	Stretched peal using modern tuning methods
St Paul's Cathedral, London	Cast Taylor 1878, no subsequent tuning	Stretched peal using old tuning methods
Preston Minster	Cast Mears 1920, 1997, 2003	Stretched peal using (in part) old tuning methods
Llandaff Cathedral	Cast Whitechapel 1992	Modern unstretched peal

Table 10-6 Summary of peals of bells analysed for stretch tuning

The analysis of these bells followed the following procedure:

- Tuning figures were retrieved from the author's database
- A regression fit was done on the nominals to determine what temperament the bells were tuned in, and the average stretch in the tuning (expressed as cents per octave)
- Variances in the nominal tuning of each bell from the ideal stretched temperament were calculated and plotted
- The predicted shift in the pitch of each bell was calculated using the simple model derived from experiment in chapter 9
- The actual stretch measured for each bell, and that calculated from the model, were plotted for comparison purposes.

- A regression fit was done on the calculated stretch to ascertain the calculated cents per octave for comparison with the actual tuning. Also, the linear correlation between actual and calculated stretch was determined.

The regression approach used to determine the temperament and the average stretch involved doing linear regressions of the cents of each bell's nominal to the nominal of the heaviest bell, against ideal figures for three possible temperaments as in Table 10-7 below:

Bell	Equal temperament	Meantone temperament	Just tuning
12	0	0	0
11	200	193	204
10	400	386	386
9	500	503	498
8	700	697	702
7	900	890	884
6	1100	1083	1088
5	1200	1200	1200
4	1400	1393	1404
3	1600	1586	1586
2	1700	1703	1698
1	1900	1897	1902

Table 10-7 Tuning figures for various temperaments

Then, for each temperament or tuning, the sum of squared differences between actual tuning figures and regression fit was calculated. The tuning with the lowest sum of squared differences was taken as the best fit tuning for the bells. Then, if c is the gradient from the regression fit for the selected tuning, the stretch in cents per octave is calculated as $1200 \times (c-1)$.

An overview of the salient features of the tuning of each peal is given in Table 10-8.

Peal	Nominal range	Temperament	Stretch in cents per octave
Melbourne	559Hz - 1714Hz	Equal	27.1
Tewkesbury	583Hz - 1786Hz	Equal	25.8
Cambridge	545Hz - 1667Hz	Equal	25.4
Cornhill	484Hz - 1467Hz	Just	20.8
St Paul's	466Hz - 1455Hz	Just	29.2
Preston	684Hz - 2072Hz	Equal	20.9
Llandaff	623Hz - 1871Hz	Equal	1.9

Table 10-8 Overview of tuning of peals analysed for stretch tuning

One peal (Melbourne) will be analysed in detail to show the technique. Summary information only will be given for the remainder.

10.3.3 Analysis of Melbourne bells

The relevant tuning figures for these bells (analysed from recordings provided by Andrew Chin) are as shown in Table 10-9. Figures in the Freq. columns are the measured frequencies in Hz. Figures in the Cents columns for Prime, Tierce and Octave Nominal are relative to the nominal of each bell. Cents figures for the nominals are relative to the nominal of the heaviest (no. 12) bell. The effect of the stretch tuning can be seen in the 5th bell. This nominal should be an octave or 1200 cents above bell no. 12; it is actually tuned 31 cents or just less than 1/3 of a semitone sharp of this.

Bell	**Prime**		**Tierce**		**Nominal**		**Octave Nominal**	
	Freq.	Cents	Freq.	Cents	Freq.	Cents	Freq.	Cents
12	266	-1286	338	-871	559	0	1143	1238
11	325.5	-1143	390	-830	630	207	1261.5	1202
10	350.5	-1220	436.5	-840	709	412	1428	1212
9	369.5	-1226	462	-839	750	509	1509.5	1211
8	406.5	-1260	515	-850	841.5	708	1696.5	1214
7	443.5	-1321	575	-871	951	920	1937	1232
6	476	-1398	646.5	-868	1067.5	1120	2138.5	1203
5	535	-1307	691.5	-862	1138	1231	2285.5	1207
4	561.5	-1428	771.5	-878	1281	1436	2559	1198
3	631.5	-1426	867	-878	1439.5	1638	2851	1183
2	683.5	-1390	916	-884	1526	1739	3028	1186
1	763	-1401	1029.5	-883	1714	1940	3351	1161

Table 10-9 Tuning figures for Melbourne

As listed in Table 10-9, these bells are tuned in equal temperament with an average stretch of 27.1 cents per octave. The actual tuning of the nominals against this temperament is shown in Figure 10-1:

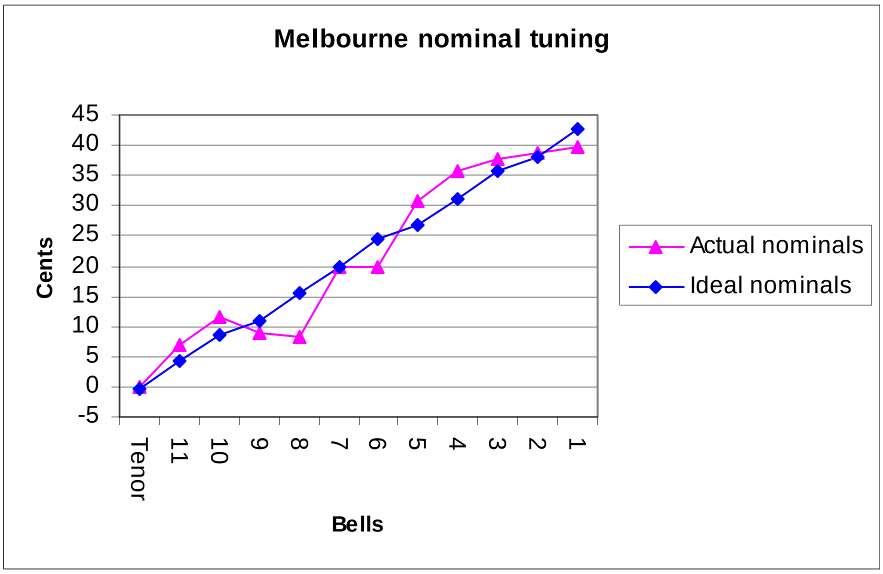


Figure 10-1 Nominals for Melbourne

The tuning of the nominals of these bells is accurate, with no nominal more than 6 cents out. The calculations to determine the theoretical pitch shift are shown in Table 10-10:

Bell	Nominal	Prime effect	Tierce effect	O/nom effect	Calculated shift	Stretch relative to no. 12
12	559	0.034	0.084	0.434	48.9	0.0
11	630	0.034	0.084	0.380	38.2	10.8
10	709	0.034	0.084	0.334	35.3	13.7
9	750	0.034	0.084	0.314	33.3	15.6
8	841.5	0.034	0.084	0.276	29.4	19.6
7	951	0.034	0.084	0.242	27.3	21.7
6	1067.5	0.034	0.084	0.213	16.1	32.8
5	1138	0.034	0.084	0.198	19.7	29.3
4	1281	0.034	0.084	0.174	11.0	37.9
3	1439.5	0.034	0.084	0.153	7.6	41.4
2	1526	0.034	0.084	0.144	8.4	40.6
1	1714	0.034	0.084	0.126	4.0	44.9

Table 10-10 Calculated pitch shifts for Melbourne

The columns headed Prime and Tierce effect are the contribution to pitch shift in cents per cent change in the tuning of the relevant partial, derived in the model in chapter 9. The figures in the Octave Nominal effect column are calculated from the model relationship $effect = 456.668 \times f^{-1.10012}$ where f is the nominal frequency. The calculated shift is the sum of the shifts due to these three partials, calculated from the tuning figures in Table 10-9, using the formula in section 9.7.

We do not know from this calculation what the absolute perceived pitch of each bell is; the experiments in chapter 9 only established shifts, not absolute values. For ease of comparison, it is appropriate to baseline the shifts to the tenor (the bell deepest in pitch) as that bell is customarily used to establish the key of a peal. The final column shows the calculated stretch required in each bell to compensate for the virtual pitch shift (obtain by subtracting the calculated shift for each bell from that of the tenor). The plot of calculated stretches compared to those actually measured is given as Figure 10-2:

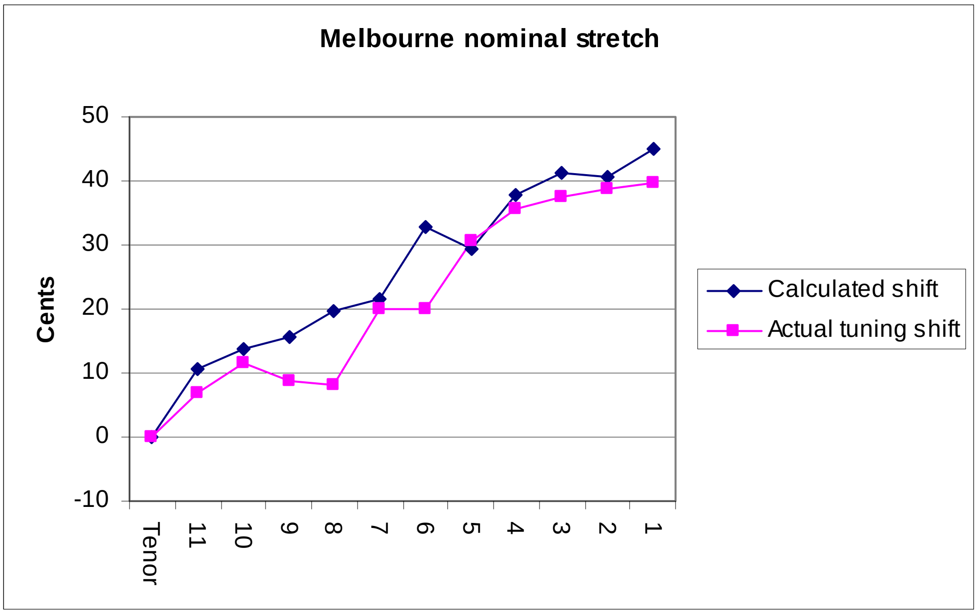


Figure 10-2 Calculated and measured stretch for Melbourne

This plot shows that the direction (i.e. positive stretch) and overall shape of the calculated stretch corresponds very well with that measured in practice. Comparison with Figure 10-1 shows that much of the discrepancy between calculated and actual tuning shift is down to discrepancies in the tuning of the nominals.

The results of the regression fits on actual and calculated nominals against temperament, and the linear correlation between them, appear in Table 10-11:

Stretch in actual nominals, versus equal temperament	27.1 cents per octave
Stretch in calculated nominals	27.3 cents per octave
Correlation between actual and calculated	0.955

Table 10-11 Comparison of actual and calculated stretch for Melbourne

For the Melbourne bells, this analysis provides a good demonstration that the pitch shift effects established in the virtual pitch experiments align closely with those experienced by the bell tuner who worked on this peal.

10.3.4 Other stretched peals tuned with modern methods

Summary plots for three more peals of twelve tuned with stretch using modern methods are given below. The calculations leading to these plots are identical to those used in the Melbourne analysis. The first plot of each pair serves only to show discrepancies in the tuning of the nominals against temperament. The second plot shows the correspondence between actual tuning shift, and that calculated from the model.

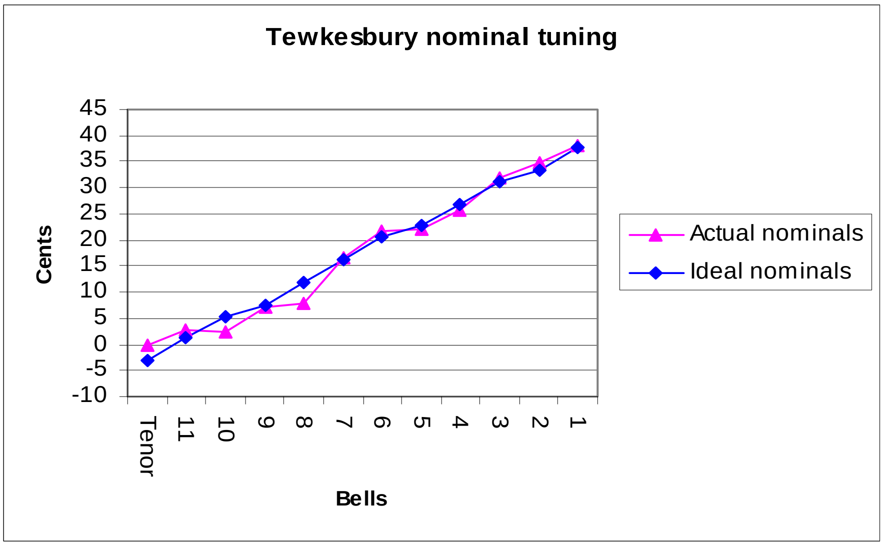


Figure 10-3 Tewkesbury nominal tuning

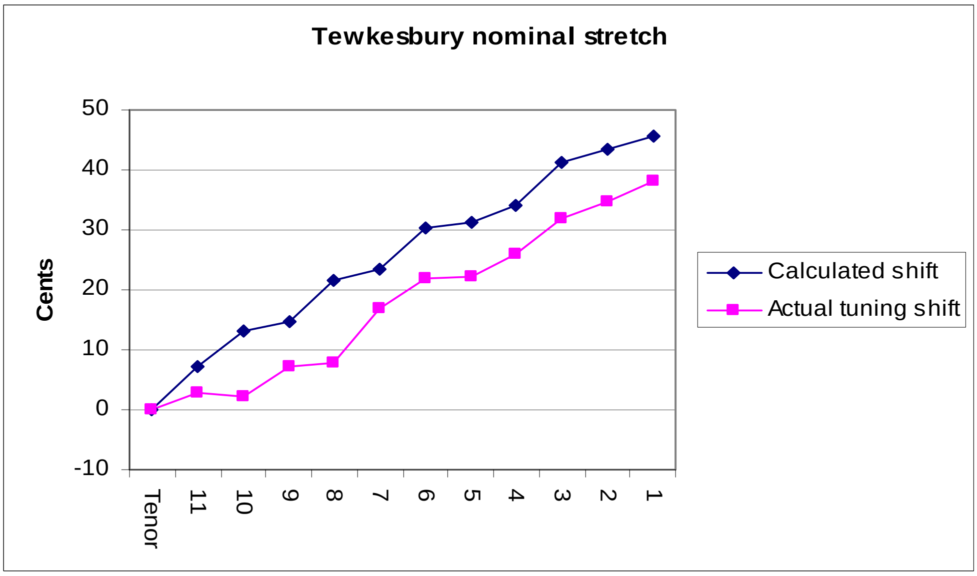


Figure 10-4 Tewkesbury stretch

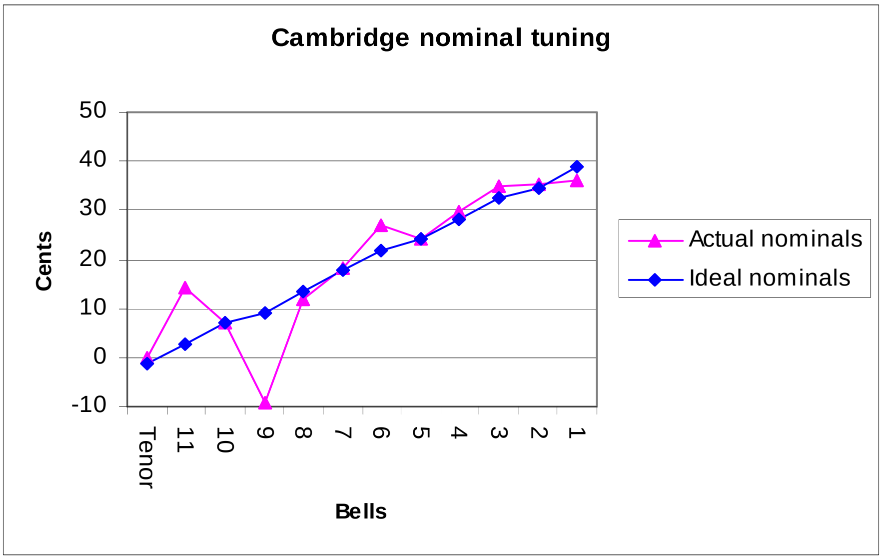


Figure 10-5 Cambridge nominal tuning

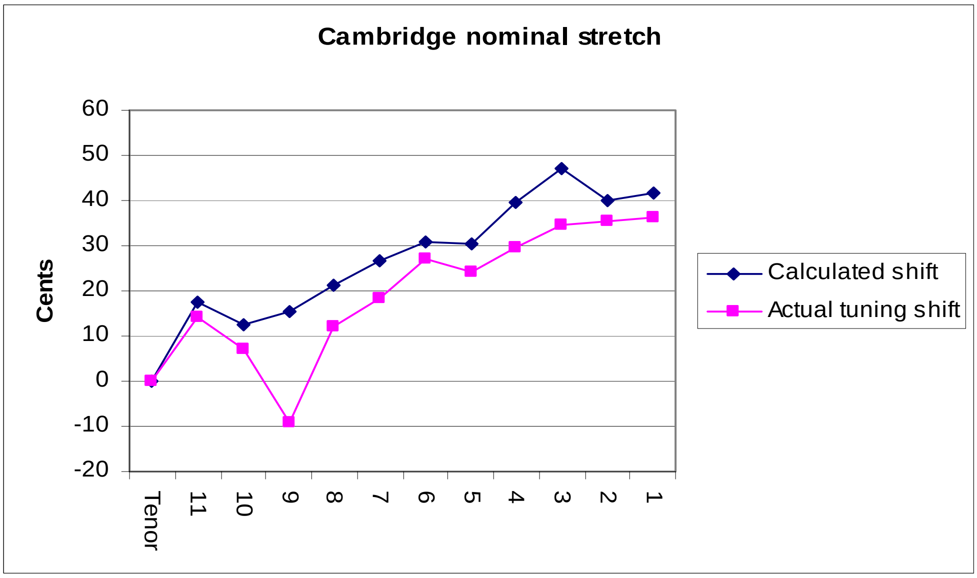


Figure 10-6 Cambridge stretch

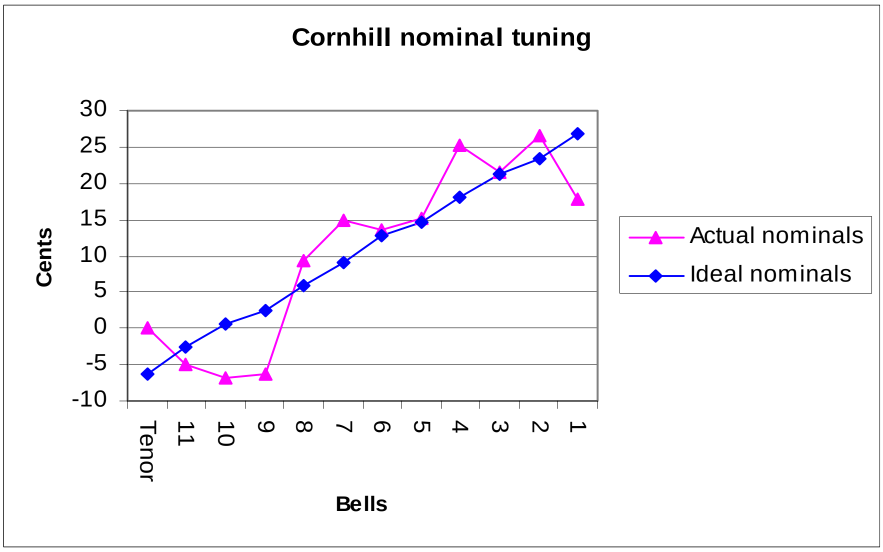


Figure 10-7 Cornhill nominal tuning

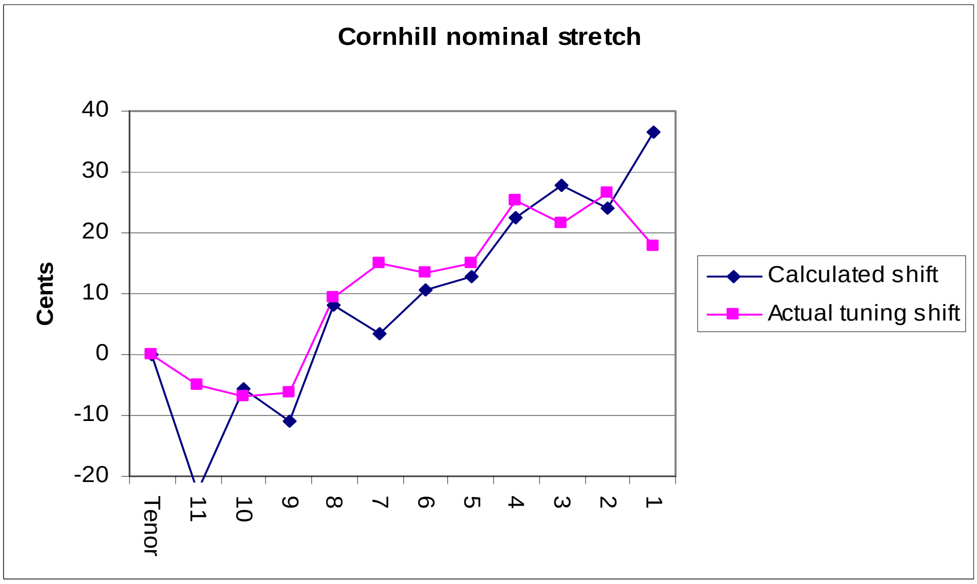


Figure 10-8 Cornhill stretch

Comments on these analyses are as follows:

- In the Tewkesbury bells, the tuner stretched less than the theoretical prediction (by 2.6 cents in the octave). This could be due to difference in pitch perception in the bell tuner's ears compared with the average of the test subjects in the virtual pitch tests, or some other cause
- In the other two peals, vagaries of the tuning of individual bells are noticeable. All of these peals include or are completely made up of pre- or early-20th century bells; characteristics of individual bells constrain what the tuner can achieve

- In particular, the 11th bell at Cornhill (off the bottom of the plot in Figure 10-8) is a Mears bell of 1910 with very different tonal characteristics to the rest of the peal.

A summary of the figures for these three peals follows as Table 10-12. The regression and correlation for the Cornhill peal was run both with and without the 11th bell due to the anomalous partials of this bell - it has much sharper upper partials, i.e. a thinner profile, than the bells around it in the peal. On the basis of this analysis, it could advantageously have been replaced when the peal was remodelled in 1960, as its timbre will not match well with the other bells.

	Tewkesbury	**Cambridge**	**Cornhill**	**Cornhill less 11th bell**
Stretch in actual nominals, versus temperament	25.8 cents	25.4 cents	20.8 cents	20.2 cents
Stretch in calculated nominals	28.4 cents	26.1 cents	30.5 cents	24.2 cents
Correlation between actual and calculated	0.976	0.909	0.876	0.904

Table 10-12 Calculated and actual stretch for Tewkesbury, Cambridge and Cornhill

These figures again show that the predictions from the virtual pitch experiments align well with the pitches heard by the bell tuner when working on these peals.

10.3.5 Stretch in St Paul's Cathedral and Preston Minster

The four peals of bells analysed so far were all tuned using modern tuning methods. The approach used to tune the St Paul's bells is unknown but would have been cruder than modern methods. The heaviest eight bells at Preston were tuned by the strike pitches rather than the nominals, as described in section 2.5. The smallest four bells at Preston were tuned using modern methods but in such a way as to fit in with the older bells. As a result, we can expect wider variations in tuning for these two peals than seen in the previous four peals.

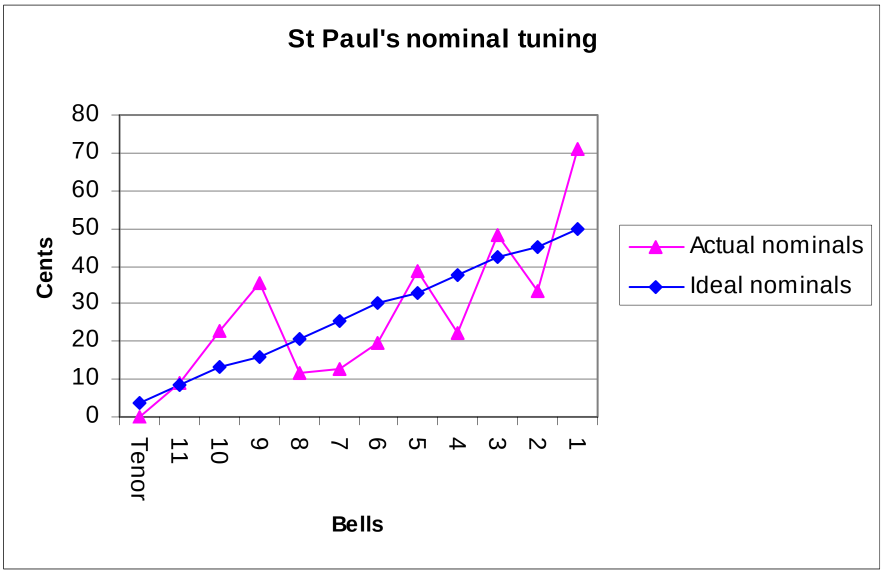


Figure 10-9 Nominals of St Paul's Cathedral, London

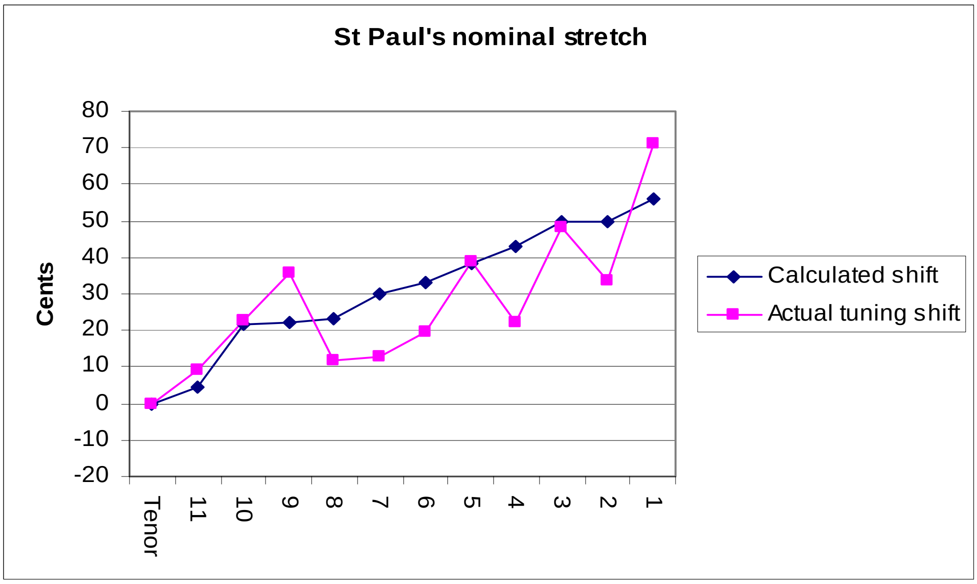


Figure 10-10 Calculated and actual stretch of St Paul's Cathedral

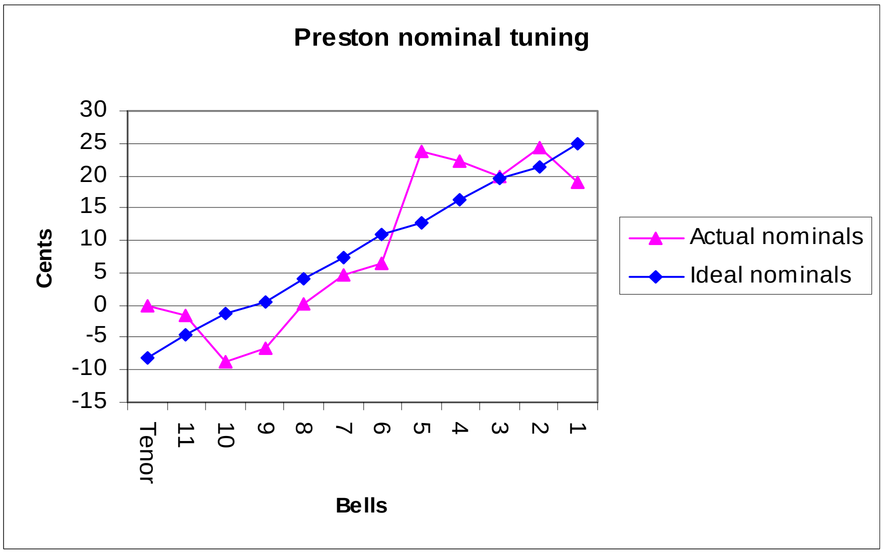


Figure 10-11 Nominals of Preston Minster

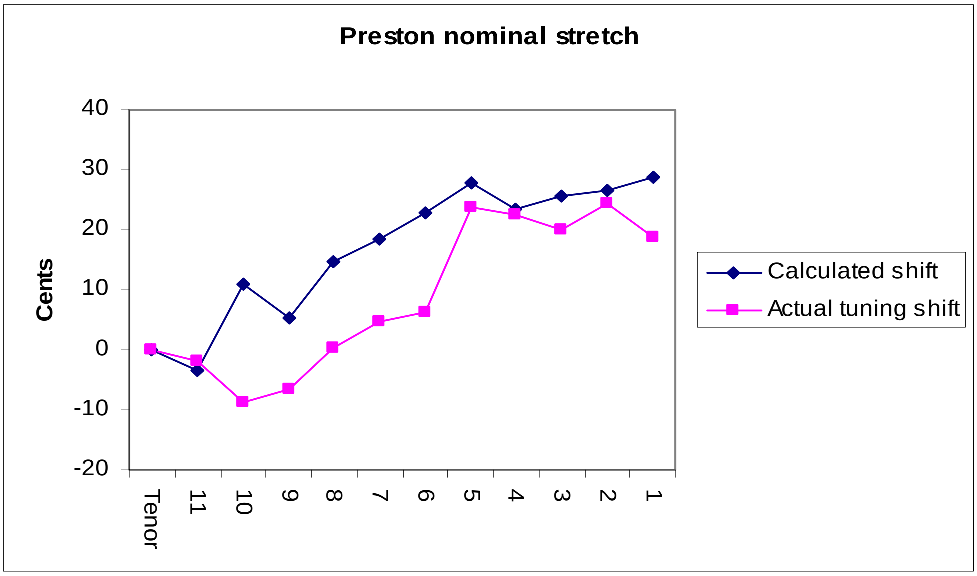


Figure 10-12 Calculated and actual stretch for Preston Minster

The regression and correlation figures for these bells follow in Table 10-13:

	St Paul's	**Preston**
Stretch in actual nominals, versus temperament	29.2 cents	20.9 cents
Stretch in calculated nominals	33.6 cents	20.2 cents
Correlation between actual and calculated	0.799	0.824

Table 10-13 Calculated and actual stretch for St Paul's Cathedral and Preston

It is clear from the plot of the St Paul's nominals (Figure 10-9) that much of the discrepancy between predicted and actual stretch for these bells is due to tuning errors. The stretch in the Preston bells is successfully predicted by the results of the virtual pitch experiments.

10.3.6 Analysis of Llandaff - a peal tuned without stretch

The bells at Llandaff are tuned without significant stretch, as can been seen from Figures 10-13 and 10-14 below. Note the diminished vertical scale for Figure 10-13: the tuning of the nominals in this peal is very accurate indeed.

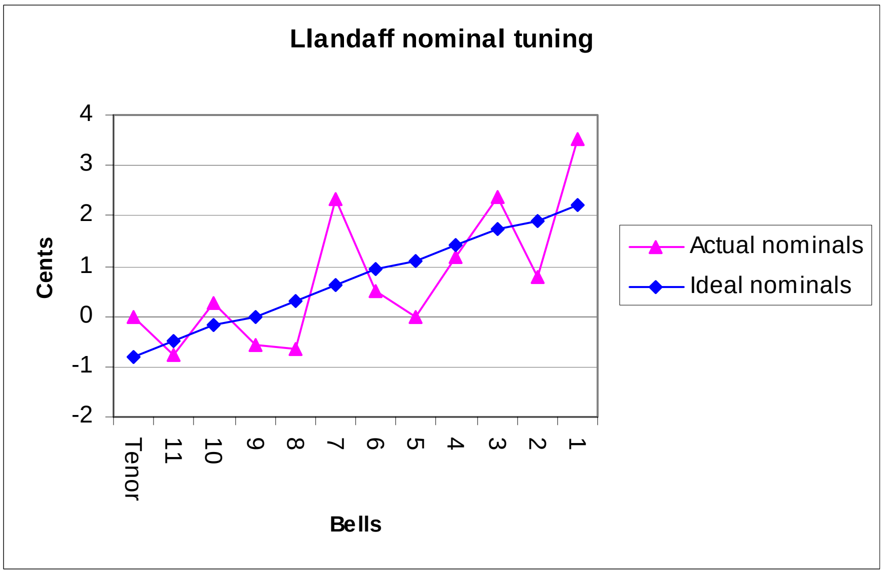


Figure 10-13 Nominals for Llandaff Cathedral

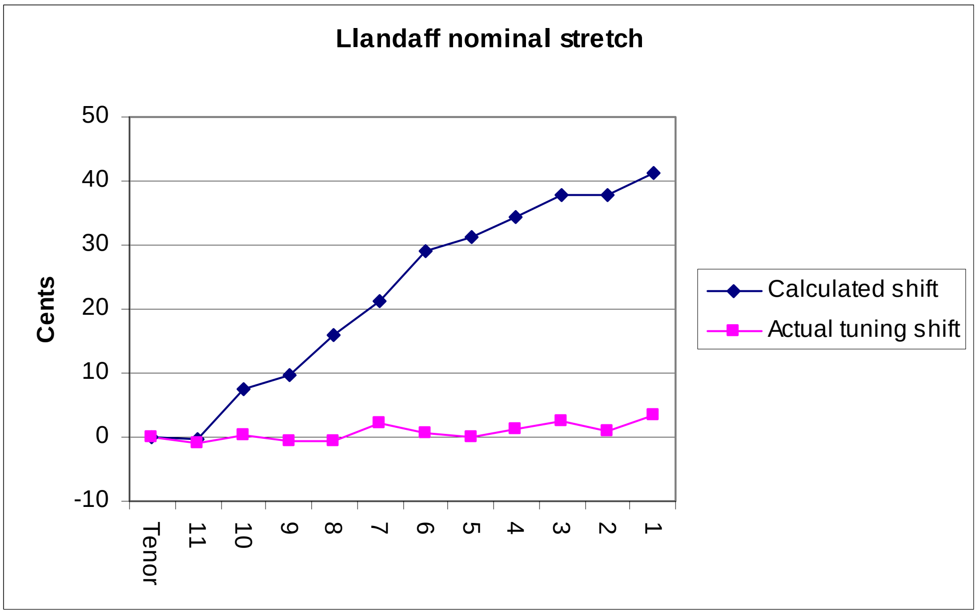


Figure 10-14 Calculated and actual stretch for Llandaff Cathedral

The second plot shows the absence of stretch in practice, compared with that suggested by the virtual pitch model. The consequence shows in the reaction of listeners to these bells, especially the highest-pitched ones. Some people admire the accuracy of the tuning; others find the trebles sound flat and dull when rung in the peal, even though they are sweet-toned bells rung alone.

10.3.7 Summary of analysis

The results of the various regressions and correlations for the stretched peals are drawn together in Table 10-14 to provide a summary of the analysis. The stretch figures are given in cents per octave:

Peal	Actual stretch	Calculated stretch	Correlation
Melbourne	27.1	27.3	0.955
Tewkesbury	25.8	28.4	0.976
Cambridge	25.4	26.1	0.909
Cornhill (less no. 11)	20.2	24.2	0.914
St Paul's Cathedral	29.2	33.6	0.799
Preston	20.9	20.2	0.824

Table 10-14 Summary of stretch tuning analysis and prediction

These figures show that the model derived from the virtual pitch experiments proves a good predictor of stretch tuning in these peals. Therefore:

- The quantified results from the virtual pitch experiment are genuine effects in real bells, as experienced by a number of different bell tuners over an extended period of time
- Virtual pitch and in particular pitch shifts provide a robust explanation of the stretch tuning used in some peals of bells.

11 CONCLUDING REMARKS

The primary objective of this research work, which was the quantification of pitch and pitch shifts in bells, has been amply met as has been described in the previous chapter. The practical application of the experimental results and the model derived from them has been proved in the comparisons with actual peals of bells tuned with stretched trebles. The contribution of the upper partials to the timbre and overall musical quality of bells has been established by the work carried out.

As ever with research of this type, there are loose ends and further areas for investigation. The contribution of the hum partial to bell pitches and pitch shifts has not been fully established, as was pointed out in the discussion of the virtual pitch experiment design. In general the pitch of small bells, in which virtual and spectral pitches may conflict, needs more investigation. However, provided the consequences of the spectral dominance region as explored in chapter 4 are born in mind, this does not detract from the usefulness of the results in practical bell design and tuning.

The contribution of other partials to the timbre of bells needs considerable further investigation. As pointed out in section 4.5, as yet no definition of timbre exists as there is for pitch which provides for quantification through experiment. Experience shows that bells with the same relative frequencies for the lower and group-I partials can sound distinctly different. Bellfounders have established shapes or profiles for the bells they cast which have predictable results for the sound of the bells they make. It has been suggested that the frequencies and relative amplitudes of partials other than the lower and group-I partials account for the difference in sound, but no investigation has to date been done in this area.

Stepping back from the specifics of bell acoustics, the experimental results showing the dependence of virtual pitch shifts on the relative frequencies of the partials which generate those pitches, are of relevance to wider research into pitch perception mechanisms. Previous work documented in the literature and experiments in this research show that partial amplitude does not significantly affect virtual pitch shifts. Therefore, although the test sounds used in these experiments are bell-specific, it is likely that results are generally applicable. The failure of the Terhardt virtual pitch algorithm to predict the shifts seen in practice is notable given the musical significance of the shifts observed. There is a place for a new model of virtual pitch to match the effects observed in practice.

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Appendix 2 – General bibliography

This appendix comprises a list of sources and documents, including those referenced from the main text of the thesis, which were consulted during the course of the research. They are divided into five areas:

- General acoustics, and specific items on musical pitches
- Bell acoustics
- CDs of bells and bellringing
- General and topographical information on bells
- Mathematical techniques including FFT, matrix and statistical techniques
- Musical temperaments and tuning.

Web references are given in numerous cases to aid the reader without immediate access to a research library, though it will be recognised that web addresses can quickly go out of date, and that material published only on the web may not have gone through a peer review process.

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Appendix 3 – Glossary of technical terms

Two lists of technical terms are given below. The first covers terms used in bell acoustics – some of these are used generally in acoustics but have special or restricted definitions when applied to bells. The second list includes an outline of some statistical terms, but is not meant to be a general introduction to statistical analysis.

Bell Acoustics

Antinode	For a particular mode of vibration, a line around the bell parallel to the rim, or a line from rim to crown, where the amplitude of the vibration is maximum. See also nodal circle and nodal line.
Beats	A warbling effect in a sound which contains two frequencies close together. Heard in bells with doublets.
Bourdon	A term often used for a heavy bell (e.g. several tons in weight) and for the bass bells in carillons.
Canons	Loops cast onto the crown of older bells allowing them to be suspended from a headstock or other support.
Carillon	A set of at least 23 bells arranged to be played from a keyboard or by an automatic chiming mechanism.
Cent	A convenient way to measure intervals by the log of the frequency ratio. 100 cents make a semitone, 1200 cents make an octave. The interval in cents between two frequencies f1 and f2 is $1200 * \log_2 (f2 / f1)$. See Equation 3-1.
Chime	To sound a bell by swinging it through a small arc either side of mouth downwards, or by striking it with an external hammer. Also, a set of bells arranged to be rung by chiming.
Clapper	The iron bar or striker, pivoted in the crown of the bell or on an external framework, which hits the soundbow to sound the bell. Usually has an enlarged portion called the ball which actually makes the impact.
Crown	The top of the bell. See Figure 3-8.
Crown partial	Strictly not a partial but a mode of vibration, synonymous with upper partial. The term arises because these partials are most easily heard by striking the bell near the crown.
Diatonic scale	The usual scale in western music, corresponding to the white notes on a keyboard.
Doublet	If a bell is not round, individual partials can split into a close pair of frequencies which beat together. Doublets can add antique charm to a bell's sound but are usually to be avoided.

Equal temperament	A way of tuning the diatonic scale such that all semitones are identical in size. Allows instruments to play in any key (and bells of a given note to be tuned to the same frequency whatever their position in the peal). See also just tuning and meantone temperament.
Extra partial	An alternative name for the prime partial.
Full-circle	A style of bellringing in which the bell turns through 360 degrees from mouth-upwards on a pivoted headstock.
Fundamental	An alternative name for the prime partial.
Harmonic series	A set of partials with frequencies in the ratios of small integers.
Headstock	An iron or wooden beam pivoted at both ends top which a bell intended for full-circle ringing or swing chiming is attached.
Higher partial	A partial higher in frequency than the nominal. Synonymous with upper partial.
Hum	The lowest in frequency of the five low partials of a bell.
Inner tuning	Tuning of an individual bell to bring its partials into a desired relationship (to ‘make it in tune with itself’).
Just tuning	A way of tuning a diatonic scale for which, unlike equal and meantone temperament, all the frequencies are related by small integers.
Low partials	The five partials of a bell lowest in pitch (hum, prime, tierce, quint and nominal).
Meantone temperament	A way of tuning the diatonic scale which represents a compromise between equal temperament and just tuning. Various meantone temperaments were the most common way of tuning musical instruments for a couple of hundred years prior to the middle of the 19th century.
Missing fundamental	A psycho-acoustical effect whereby the ear creates a pitch for a musical sound which does not correspond to a partial frequency in the sound. See also virtual and spectral pitch.
Mode	A way in which a bell can vibrate and produce sound. Different vibrational modes have different nodal lines and circles. Different modes give rise to different partial frequencies in the bell's sound.
Mouth	The opening at the rim of the bell.
Nodal circle	A circle around the bell (i.e. parallel to the rim) which does not move for a particular mode of vibration. Different modes can have different numbers of nodal circles at different heights above the rim.

Nodal line / nodal meridian	A line from rim to crown which does not move for a particular mode of vibration. Different modes can have different numbers of nodal lines around the bell - the number is always even.
Node	For a particular mode of vibration, a part of the bell which is stationary (or at least not moving in the plane of the vibration). See nodal line / meridian and nodal circle.
Nominal	The highest in frequency of the five low partials. Given this name because the strike pitch of a bell is usually about an octave below the nominal; hence this partial provides the note-name of the bell.
Octave nominal	The partial roughly an octave above the nominal. Also called the double octave, especially in the low countries.
Octave stretch	A way in which peals of bells (especially larger numbers) are tuned whereby the smaller bells are sharper than the diatonic scale would suggest they should be. Also seen in other instruments such as pianos.
Old style	Used to describe bells which are not tuned on true-harmonic principles. UK old-style bells usually have hums which are sharp of true harmonic, and almost as often, primes which are flat of true-harmonic.
Outer tuning	Tuning of a set of bells together to bring their strike pitches into a desired relationship (usually as the notes of a diatonic scale). See inner tuning.
Partial	An identifiable frequency in the sound of a bell, arising from a mode of vibration of the bell. If the partial is a doublet (q.v.) it consists of two frequencies close together.
Peal	A set of bells arranged for full-circle ringing. Also, a bellringer’s term for an extended performance of ringing meeting certain criteria.
Pitch	The note-name assigned by a listener to a bell or other musical sound. The formal definition of pitch is ‘that characteristic of musical sounds allowing them to be ordered in a musical scale’.
Prime	The second lowest of the five low partials of a bell. This partial is also called the fundamental, the extra or the second partial.
Profile	This term has two different usages, either the cross-sectional shape of a bell (i.e. the inner and outer form together with the thickness of metal), or the centre line of the cross-sectional shape of a bell. According to the second definition, two bells can have the same profile but different thicknesses. In this thesis, the first definition is used exclusively.
Quint	One of the five low partials of a bell, very roughly a fifth above the strike pitch, i.e. a fourth below the nominal.

Rim	The lip of the bell’s mouth. See Figure 3-8.
Rim partial	Strictly not a partial, but a mode of vibration in which there is a radial antinodal circle at or near the bell's rim. These modes of vibration are stimulated directly by the impact of the clapper and are of great importance in forming the strike pitch of a bell.
Secondary strike	An effect heard in large bells of a second strike pitch, often about a fourth above the primary strike pitch.
Shoulder	The point where the waist of a bell turns over to form the crown. See Figure 3-8.
Simpson	Canon A. B. Simpson, who wrote two important papers on the sound of bells in the 1890s. True-harmonic tuning is sometimes called Simpson tuning.
Soundbow	The thickened portion of a bell at the bottom of the waist, immediately above the rim. See Figure 3-8.
Spectral pitch	The hearing by the ear of a pitch determined by the frequency of a single partial. See also virtual pitch.
Stock	See headstock.
Stretch	See octave stretch.
Strike note	An alternative term for the strike pitch.
Strike pitch	The pitch assigned to a bell by the human ear. It does not correspond to any particular partial except in small bells (though it is often close to an octave below the nominal) and is a virtual pitch effect.
Superquint	One of the names given to the partial roughly a fifth above the nominal. Also known as the twelfth or the octave quint.
Swing-chiming	A style of bell-ringing in which the bell is hung from a pivoted headstock and swings up to 90 degrees either side of mouth down.
Temperament	A way of tuning a diatonic scale. See the entries for equal and meantone temperaments and just tuning.
Tenor	The heaviest and deepest-pitched bell in a peal or chime.
Tierce	One of the five low partials of a bell, usually about a minor third above the strike pitch, i.e. a major sixth below the nominal.
Timbre	The overall effect or quality of a bell's sound.
Treble	The lightest and highest-pitched bell or bells in a peal, chime or carillon.

True harmonic	A system of inner tuning whereby the hum, prime and nominal partials are in octaves.
Tuning	The process of bringing the partials of a bell into harmonious arrangement (inner tuning) and/or the strike pitches of a set of bells into harmonious arrangement (outer tuning). Achieved by removing metal from the inside of the bell. Also used to refer to the relationship of various partial frequencies in a bell, even if in fact they have not been tuned.
Twelfth	An alternative name for the superquint partial.
Upper partial	A partial higher in frequency than the nominal. Synonymous with higher partial.
Virtual pitch	The sensation of pitch created in the ear by multiple partials acting together; the dominant method of pitch perception for bells. See also spectral pitch.
Waist	The area of the bell between the crown and the soundbow. See Figure 3-8.

Statistics

Confidence limit, confidence interval	If an experiment results in a measured value v with 95% confidence limit l, then if the experiment were to be repeated multiple times, we would expect that the measured value for each experiment run would lie in the range $v-l$ to $v+l$ for 95% of the runs. $v \pm l$ is the 95% confidence interval.
Contrast	In a set of experiments with factors at different levels, a contrast is a linear combination of the means of experiment results with factors at the same levels, such that the coefficients of the various means sum to zero. Contrasts are used to isolate the contribution of one factor or a combination of factors to the overall results.
Correlation coefficient	A quantity that, for two series of data values, indicates the strength and direction of any linear relationship between them.
Factor	An independent variable in an experiment.
Mean	The sum of a set of observations divided by the number of observations. Otherwise known as the arithmetic average.
Model	A mathematical relationship between variables used as an approximation for the behaviour of a system being studied.
Normal distribution	A probability distribution which approximates well the distribution of many physical and psychological measurements in the presence of noise and random error.

Regression	The process of fitting a model to a set of observations. Unless otherwise specified, the fitting process generally involves minimising the sum of the squared residuals by adjusting model parameters.
Replicate	In an experiment with factors at different levels, a single set of test runs with all desired factors and factor combinations.
Residual	The difference between the observed or measured value of a quantity, and the value predicted by a model.
Standard deviation	A measure of the spread of a set of values of a population, calculated as the root mean square (RMS) deviation of values from the population’s arithmetic mean.
Standard error	An estimate of the standard deviation from a set of data
Treatment	In an experiment with factors at different levels, a particular set of values for the factors.
Variance	The square of the standard deviation of a set of values.

Appendix 4 – Extracts from Isaac Beeckman's 1633 Journal

Isaac Beeckman (1588 – 1637) was a Dutch philosopher and scientist, one time mentor of René Descartes and a voluminous correspondent with most of the eminent scientists of his age. Though no important scientific discoveries can be uniquely attributed to him (perhaps because he did not publish his work), he acted as an information exchange and had a hand in most of the scientific advances of his time.

His journal and letters are written in a mixture of Dutch and Latin. They were transcribed and published in 1945, with French footnotes – see bibliography entry (Beeckman 1633a, 1633b). The extracts below were translated for me by Henry Wickens.

Jacob van Eyck (c.1590 – 1657) was a blind carillon player most noted by musicians today for the composition of a collection of virtuoso solo pieces for the recorder. Along with the Hemony brothers François and Pieter, he discovered the techniques of true-harmonic tuning of bells. As a result, the Hemony brothers became the dominant founders of carillons in the low countries in the 17th century, and the quality of their work is still admired today.

The first extracts are taken from Beeckman's journal (Beeckman 1633a pp310, 311) and are the earliest reference I have been able to find to the existence of multiple partial frequencies in a bell. The three partials referred to are the hum (lowest), prime (middle, the one said to be often out of tune) and the nominal (called here the strike). Van Eyck was said to be able to stimulate individual partial frequencies in a bell by whistling them. The meaning of the last sentence is a little obscure but I think refers to the fact that the tierce of a bell is a minor rather than a major third. In a vibrating string, the lowest partial sounding a third is major.

Mr van Eyck, bellringer at Utrecht, who is blind and very famous in these matters, told me on 24 Sept. 1633 that the bells always ring 2 octaves under the strike, in other words the strike is the highest, of which the middle (one) is often out of tune, and it is fortunate if this additional sound is in tune; nevertheless, he says that he has discovered the true art of casting bells, which apparently lies in the correct shape.

He also says that he can make each of the aforesaid octaves sound separately without touching the bells and without visibly knocking them with anything. He also says that a minor third under the strike is always heard, which sounds to be a twelfth above the strike. He also says that no man knows nor can know the reason for these resonances, and that God alone knows it.

The aforesaid Mr van Eyck says that the bells do not sound ut mi sol [C E G] but re fa la [D F A], though I think I have recently proved the contrary in a letter to D. Mersenne.

The second extract is taken from a letter from Beeckman to Marin Mersenne written on 30 May 1633 (Beeckman 1633b p214). Mersenne is most famous today for his discoveries about prime numbers, but also wrote one of the early textbooks on musical acoustics (Harmonie Universelle published in 1636). The extract shows the difficulty being experienced at the time in understanding how a vibrating body could stimulate more than one frequency of sound simultaneously. The demonstration by van Eyck later the same year of bell partials was proof that this could occur. It is also possible to infer that octave ambiguities were being experienced, as discussed in chapter 4 of this thesis.

However, I shall say what I think. The string, dispersing the air with its oscillation, breaks it into almost equal particles; however, since all the parts of the string are oscillating at the same frequency, but not at the same speed, and perhaps some particles of air are more easily broken than others, and the thickness of the string is not exactly equal in all parts, it happens that their particles are broken into two, three, four etc. parts. Those which are broken into two represent an octave to the ears, because they have a simultaneous effect time with a double attack; this, however, the mind does not perceive because [the sound], being narrow, is so similar to a unison that it is taken to be the same. Those which go into three parts, which are more dissimilar to each other, appear more clearly to the mind. Those which are broken into four parts are the same as those in two; but those which are cut into five parts are perceived quite clearly, so that because of their similarity a twelfth is taken for a fifth, a seventeenth for a third, hence it happens that bells, all well cast, seem to sound the consonance ut mi sol (C E G). The fact that the fifth, which is a twelfth, is heard either only on certain instruments (e.g. on a string, as you suggested), or especially as in a bell, is not so much because several particles are broken into three than into five parts, because all of them, including those which are broken into six parts, represent a nineteenth (which is also a fifth).

Appendix 5 – List of bells analysed in this research

The listings on the following pages show all the bells analysed during this research. They are all included in the investigation into partial frequencies in Chapter 5. Those for which weights are available were also included in the analysis in Chapter 3. The number of bells listed for each location is the number of recordings in the collection, not necessarily the number of bells installed. Ranges of weights (in kg), nominals (in Hz) and date of casting are included where this information is available.

The information on founders is intended only to be indicative of the wide range of founders covered by the investigation. No attempt has been made, for space reasons, to give specific information on founders, nor to clarify the history of bellfoundries (such as that at Whitechapel in London) which have had a number of proprietors over the centuries.

As well as the author's own investigations in towers and in books listed in the general bibliography, details of a number of bells were obtained from the prototype national bell register included in the online version of Dove's Guide (Dove 2008), and George Dawson's listings (Dawson 2008).

Where a bell is notable, its name or other significant details are given in parentheses after the founder.

Place	Building	Bells	Weights (kg)	Nominals (Hz)	Dates	Founders and notes
André Lehr's papers (Lehr 1965)		2	428 - 567	820.6 - 845.6	-	
André Lehr's papers (Lehr 1986)		11	67 - 1,291	614 - 2,611	?? - 1986	Both, Bourlet, Van Den Ghein, Eijsbout
Experimental and trial bells		16	-	654 - 3,603	1887 - 1999	Unknown, Warner, Taylor, Eijsbout
Albury, Surrey, UK	SS Peter & Paul	6	259 - 792	679 - 1,143	1935	Taylor
Allesley, West Midlands, UK	All Saints	8	159 - 610	808.5 - 1,619.5	1580 - 1946	Newcombe, Smith, Taylor
Amsterdam, Amsterdam, Holland	Munttoren	38	-	541.4 - 6,425	1651 - 1992	Hemony, Petit & Fritsen, Eijsbout
Andover, Hampshire, UK	St. Mary	8	260 - 772	679 - 1,355.5	1758 - 1947	Wells, Lester & Pack, Taylor
Appleton, Oxfordshire, UK	St. Laurence	10	142 - 665	670.5 - 1,700	1817 - 1975	Mears & Stainbank
Arnhem, Holland	St Eusebius	1	-	327	-	
Ashchurch, Gloucestershire, UK	St. Nicholas	6	266 - 595	731 - 1,225.4	1759 - 1763	Rudhall (old six)
Askerswell, Dorset, UK	St. Michael	6	186 - 442	831 - 1,402.5	1619 - 1960	Purdue, Warner, Mears, Taylor
Banstead, Surrey, UK	All Saints	8	254 - 1,079	618.5 - 1,235.5	1921 - 1925	Gillett & Johnston
Barcelona, Catalonia, Spain	S. Maria del Mar	1	-	585.5	-	
Barford St Martin, Wiltshire, UK	St Martin	6	230 - 590	712 - 1,241.5	1732 - 1906	Cockey, Mears & Stainbank
Barham, Kent, UK	St. John Baptist	5	259 - 598	724.5 - 1,097.5	1633 - 1947	Wilnar, Knight, Mears & Stainbank
Barmby Moor, East Yorkshire, UK	St. Catherine	6	165 - 382	885 - 1,483	1598 - 1998	Smith, Mears, Taylor
Barton Turf, Norfolk, UK	St. Michael	6	235 - 540	724.5 - 1,206	1499 - 1672	Darbie, Brend, Brasyers of Norwich
Beaumaris, Anglesey, Wales	St Mary	8	620 - 620	817.5 - 1,668	1819 - 1904	Dobson, Barwell
Beckley, East Sussex, UK	All Saints	6	614 - 614	706.5 - 1,184	1708 - 1910	Phelps, Bowell, Mears
Berlin, Germany	Rathaus Schöneberg	1	9,660	335.2	1950	Gillett & Johnston
Berrow, Worcestershire, UK	St. Mary Magdalen	5	170 - 435	860 - 1,290.5	1914	Gillett & Johnston
Berrynarbor, Devon, UK	St Peter	6	252 - 510	770 - 1,308.5	1722 - 1893	Evans, Taylor
Bethersden, Kent, UK	St. Margaret	6	278 - 790	695 - 1,165.5	1335 - 1953	de Weston, Hatch, Mears
Beverley, East Yorkshire, UK	Minster of St. John	1	7,151	395	1901	Taylor (Great John)
Bickleigh, Devon, UK	St Mary the Virgin	6	236 - 508	687.5 - 1,151.5	1765 - 1937	Pennington, Pannell, Gillett & Johnston
Bildeston, Suffolk, UK	St. Mary Magdalen	6	260 - 723	651.2 - 1,095.5	1450 - 1718	Gardiner, Pleasant, Graye, Gilpin, Danyell, Brasyers of Norwich
Birmingham, West Midlands, UK	St. Martin	16	294 - 2,003	523 - 2,363.1	1989 - 1991	Whitechapel
Bishopstoke, Hampshire, UK	St Mary	10	164 - 572	786.7 - 1,978.5	1995	Whitechapel
Blackheath, Kent, UK	All Saints	2	30 - 556	873.5 - 2,284	1858 - 1879	Warner, Taylor
Blythburgh, Suffolk, UK	Holy Trinity	6	213 - 525	778.5 - 1,301.5	1608 - 1950	Mears & Stainbank
Bolzano, Italy	San Paolo di Apiano	1	-	450.5	-	
Bornheim-Brenig, , Germany	St Evergislus	1	-	563.6	-	
Bradpole, Dorset, UK	Holy Trinity	8	221 - 775	741.5 - 1,484.5	1981 - 1981	Whitechapel
Breisach am Rhein, , Germany		1	-	547.5	-	
Bremhill, Wiltshire, UK	St. Martin	5	275 - 891	664.5 - 1,109.5	1685 - 1770	Keene, Wells, Rudhall (before tuning)

Place	Building	Bells	Weights (kg)	Nominals (Hz)	Dates	Founders and notes
Bremhill, Wiltshire, UK	St. Martin	6	-	694.9 - 1,150.5	1685 - 1826	Keene, Wells, Rudhall (after tuning)
Brentor, Devon, UK	St. Michael	5	166 - 289	1100 - 1,651	1909	Mears & Stainbank
Brewood, Staffordshire, UK	SS Mary and Chad	8	337 - 1,098	615 - 1,233.5	1896	Taylor
Bridport, Dorset, UK	St. Mary	8	296 - 1,067	631.7 - 1,279.5	1924	Mears & Stainbank
Brightling, East Sussex, UK	St. Thomas of Canterbury	8	251 - 626	761 - 1,522	1815 - 1818	Mears & Stainbank
Bristol, Avon, UK	Cathedral	1	410	1,081	1300	Ancient bell in central tower
Brockhampton, Herefordshire, UK	Chapel	3	160 - 260	1,190.5 - 1,505.5	1809 - 1913	Rudhall, Barwell
Bromham, Bedfordshire, UK	St. Owen	8	267 - 1,164	588.5 - 1,185.5	1907 - 1934	Taylor
Brompton by Sawdon, North Yorkshire, UK	All Saints	5	211 - 415	825 - 1,224.5	1500 - 1991	York foundry, Conyers, Whitechapel
Bromsgrove, Worcestershire, UK	St. John Baptist	10	210 - 884	664 - 1,684.5	1701 - 1897	Rudhall, Barwell, Mears & Stainbank
Brundish, Suffolk, UK	St Lawrence	1	-	1,059.5	-	
Brussells, Belgium	St Gudule Cathedral	2	3,300 - 7,000	387 - 467.5	-	
Bryanston, Dorset, UK	St Martin (school chapel)	8	288 - 863	609.5 - 1,231	1898 - 1995	Warner, Whitechapel
Buckfast, Devon, UK	Abbey Church of St Mary	1	7,578	346	1936	Taylor (Hosanna)
Buckland Dinham, Somerset, UK	St. Michael & AA	8	194 - 542	682 - 1,366	1530 - 1896	Bristol foundry, Lott, Rudhall, Warner
Bucklebury, Berkshire, UK	St Mary	8	180 - 658	768 - 1,536	1610 - 1926	Knight, Mears, Bowell
Burton Bradstock, Dorset, UK	St. Mary	6	224 - 491	720 - 1,206	1615 - 1928	Bilbie, Purdue, Mears
Calne, Wiltshire, UK	St. Mary the Virgin	8	?? - 1,202	588 - 1,198	1707 - 1989	Rudhall, Wells, Mears, Whitechapel
Cambridge, Cambridgeshire, UK	Great St. Mary	12	264 - 1,378	544.6 - 1,666.5	1722 - 1952	Phelps, Pack & Chapman, Dobson, Taylor
Castleton, Derbyshire, UK	St. Edmund	8	230 - 584	632.5 - 1,259.5	1803 - 1812	Harrison
Cattistock, Dorset, UK	SS Peter and Paul	8	197 - 704	717.5 - 1,442.5	1950	Whitechapel
Chagford, Devon, UK	St. Michael	8	215 - 924	677.5 - 1,357.5	1914	Taylor
Chalford, Gloucestershire, UK	Christ Church	6	510 - 510	735.5 - 1,258	1857 - 1859	Naylor-Vickers (steel bells)
Chapel-en-le-Frith, Derbyshire, UK	St. Thomas a Becket	6	560 - 560	790 - 1,336.5	1733	Rudhall
Charing, Kent, UK	SS Peter & Paul	6	264 - 640	768.6 - 1,296.5	1878	Taylor
Charminster, Dorset, UK	St. Mary the Virgin	1	358	950	1575	Purdue (old 7th bell)
Charminster, Dorset, UK	St. Mary the Virgin	10	186 - 753	691.5 - 1,764.5	1663 - 1987	Purdue, Mears, Taylor (ringing peal)
Chartres, France	Cathédrale	1	-	481.5	-	
Cheadle, Greater Manchester, UK	St. Mary	8	231 - 694	712 - 1,423	1749 - 1882	Rudhall, Taylor
Checkendon, Oxfordshire, UK	SS Peter and Paul	8	190 - 530	761.5 - 1,543.5	1765 - 1967	Lester & Pack, Mears & Stainbank
Cheltenham, Gloucestershire, UK	St. Mark	5	308 - 826	727 - 1,100	1884	Taylor (old five)
Cheltenham, Gloucestershire, UK	St. Mary	12	230 - 1,150	591.5 - 1,799.5	1823 - 1911	Rudhall, Mears & Stainbank
Chilcompton, Somerset, UK	St. John the Baptist	10	216 - 981	629.5 - 1,565.5	2000 - 2001	Taylor
Chobham, Surrey, UK	St. Lawrence	8	153 - 458	810.1 - 1,616.2	1520 - 1892	Colverden, Eldridge, Mears & Stainbank
Church Knowle, Dorset, UK	St. Peter	3	230 - 279	1038.5 - 1,312	1804 - 1928	Wells, Bond

Place	Building	Bells	Weights (kg)	Nominals (Hz)	Dates	Founders and notes
Churchstow, Devon, UK	St. Mary	6	790 - 790	682 - 1,158	1877	Warner
Città di Castello, Umbria, Italy	Duomo	3	-	599.4 - 775	-	
Cleeve Prior, Worcestershire, UK	St Andrew	1	456	791	1658	Bagley
Cologne, Germany, Germany	Dom	3	5,600 - 24,200	252.3 - 429.6	1448 - 1923	Broderman, Hoernken, Ulrich
Corfe Castle, Dorset, UK	St. Edward Martyr	2	247 - 589	701 - 1,175.8	1828 - 1999	Dobson, Whitechapel
Corley, Warwickshire, UK	Unknown	5	100 - 274	1041 - 1,456.8	1410 - 1937	de Colsale, Hancock, Watts, Taylor
Cothelstone, Somerset, UK	St. Thomas of Canterbury	6	938 - 938	660 - 1,110	1632 - 1897	Purdue, Bayley, Mears
Coventry Cathedral, West Midlands, UK	Cathedral of St. Michael	13	216 - 1,717	548.5 - 1,638	1927	Gillett & Johnston
Cowley, Oxfordshire, UK	St. Luke	4	305 - 1,222	610 - 1,222	1938	Gillett & Johnston (now at Hampstead)
Crediton, Devon, UK	Holy Cross	1	1,357	591.5	2004	Whitechapel (tenor bell)
Croydon, Greater London, UK	St. Peter, South Croydon	10	294 - 1,533	592.5 - 1,473	1911	Gillett & Johnston
Danehill, East Sussex, UK	All Saints	6	?? - 396	1018.5 - 1,723	1897 - 1959	Taylor
Disley, Cheshire, UK	St. Mary the Virgin	6	?? - 372	877.6 - 1,490.8	1837	Mears
Doddiscombsleigh, Devon, UK	St. Michael	6	183 - 540	863 - 1,446.5	1654 - 2001	Pennington, Taylor
Dorchester, Dorset, UK	St. Peter	8	310 - 1,060	644.6 - 1,311.7	1734 - 1889	Bilbie, Warner
Dorking, Surrey, UK	St. Martin	8	279 - 1,168	594.5 - 1,229.5	1626 - 1955	Wilner, Catlin, Mears (old peal)
Dorking, Surrey, UK	St. Martin	10	220 - 935	654.1 - 1,649.5	1998	Taylor (new peal)
Dover, Kent, UK	St. Mary the Virgin	8	242 - 792	631 - 1,272.5	1724 - 1898	Knight, Warner
Dunham Massey, Greater Manchester, UK	St. Margaret	10	302 - 1,306	516.5 - 1,307	1854	Taylor
East Hardwick, West Yorkshire, UK	St Stephen	3	81 - 140	1324 - 1,529	1874	Naylor-Vickers (steel bells)
Edwardstone, Suffolk, UK	St. Mary the Virgin	6	227 - 507	790.2 - 1,301.8	1640 - 1986	Waylett, Graye, Taylor
Egham, Surrey, UK	St. John Baptist	10	165 - 863	707.5 - 1,780.5	1912 - 1971	Gillett & Johnston, Whitechapel
Epsom Common, Surrey, UK	Christ Church	8	150 - 433	880.5 - 1,758	1992	Whitechapel
Epsom, Surrey, UK	St. Martin	10	211 - 814	691.5 - 1,769	1921	Mears & Stainbank
Erfurt, Germany	St Bonifatius	3	430 - 910	765.8 - 985.1	1954	Schilling (steel bells)
Erfurt, Germany	Dom	1	11,367	329.8	1497	Gerhardus Wou de Campis (Gloriosa)
Evesham, Worcestershire, UK	The Bell Tower	14	249 - 1,825	553 - 1,881	1951 - 1992	Taylor
Ewhurst, Surrey, UK	SS Peter & Paul	8	148 - 588	832.5 - 1,661.5	1400 - 1938	Wokingham foundry, Eldridge, Taylor
Exeter, Devon, UK	Cathedral of St. Peter	1	4,064	448.6	1676	Purdue (Great Peter)
Exeter, Devon, UK	Cathedral of St. Peter	1	3,684	460	1902	Taylor (tenor of ringing peal)
Eythorne, Kent, UK	SS Peter & Paul	5	214 - 410	913 - 1,371	1440 - 1924	Chamberlain, Hatch, Warner, Mears
Fairwarp, East Sussex, UK	Christ Church	8	228 - 787	729 - 1,461.5	1936	Gillett & Johnston
Faversham, Kent, UK	St. Mary of Charity	8	260 - 860	663.5 - 1,332.5	1748 - 1930	Catlin, Warner, Mears, Gillett & Johnston
Fewston, Yorkshire, UK	SS Michael & Lawrence	4	260 - 260	1,280.5 - 1,750.5	1808	Mears
Findon, West Sussex, UK	St. John Baptist	6	195 - 489	808 - 1,345	1530 - 1958	Tonne, Cole, Mears

Place	Building	Bells	Weights (kg)	Nominals (Hz)	Dates	Founders and notes
Firenze, Italy	Duomo	1	-	429.4	-	
Flamborough, East Yorkshire, UK	St. Oswald	6	201 - 386	888 - 1,493	1898 - 1990	Mears, Taylor
Folkestone, Kent, UK	Cheriton, St. Martin	8	147 - 371	879 - 1,770.6	1881	Mears & Stainbank
Frankfurt, Germany	Kaiserdom St. Bartholomäus	1	11,850	324.5	1877	Grosse
Freiburg, Breisgau, Germany	Freiburger Münster	1	3,290	365.8	1258	Unknown (Hosanna)
Fürstenfeldbruck, Germany	Erlöserkirche	3	-	694 - 967.6	1926	Bochumer (steel bells)
Gidleigh, Devon, UK	Holy Trinity	5	171 - 392	876 - 1,225	1450 - 1923	Exeter foundry, Pennington, Taylor
Glanvilles Wootton, Dorset, UK	St. Mary the Virgin	1	606	788.5	1996	Whitechapel (tenor bell)
Godmersham, Kent, UK	St. Lawrence	6	225 - 616	740.5 - 1,239	1687 - 1998	Hodson, Whitechapel
Gossau, Switzerland		1	8,695	345.9	1958	Rüetschi
Grappenhall, Cheshire, UK	St. Wilfrid	8	186 - 574	746.4 - 1,493	1700 - 1899	Bagley, Mears & Stainbank, Taylor
Graz, Austria	Herz-Jesu Kirche	2	-	487.3 - 578.6	-	(One steel bell)
Great Bookham, Surrey, Surrey, UK	St. Nicolas	2	-	956 - 1,296.5	1400 - 1675	Burford, Eldridge
Great Driffield, East Yorkshire, UK	All Saints	6	308 - 597	726.5 - 1,222.5	1593 - 1880	York foundry, Smith, Warner
Great Glemham, Suffolk, UK	All Saints	5	660 - 660	719 - 1,064	-	
Great Yarmouth, Norfolk, UK	St. Nicholas	13	248 - 1,551	582 - 1,761.5	1958	Mears & Stainbank
Greenwich, Greater London, UK	St. Alfege	10	274 - 1,200	590.5 - 1,480.5	1824 - 1954	Mears & Stainbank
Halberstadt, Germany	Dom	2	2,290 - 8,320	394 - 568.2	1997 - 1999	Giesserei
Hale, Cheshire, UK	St. Mary	8	134 - 415	883 - 1,765.5	1866 - 1987	Naylor-Vickers, Eijsbout (6 steel bells)
Halesowen, West Midlands, UK	St. John the Baptist	8	240 - 846	613.5 - 1,232.5	1707 - 1864	Smith, Lester & Pack, Warner
Halesworth, Suffolk, UK	St. Mary	8	266 - 926	644 - 1,331.5	1440 - 1824	Church, Brend, Lester & Pack, Pack & Chapman
Hallow, Worcestershire, UK	SS Philip and James	8	286 - 1,100	654 - 1,308.5	1900	Taylor
Halwell, Devon, UK	St. Leonard	6	?? - 620	720 - 1,198.5	1829 - 1932	Pennington, Hambling
Haselbury Plucknett, Somerset, UK	St. Michael and All Angels	6	?? - 349	887 - 1,490	1735 - 1949	Bilbie, Llewellins & James, Mears
Haughley, Suffolk, UK	St. Mary	5	?? - 756	700.2 - 1,060	1572 - 1885	Tonne, Warner
Hazelbury Bryan, Dorset, UK	SS Mary and James	6	222 - 766	701.5 - 1,173.5	1300 - 1934	14th & 15 cent., Wallis, Mears, Taylor
Heathfield, East Sussex, UK	All Saints	8	562 - 562	776.5 - 1,592	1920	Mears & Stainbank
Helensburgh, Scotland	St Columba	1	-	662	1861	Duff
Henfield, West Sussex, UK	St. Peter	8	203 - 841	669 - 1,344	1913	Taylor
Herrenberg, Germany	Stiftskirche	1	1,250	706.5	1483	
Heywood, Greater Manchester, UK	St. Luke	8	261 - 1,109	625 - 1,250.5	1921	Gillett & Johnston
High Halden, Kent, UK	St. Mary the Virgin	6	225 - 673	761 - 1,241.4	1609 - 1955	Hatch, Bowell, Mears & Stainbank
Hilmarton, Wiltshire, UK	St. Laurence	6	292 - 668	650 - 1,136.5	1424 - 1874	Bristol foundry, Purdue, Rudhall, Whitechapel
Holborn, Greater London, UK	Italian Church of St. Peter	1	4,420	395.5	1862	Naylor-Vickers (largest steel bell in UK)
Holborn, Greater London, UK	St. Giles in the Fields	8	235 - 718	723 - 1,462.5	1635 - 1736	Knight, Wightman, Phelps

Place	Building	Bells	Weights (kg)	Nominals (Hz)	Dates	Founders and notes
Holcombe Burnell, Devon, UK	St. John the Baptist	6	206 - 571	831.5 - 1,394.5	1450 - 1989	Pennington, Taylor
Horley, Surrey, UK	St. Bartholomew	8	213 - 608	740 - 1,492.5	1812 - 1927	Shore, Mears, Mears & Stainbank
Hothfield, Kent, UK	St. Margaret	6	191 - 430	866 - 1,487	1762 - 1927	Lester & Pack, Bowell
Houston, Texas, USA	Palmer Church	8	123 - 315	983 - 1,966	2005	Whitechapel
Houston, Texas, USA	St Thomas	8	112 - 359	974.5 - 1,977.5	1971	Taylor
Houston, Texas, USA	St. Paul's Methodist	8	?? - 560	782.3 - 1,568.7	2001	Whitechapel
Hovingham, North Yorkshire, UK	All Saints	6	220 - 572	790.5 - 1,331.5	1878	Taylor
Hythe, Kent, UK	St. Leonard	10	250 - 1,005	644 - 1,621	1914 - 1992	Mears & Stainbank, Whitechapel
Innsbruck, Austria	St Jacob Cathedral	1	19	2382	-	Grassmayr (Tirolean bell)
Innsbruck, Austria	St Jacob Cathedral	8	202 - 7,168	389 - 1,184.2	1846 - 1961	Grassmayr
Interlaken, Bernese Oberland, Switzerland	Schlosskirche	3	-	566 - 834.5	1926	Rüetschi
Ipswich, Suffolk, UK	St. Mary le Tower	14	203 - 1,768	546 - 1,633.5	1610 - 1999	Graye, Darbie, Warner, Whitechapel, Taylor
Kenn, Devon, UK	St. Andrew	6	277 - 684	702 - 1,173.5	1826 - 1826	Mears
Kensington, South, Greater London, UK	Imperial College	10	337 - 1,950	553.6 - 1,405.4	1892	Taylor
Kevelaer, Germany		1	-	430.5	1954	Bochumer (steel bell)
Kidderminster, Worcestershire, UK	St. Mary and All Saints	12	255 - 1,255	585 - 1,741.5	1754 - 1935	Rudhall, Mears, Taylor, Gillett & Johnston (old peal)
Kingston upon Hull, East Yorkshire, UK	Guildhall	1	1,930	480.5	1865	Warner
Kingston, Dorset, UK	St. James	10	318 - 1,366	603 - 1,517	1878 - 2000	Taylor
Krakow, Poland	Wawel Royal Castle	1	11,000	385.2	1520	Beham (Sigismund)
Kronplatz, Tirol, Italy		1	18,100	283	2000	Oberascher (Concordia)
La Vinzelle, Aveyron, France	Church	1	-	605.6	1870	Triadou Cazes Pourcel
Lahore, Pakistan	Cathedral of the Resurrection	8	274 - 902	686 - 1,850.5	1862 - 1903	Taylor
Laigueglia, SV, Italy		1	-	543.5	1872	Mazzola
Lamerton, Devon, UK	St. Peter	6	255 - 596	811 - 1,370	1878	Warner
Lavenham, Suffolk, UK	Guildhall	1	-	2,591	1896	Goslin (ex school bell)
Leatherhead, Surrey, UK	SS Mary and Nicholas	10	315 - 971	615.5 - 1,544	1816 - 1877	Mears, Warner
Leverkusen, Germany	Herz Jesu	1	3,300	441	1902	Bochumer (steel bell)
Linz, Austria	Neuer Dom	1	8,120	348.2	1901	Gugg (Immaculata)
Little Chart, Kent, UK	St. Mary of Holy Rood	6	136 - 314	980 - 1,666	1956	Taylor
Little Horsted, East Sussex, UK	St. Michael and All Angels	6	242 - 594	772 - 1,294	1440 - 1863	Chamberlain, Mears
Little Somerford, Wiltshire, UK	St John the Baptist	3	-	823 - 969.5	1725 - 1753	Burrough, Tosier
Litton Cheney, Dorset, UK	St. Mary	8	234 - 696	693.5 - 1,399	1400 - 1950	15th cent., Baker, Purdue, Mears, Whitechapel
Liverpool, Merseyside, UK	Cathedral Church of Christ	1	15,013	277	1940	Taylor (Great George)
Liverpool, Merseyside, UK	Cathedral Church of Christ	12	481 - 4,171	413.1 - 1,248	1938	Whitechapel (ringing peal)
Liverpool, Merseyside, UK	All Saints, Childwall	6	245 - 650	773 - 1,302	1912	Warner

Place	Building	Bells	Weights (kg)	Nominals (Hz)	Dates	Founders and notes
Llandaff, South Glamorgan, UK	Llandaff Cathedral	13	242 - 1,239	623.2 - 1,871.3	1992	Whitechapel
Loddiswell, Devon, UK	St. Michael	6	210 - 478	771 - 1,317	1782 - 1910	Pennington, Warner, Taylor
London, Great London, UK	Cornhill, St Michael	12	283 - 2,136	484 - 1,467	1728 - 1968	Phelps, Lester, Mears & Stainbank
London, Greater London, UK	St Paul's Cathedral	1	17,002	317.2	1881	Taylor (Great Paul - heaviest bell in the UK)
London, Greater London, UK	St Paul's Cathedral	1	5,205	425.5	1716	Phelps (Great Tom)
London, Greater London, UK	St Paul's Cathedral	12	421 - 3,130	465.5 - 1,455	1878	Taylor (ringing peal)
London, Greater London, UK	Houses of Parliament	5	1,321 - 13,762	335.1 - 869.5	1856 - 1858	Warner, Mears (including Big Ben)
London, Greater London, UK	St. Mary le Bow	12	284 - 2,131	521.5 - 1,565.6	1956	Whitechapel
Loughborough, Leicestershire, UK	Taylor's Bellfoundry	1	-	1,171.24	1982	The bell experimented on by Perrin, Charnley & DePont
Louviers, France		1	-	727.4	-	
Lundy Island, Devon, UK	St Helena	10	248 - 681	742 - 1,871.5	1897 - 2004	Carr, Taylor
Lustenau, Vorarlberg, Austria	Kath SS Peter & Paul	5	-	489.5 - 790	-	(Steel bells)
Lyminge, Kent, UK	SS Mary and Ethelburga	8	-	685.5 - 1,383	1631 - 1904	Wilnar, Knight, Lester & Pack, Mears (before tuning)
Lyminge, Kent, UK	SS Mary and Ethelburga	8	248 - 811	660 - 1,326.5	1631 - 1904	Wilnar, Knight, Lester & Pack, Mears (after tuning)
Lymm, Cheshire, UK	St Mary V	8	305 - 1,515	598.5 - 1,199.5	1891	Taylor
Lyndhurst, Hampshire, UK	St Michael	8	196 - 559	781.5 - 1,569	1868 - 1948	Warner, Taylor
Madresfield, Worcestershire, UK	St. Mary Virgin	5	496 - 699	826 - 1,449.5	1867	Taylor
Maitland, NSW, Australia	St Paul	6	720 - 720	901.5 - 1,490	1868 - 1869	Naylor-Vickers (steel bells)
Malborough, Devon, UK	All Saints	6	196 - 486	835 - 1,413	1936 - 1973	Gillett & Johnston, Taylor
Manchester, Greater Manchester, UK	Manchester Town Hall	1	8,269	378.5 - 379.4	1882	Taylor (Great Abel)
Manchester, Greater Manchester, UK	Manchester Town Hall	22	100 - 2,170	505.1 - 1,906.5	1937	Taylor (ringing peal and carillon)
Manchester, Greater Manchester, UK	Holy Innocents, Fallowfield	8	241 - 694	711 - 1,425	1863	Mears
Mappowder, Dorset, UK	SS Peter & Paul	6	210 - 638	718 - 1,196.5	1735 - 1998	Knight, Whitechapel
Marston Bigot, Somerset, UK	St. Leonard	3	190 - 365	973.5 - 1,456.5	1912 - 2002	Warner, Whitechapel (3 of 8 bells)
Mary Tavy, Devon, UK	St. Mary the Virgin	5	527 - 527	767 - 1,176	1720 - 1878	Pennington, Warner
Meersburg, Germany		1	-	534.2	-	
Melbourne, Victoria, Australia	Cathedral of St. Paul	13	249 - 1,491	559 - 1,714	1889	Mears & Stainbank
Merthyr Tydfil, Mid Glamorgan, Wales	St. Tydfil	8	253 - 1,023	690.1 - 1,383.7	1893 - 1895	Taylor
Milton Abbey, Dorset, UK	St. Sampson	8	180 - 510	813.5 - 1,626	1861 - 1908	Warner, Llewellins & James
Mirfield, Yorkshire, UK	St Mary	10	318 - 1,547	591.9 - 1,487.2	1869	Taylor
Monkton Farleigh, Wiltshire, UK	St Peter	1	-	1,358.50	-	
Montalcino, Italy		2	-	854.5 - 904	-	
Montserrat, Barcelona, Spain	Monastery	1	-	864.1	-	
Monza, Italy	Duomo	2	-	431.4 - 489.6	1741	Bozzio
Moretonhampstead, Devon, UK	St. Andrew	8	215 - 765	701.5 - 1,405.5	1922	Gillett & Johnston

Place	**Building**	**Bells**	**Weights (kg)**	**Nominals (Hz)**	**Dates**	**Founders and notes**
Mösern, Tirol, Austria		1	10,180	306	1997	Grassmayr
Moscow, Russia	Kremlin	1	65,320	204.4	1817	Zavayalov & Rusinov
Neustadt an der Weinstraße, Germany	Stiftskirche	1	14,000	309.6	1949	Bochumer (Kaiser Ruprecht, the largest steel bell in Europe)
New Alresford, Hampshire, UK	St. John Baptist	7	313 - 808	672 - 1,355	1811 - 1936	Mears, Taylor
New Buckenham, Norfolk, UK	St. Martin	6	598 - 598	826 - 1,387.5	1814	Dobson
Newcastle upon Tyne, Tyne and Wear, UK	Newcastle Cathedral	1	6,020	412.9	1891	Taylor (Major)
Newcastle upon Tyne, Tyne and Wear, UK	Newcastle Cathedral	3	-	619 - 832	1400 - 1425	York 15th cent., Dawe (disused bells)
Newcastle upon Tyne, Tyne and Wear, UK	Newcastle Cathedral	14	241 - 1,913	545 - 1,848.5	1892 - 2000	Taylor (ringing peal)
Newport, Kentucky, USA		1	33,285	222.1	1998	Paccard (Peace Bell)
Newton le Willows, Merseyside, UK	St. Peter	10	259 - 1,247	620.4 - 1,558.4	1903 - 2001	Taylor
Northiam, East Sussex, UK	St. Mary	6	210 - 757	731 - 1,215.5	1737 - 1914	Phelps, Gillett & Johnston
Norwich, Norfolk, UK	St. Peter Mancroft	14	321 - 1,925	502 - 1,697.5	1775 - 1997	Pack & Chapman, Mears, Whitechapel
Nottingham, Nottinghamshire, UK	Council House	1	10,528	308	1928	Taylor (Little John)
Ockley, Surrey, UK	S Margaret	6	688 - 688	720 - 1,216	1701 - 1981	Phelps, Whitechapel
Odcombe, Somerset, UK	SS Peter and Paul	6	159 - 605	748 - 1,268.5	1420 - 1791	Bristol foundry, Webb
Old Wolverton, Buckinghamshire, UK	Holy Trinity	6	270 - 620	774.7 - 1,329.5	1820	Briant
Ospringe, Kent, UK	SS Peter & Paul	8	197 - 880	729 - 1,469.6	1891	Taylor
Oxford, Oxfordshire, UK	Christ Church	1	6,325	450.6	1680	Hodson (Great Tom)
Oxford, Oxfordshire, UK	Christ Church	12	245 - 1,585	574.5 - 1,738.5	1410 - 1952	Unknown, Bird, Knight, Rudhall, Mears & Stainbank (ringing peal)
Oxford, Oxfordshire, UK	St Margaret	1	210	1,674.70	1857	Warner
Oxford, Oxfordshire, UK	St. Mary the Virgin	1	1,372	591	1639	Knight (tenor of peal of 6)
Oxford, Oxfordshire, UK	St. Mary Magdalen	2	-	1,665.5 - 1,766	1973	Whitechapel (previous bells, now recast)
Oxford, Oxfordshire, UK	St. Mary Magdalen	10	117 - 374	879 - 2,224	1988 - 2001	Taylor (new peal of 10)
Oxford, Oxfordshire, UK	St. Thomas the Martyr	6	-	998.5 - 1,663.5	1706 - 1806	Rudhall, Mears (old peal of 6)
Oxford, Oxfordshire, UK	St. Thomas the Martyr	10	115 - 587	801 - 2,024.6	1992 - 1996	Taylor (new peal of 10)
Oxford, Oxfordshire, UK	Magdalen College	10	260 - 970	653.5 - 1,668.5	1410 - 1848	Dawe, Knight, Rudhall, Taylor
Oxford, Oxfordshire, UK	New College	10	210 - 1,020	601 - 1,546	1655 - 1723	Darbie, Knight, Rudhall
Paderborn, Germany	Dom	5	620 - 2,800	433.5 - 729.5	1951	Bochumer (steel bells)
Painswick, Gloucestershire, UK	St. Mary the Virgin	14	249 - 1,304	582 - 1,988.7	1731 - 1993	Rudhall, Whitechapel
Paris, France	Notre Dame	1	12,800	357.3	1685	Chapelle, Gillott & Moreau
Penmon, Anglesey, Wales	Lighthouse	1	-	844	-	
Pescina, Italy	San Berardo	1	-	555.2	-	
Peter Tavy, Devon, UK	St. Peter	6	510 - 510	800.5 - 1,409	1722 - 1882	Pennington, Llewellins & James
Petham, Kent, UK	All Saints	6	167 - 318	936.5 - 1,590.5	1923	Mears & Stainbank
Philadelphia, Pennsylvania, USA	Wanamaker's Store	1	15,774	288.5	1926	Gillett & Johnston

Place	Building	Bells	Weights (kg)	Nominals (Hz)	Dates	Founders and notes
Philadelphia, Pennsylvania, USA	Christ Church	11	299 - 925	720 - 1,609	1754 - 1953	Lester & Pack, Mears, Paccard
Plymouth, Devon, UK	St. Budeaux	6	233 - 621	738 - 1,245.5	1780 - 1931	Pennington, Mears & Stainbank, Taylor
Plymouth, Devon, UK	Laira, SMV	8	189 - 565	786 - 1,573	2000	Whitechapel
Pocklington, East Yorkshire, UK	All Saints	1	864	658.5	1914	Warner (tenor of peal of 8)
Poplar, Greater London, UK	Christ Church, Cubitt Town	8	191 - 558	740 - 1,491.5	1854 - 1907	Mears
Poynton, Cheshire, UK	St. George	6	225 - 694	758.1 - 1,274.5	1887	Taylor
Preston Plucknett, Somerset, UK	St. James	6	210 - 474	793.5 - 1,349.5	1588 - 1869	Wiseman, Mears
Preston, Lancashire, UK	St. John	12	229 - 881	684 - 2,072.1	1920 - 2003	Mears & Stainbank, Whitechapel
Ranmore, Surrey, UK	St. Barnabas	9	?? - 1,000	641 - 1,300	1859	Mears
Robbio, Italy	S. Michele	5	210 - 680	677.2 - 1,015.6	1886 - 1926	Mazzola
Rodmersham, Kent, UK	St. Nicholas	6	269 - 483	818 - 1,386.5	1380 - 1893	Burford, Wilnar, Darbie, Mears & Stainbank
Rolvenden, Kent, UK	St. Mary the Virgin	8	243 - 765	699.5 - 1,402	1819	Mears
Rome, Italy	St Peter's Basilica	1	9,800	326.9	1786	Valadier
Ronca, VR, Italy	Chieseta S Margherita	1	-	1,467	1865	Cavadini
Rostherne, Cheshire, UK	St. Mary	6	?? - 1,016	613.1 - 1,063	1630 - 1782	Hutton, Oldfield, Rudhall
Rostov, Russia	Uspensky Cathedral	12	106 - 32,760	252.3 - 2,150.5	1600 - 1911	Various Russian founders
Rovereto, Italy	Campana dei Caduti	1	22,639	249.9	1964	Capanni (Maria Dolens)
Rusper, West Sussex, UK	St. Mary Magdelen	8	200 - 720	710 - 1,425	1699 - 1897	Eldridge, Warner
Saalfeld, Thuringia, Germany	Unknown	1	-	495	-	(Seigerglocke)
Saltash, Cornwall, UK	SS Nicholas and Faith	6	269 - 681	706 - 1,176	1760 - 1959	Bilbie, Mears & Stainbank, Taylor
Salzburg, Salzburg, Austria		2	?? - 14,256	316.2 - 475	1961	Oberascher (Salvator Mundi)
Scalby, North Yorkshire, UK	St. Laurence	6	157 - 483	824 - 1,381	1674 - 1961	Smith, Taylor
Schoneberg, Berlin, Germany	Apostel Paulus Kirche	1	-	423	-	
Scilly Isles, St Mary's, Cornwall, UK	St Mary, Hugh Town	1	413	899	1869	Mears & Stainbank
Scottow, Norfolk, UK	All Saints	6	200 - 540	818 - 1,409	1713 - 1936	Newman, Bowell
Sens, France	Cathédrale	2	13,660 - 16,230	294 - 338.5	1560 - 1563	Mongin-Viard (Potentienne and Savinienne)
Sepino, Italy		1	-	570.4	-	
Shalford, Surrey, UK	St Mary V	8	204 - 689	710 - 1,421.5	1936	Taylor
Sheffield, South Yorkshire, UK	St. James the Great, Norton	8	227 - 790	715.7 - 1,437.5	1896	Taylor
Sheldwich, Kent, UK	St. James	8	130 - 367	934 - 1,865.5	1998	Whitechapel
Shipton Gorge, Dorset, UK	St. Martin	6	132 - 252	1067 - 1,794.5	1655 - 1996	Purdue, Warner, Whitechapel
Silandro, Valvenosta, Italy		1	2,650	554.6	1800	Zach
Slancio, Tirol, Italy		1	2,750	513.5	1726	Grassmayr
Smarden, Kent, UK	St. Michael	6	307 - 937	676.5 - 1,156	1922	Bowell
Sopley, Hampshire, UK	St Michael & All Angels	6	166 - 371	923 - 1,568	1784 - 1963	Patrick, Gillett & Johnston, Taylor

Place	Building	Bells	Weights (kg)	Nominals (Hz)	Dates	Founders and notes
South Littleton, Worcestershire, UK	S Michael Archangel	3	211 - 420	859 - 1,056	1420 - 1901	Worcester foundry, Taylor
South Milton, Devon, UK	All Saints	6	620 - 620	743.5 - 1,263.5	1766 - 1886	Pennington, Warner
Southwold, Suffolk, UK	St. Edmund K&M	8	257 - 551	707.5 - 1,419.3	1500 - 1881	Brasyers of Norwich, Barker, Dobson, Darbie, Moore Holmes & McKenzie
St Gallen, Switzerland	St Gallen Cathedral	1	7,500	322.5	-	
St. Emillion, France	Monococh Church	1	-	667.5	-	
St. Germans, Cornwall, UK	St. Germanus	8	204 - 727	686 - 1,379	1775 - 1984	Pennington, Mears & Stainbank, Taylor
Staveley, Yorkshire, UK	All Saints	3	80 - 140	1451.5 - 1,592	1864	Naylor-Vickers (steel bells)
Steyning, West Sussex, UK	St. Andrew	8	225 - 630	739.5 - 1,496.5	1889	Mears & Stainbank
Stockport, Greater Manchester, UK	St. George	10	309 - 1,517	574.5 - 1,467	1896	Mears & Stainbank
Stoke by Nayland, Suffolk, UK	St. Mary the Virgin	8	268 - 1,123	582.2 - 1,177.4	1380 - 1956	Unknown, Sturdy, Pleasant, Gardiner, Mears, Taylor
Stratton-on-Fosse, Somerset, UK	Downside Abbey	1	5,423	409.4	1900	Taylor (Great Bede)
Tallinn, Estonia	Alexander Nevsky Cathedral	1	15,665	338.7	1898	Orlov
Tavistock, Devon, UK	St. Eustachius	10	258 - 1,222	604 - 1,525.5	1925 - 1998	Taylor
Teignmouth, Devon, UK	St. Michael	7	261 - 733	703.5 - 1,251.2	1886 - 1887	Llewellins & James
Tenterden, Kent, UK	St. Mildred	8	363 - 1,420	540 - 1,108.5	1769 - 1961	Lester & Pack, Mears, Warner, Bowell
Termeno, BZ, Italy	Slancio Tirolese	3	?? - 2,750	515.5 - 833	?? - 1726	Grassmayr
Tewkesbury, Gloucestershire, UK	Tewkesbury Abbey	4	285 - 728	656.5 - 1,115	1696 - 1725	Rudhall (disused bells)
Tewkesbury, Gloucestershire, UK	Tewkesbury Abbey	13	251 - 1,387	583 - 1,785.8	1962	Taylor
Teynham, Kent, UK	St. Mary	6	245 - 549	810 - 1,360	1743 - 1924	Catlin, Mears & Stainbank
Thatcham, Berkshire, UK	St. Mary	10	178 - 671	696.5 - 1,757	1624 - 1969	Knight, Mears, Taylor
Throwley, Kent, UK	St. Michael & All Angels	8	200 - 626	748 - 1,494.5	1780 - 1933	Pack & Chapman, Mears & Stainbank
Toller Porcorum, Dorset, UK	St. Andrew	6	220 - 439	770.5 - 1,290.5	1550 - 1990	Unknown, Purdue, Bilbie, Pyke, Taylor
Tong, Shropshire, UK	St. Bartholomew	1	2,350	528	1892	Taylor
Torquay, Devon, UK	St. Mary Church, St. Mary	10	265 - 887	658 - 1,651.5	1877 - 1989	Warner, Whitechapel
Towcester, Northamptonshire, UK	St. Lawrence	12	226 - 1,188	648.1 - 1,952.7	1897 - 1989	Taylor
Trondheim, Norway	St Nidaros	1	-	755.5	-	
Tuebrook, Merseyside, UK	St. John Baptist	1	613	718.5	1869	Warner (bell no. 7, part tuned)
Tunstall, Kent, UK	St. John the Baptist	8	203 - 524	792 - 1,589	1995	Taylor
Turners Hill, West Sussex, UK	St. Leonard	8	?? - 711	714 - 1,445.5	1924 - 1926	Mears & Stainbank
Tytheringon, Gloucestershire, UK	St. James Greater	6	172 - 460	789.4 - 1,335.2	1617 - 1959	Purdue, Rudhall, Llewellins & James, Taylor
Ullenhall, Warwickshire, UK	St Mary the Virgin	8	70 - 164	1,219.5 - 2,467	1874	Warner
Verona, Italy	Catthedrale	1	-	448.2	-	(Old tenor bell)
Verona, Italy	Catthedrale	9	337 - 4,566	420.5 - 947.8	1931 - 2003	Cavadini, De Poli
Vicenza, Italy	Trissino	1	1,480	511.5	-	Cavadini
Viellevie, Aveyron, France		4	-	1,020.5 - 1,401.5	1582 - 1808	

Place	Building	Bells	Weights (kg)	Nominals (Hz)	Dates	Founders and notes
Vienna, Austria	Stephansdom	2	5,700 - 20,132	263.8 - 405.4	?? - 1951	Geisz (Pummerin) and (Stephanus)
Wallingford, Oxfordshire, UK	St. Mary le More	8	274 - 964	614.5 - 1,231	1738 - 1887	Phelps, Mears & Stainbank
Waltham, Kent, UK	St. Bartholomew	6	217 - 433	884 - 1,475.5	1602 - 2000	Hatch, Taylor
Walton, Somerset, UK	Holy Trinity	5	340 - 839	610.7 - 915	1569 - 1935	Purdue, Austen, Mears
Wantage, Oxfordshire, UK	SS Peter & Paul	8	308 - 1,099	597.5 - 1,199	1669 - 1892	Knight, Lester & Pack, Taylor, Mears & Stainbank
Wargrave, Berkshire, UK	St. Mary	8	282 - 894	678.9 - 1,380.4	1915	Mears & Stainbank
Warnham, West Sussex, UK	St. Margaret	10	223 - 742	717 - 1,810	1897 - 1998	Mears & Stainbank, Whitechapel
Waterloo, Greater London, UK	St. John, Waterloo Road, Lambeth	8	289 - 895	655.2 - 1,312	1825	Mears
Wells, Somerset, UK	Cathedral of St. Andrew	10	399 - 2,864	519.1 - 1,318	1757 - 1964	Rudhall, Taylor, Mears & Stainbank
Welton, Northamptonshire, UK	St. Martin	5	374 - 740	709 - 1,087.8	1629 - 1825	Watts, Taylor
Wenhaston, Suffolk, UK	St. Peter	6	205 - 612	756 - 1,291.4	1450 - 1956	Brasyers of Norwich, Mears & Stainbank, Taylor
West Alvington, Devon, UK	All Saints	6	820 - 820	718 - 1,203.5	1775	Bilbie
Westcote, Gloucestershire, UK	St Mary	1	279	1,073	1614	Farmer
Whitchurch Canonicorum, Dorset, UK	St. Candida and Holy Cross	8	323 - 872	588.5 - 1,197	1603 - 1912	Purdue, Wiseman, Mears & Stainbank, Warner
Wilmslow, Cheshire, UK	St. Bartholomew	6	690 - 690	727.5 - 1,221	1733	Rudhall
Winterborne Kingston, Dorset, UK	St. Nicholas	6	198 - 479	771.5 - 1,284	1600 - 1998	Wallis, Ellery, Whitechapel
Winterborne Whitechurch, Dorset, UK	St. Mary	6	249 - 563	757 - 1,261	1300 - 1990	Salisbury 14th cent., Purdue, Tosier, Warner, Taylor
Winterton, Lincolnshire, UK	All Saints	1	480	817	1734	Hedderley (tenor of 6)
Wolverhampton, West Midlands, UK	St Peter	12	246 - 1,686	543.5 - 1,623.4	1911	Gillett & Johnston
Wonersh, Surrey, UK	St John Baptist	8	180 - 596	779.5 - 1,566	1958	Mears & Stainbank
Woodchurch, Kent, UK	All Saints	6	310 - 812	697 - 1,179.5	1608 - 1905	Hatch, Lester & Pack, Mears & Stainbank
Woolhampton, Berkshire, UK	Douai Abbey	1	-	1,808	-	French founder
Woolverstone, Suffolk, UK	St Michael	3	-	1,170 - 1,455.5	-	
Worcester, Worcestershire, UK	Worcester Cathedral	12	347 - 2,439	490.5 - 1,471	1928	Taylor
Worth Matravers, Dorset, UK	St. Nicholas of Myra	6	124 - 308	1,046.5 - 1,759.5	1951 - 1997	Gillett & Johnston, Taylor
Yeovil, Somerset, UK	St. John Baptist	10	395 - 2,068	513.5 - 1,294	1626 - 1931	Bilbie, Warner, Mears & Stainbank
York, North Yorkshire, UK	Minster	1	11,009	305.8	1927	Taylor (Great Peter)
York, North Yorkshire, UK	Minster	3	408 - 3,020	454 - 1,365.5	1925	Taylor (1, 5 and 12 of ringing peal)
York, North Yorkshire, UK	All Saints North St.	2	213 - 242	1,034 - 1,158	1627 - 1640	Oldfield (disused bells)
York, North Yorkshire, UK	All Saints North St.	6	149 - 382	905.5 - 1,523	1920 - 1989	Taylor
York, North Yorkshire, UK	St. Andrew, Bishopthorpe	6	203 - 720	733 - 1,221	1672 - 1990	Smith, Taylor
York, North Yorkshire, UK	St. Olave, Marygate	6	222 - 464	826.5 - 1,385	1789	Dalton
Youlgreave, Derbyshire, UK	All Saints	10	308 - 1,205	587.5 - 1,478	1870 - 1997	Mears & Stainbank, Taylor
Zurich, Switzerland	Church of St Peter	1	-	628	-	
Zurich, Switzerland		2	-	523 - 655.8	-	

Appendix 6 – Ringing World article on true-harmonic tuning

This article was originally published in *The Ringing World* of 29 March 2002 (pages 312-3). It documents some of my early investigations into the rediscovery of true-harmonic tuning by Taylor's bellfoundry at the end of the 19th century.

Taylors of Loughborough and true-harmonic tuning

My thanks are due to David Bryant, Steve Ivin and Chris Pickford for wittingly or unwittingly providing insights and information which led to the analysis below. Errors and unsubstantiated opinions are of course my own.

In the late 19th and early 20th centuries, Taylor's bellfoundry was the first in the UK to adopt 'true-harmonic' tuning. A working definition of this tuning style is that the prime and hum of each bell are in octaves with the nominal. The innovations made by Taylors eventually transformed the business of UK bellfounding. At the same time, Canon Simpson was publishing the results of his research. To what extent did Simpson influence Taylors in their work? This investigation tries to answer that question by looking at the tuning of Taylor peals from the period, with interesting and unexpected results.

A history of Taylor's activities in the period from the 1870s onwards appears in Jenning's book 'Master of my Art'. The sequence of events given by Jennings in chapter 8, supplemented with other information, is as follows:

- up to the 1870s, Taylors were casting peals of bells to a very thick scale on the recommendation of Beckett Denison
- in 1870, they purchased their first set of tuning forks
- in 1875, Ellis published the first UK edition of Helmholtz's 'Sensations of Tone'.
- by 1878, Taylors had begun to abandon Denison's designs
- in the 1880s, it is said that Taylors were making visits abroad to inspect continental bells
- in 1890, Lord Rayleigh published the results of his research into bell acoustics
- in about 1894, Taylors purchased a set of forks in 4 Hz steps from 340 to 540 Hz
- in 1894, Canon Simpson first wrote to Taylors to explain his ideas
- in the mid-1890s, Taylors established a tuning shop and purchased a full set of forks
- by May 1895, Taylors were routinely recording the figures for hum, prime, tierce and nominal in their tuning books
- in autumn 1895, Simpson published his first paper
- in February 1896, Taylors took delivery of a new tuning machine
- in autumn 1896, Simpson published his second paper

With these events as a background, it is illuminating to analyse the tuning of Taylor peals which survive from this period. I have taken the following peals from my collection of recordings to illustrate the changes in tuning which took place:

- 1876 - Hovingham, N. Yorks (6 bells)
- 1878 - St Paul's Cathedral, London (12 bells)
- 1887 - Poynton, Cheshire (6 bells)

- 1892 - Newcastle Cathedral (10 original bells, one being replaced in 1928)
- 1892 - Knightsbridge, Imperial Institute (10 bells)
- 1897 - Towcester, Northants (8 bells, originally at Todmorden)
- 1903 - Lahore, Pakistan (6 bells)
- 1908 - Bromham, Bedfordshire (6 bells, plus two of 1931, recording thanks to Steve Ivin)
- 1913 - Henfield, West Sussex (8 bells)

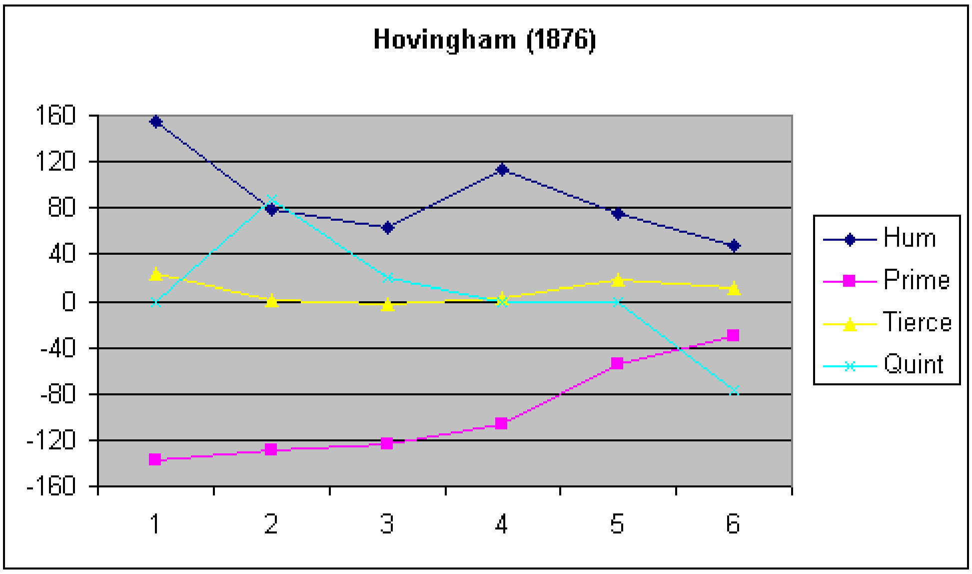
The evidence from these peals is that Taylors went through a gradual transition in the way they tuned their bells, lasting many years, but with sudden steps forward. The advances made by Taylors over this period probably involved significant changes in the design of their bells (i.e. proportions, shape and thickness) as well as in the methods used to tune them. These changes in shape could potentially have an impact on the relative intensity of partials, which significantly affects the sound, but this aspect has not yet been investigated, primarily due to lack of recordings taken on a consistent basis. What is investigated here is just the inner and outer tuning of the lowest five partials of the peals.

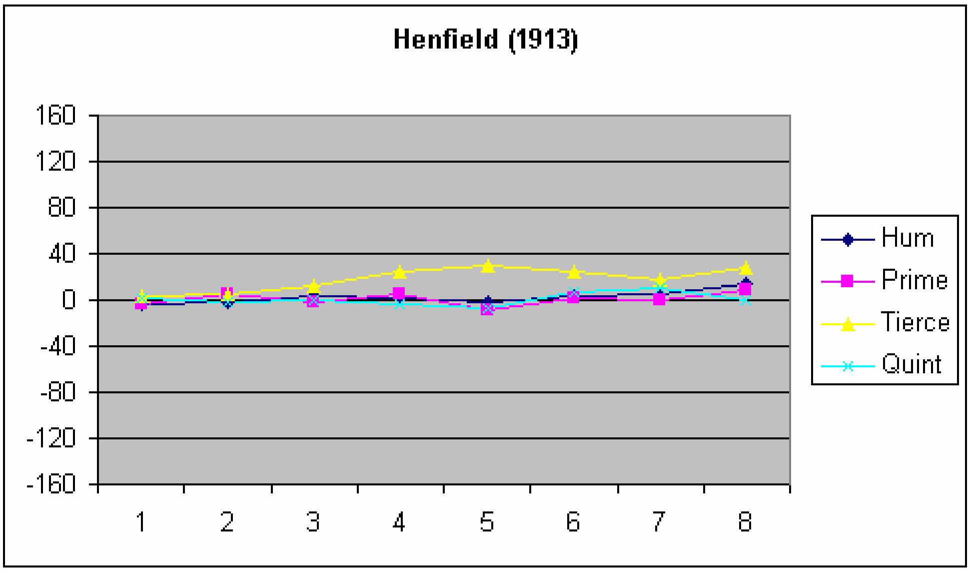
Inner tuning is concerned with the relationship of the partials within a single bell. Outer tuning is concerned with the relationship of the pitches of all the bells in a peal. The first section below deals with inner tuning.

Inner tuning of Taylor peals

The analysis of the chosen peals of bells will only cover five partials: the relationship of hum, prime, tierce and quint to the nominal partial. Other partials could with advantage be investigated, because not only do they affect the sound of the bell, but give clues to changes in shape. This is for later. In a true-harmonic bell, the hum is two octaves or 2400 cents below the nominal, and the prime one octave or 1200 cents below this partial. As tuning skills and bell designs advanced it also became custom to tune the tierce to a minor third above the strike pitch (i.e. 900 cents below the nominal) and the quint to a fifth above the strike (i.e. 500 cents below the nominal). Though this is sometimes known as 'Simpson' tuning, Simpson himself made no such recommendation about the tierce, suggesting that either major or minor thirds were acceptable. The main determinant of tierce and prime tuning is the design of the bell. It is not practical to make major changes to these partials independent from the others on the tuning machine. The prime is most affected by the thickness of the shoulder, the tierce by the design of the soundbow.

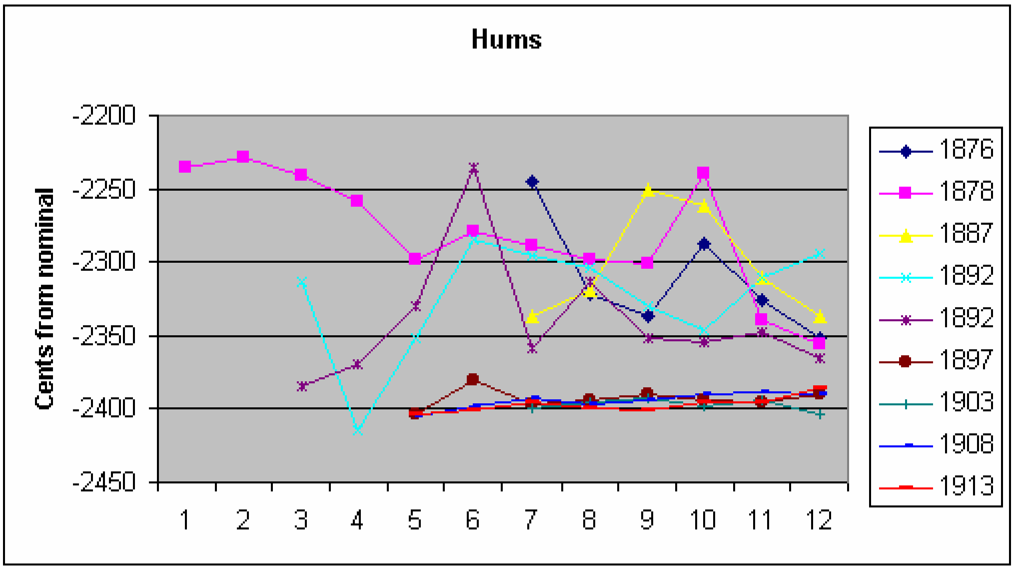
The extent of the changes made by Taylors in their tuning methods is easily seen by comparing the first and last peals in the chosen series. The two plots below are of Hovingham (1876) and Henfield (1913). The plot shows the deviation in cents from these theoretical figures for the four chosen partials across the bells in the peals:

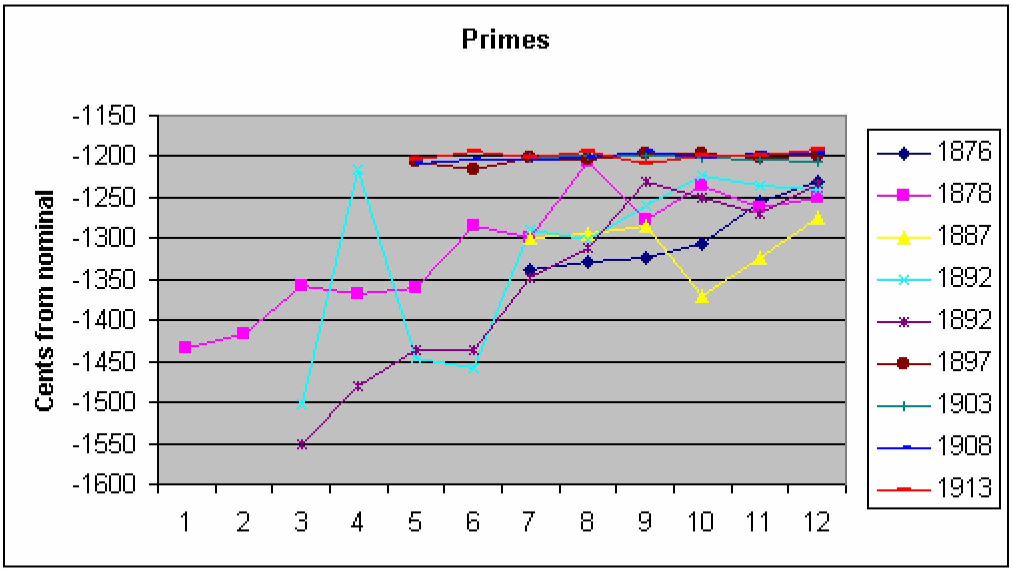


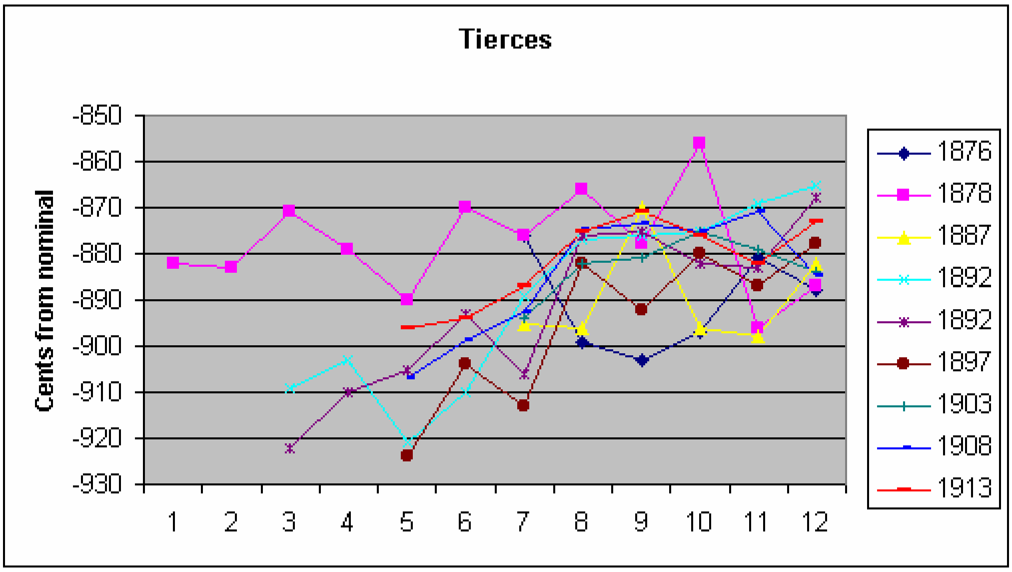


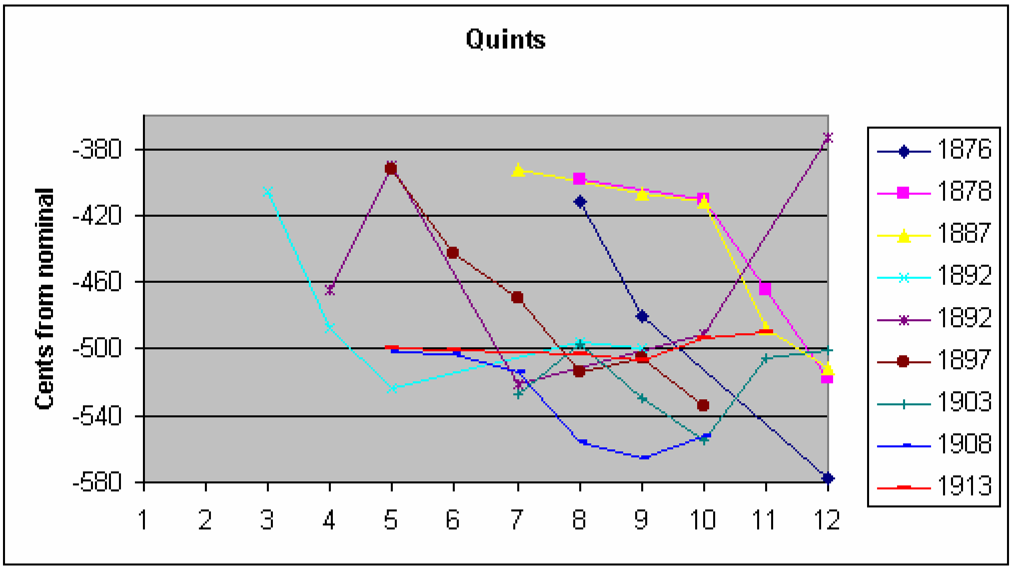
The Hovingham bells are classic 'old-style': hums sharp by up to one and a half semitones (150 cents) and primes flat by a similar amount in the trebles. The variations are less extreme in the tenors. The quint wanders about with no real pattern, but in contrast to all this, the tierce is really quite well controlled in all the bells. Henfield are a complete contrast. Hum, prime and quint are all essentially spot on, but still with variation in the tierce.

To show how Taylors changed the tuning of the various partials, below I show four plots, for hum, prime, tierce and quint. Each plot shows all nine peals of bells. The peals are aligned according to the heaviest bell, which appears as point 12 in the plot. It is to be understood that the point labelled '12' is actually the 10th of 10, 8th of 8 etc. There are two peals from 1892 - Newcastle is the light blue line, Imperial College is dark purple. The bell labelled '4' at Newcastle (actually the second of the old ten) was replaced in 1928, and stands out clearly in all the plots.


Hums
Cents from nominal
-2200
-2250
-2300
-2350
-2400
-2450
1
2
3
4
5
6
7
8
9
10
11
12
1876
1878
1887
1892
1892
1897
1903
1908
1913


Primes
Cents from nominal
-1150
-1200
-1250
-1300
-1350
-1400
-1450
-1500
-1550
-1600
1
2
3
4
5
6
7
8
9
10
11
12
1876
1878
1887
1892
1892
1897
1903
1908
1913


Tierces
Cents from nominal
-850
-860
-870
-880
-890
-900
-910
-920
-930
1
2
3
4
5
6
7
8
9
10
11
12
1876
1878
1887
1892
1892
1897
1903
1908
1913



It is clear, looking at the plots for hums and primes, that a change of considerable significance took place between 1892 and 1897. Both hums and primes in the peals up to 1892 show great scatter, but from 1897 onwards the tuning is quite exact (with the exception perhaps of the bell labelled '6' at Towcester, the 1897 peal). The hums in the back six of the two 1892 peals are perhaps slightly better than in the earlier peals, and the heaviest bells of the second 1892 peal (Imperial) are better than those of the first (Newcastle). It is thought that the Imperial bells were cast later but research is needed to confirm this. On the other hand, the primes of the 1892 peals are significantly and consistently worse than the earlier peals, especially in the trebles. Analysis of additional peals between 1892 and 1897 is clearly going to be of great interest. In 1892, Taylors had perhaps begun to address the tuning of hums and primes, though the evidence is inconclusive. By 1897, they had achieved the true-harmonic ideal.

Tuning now to the tierces, a quite different picture emerges. The 1878 bells at St Paul's show moderately consistent tierce tuning in all but the back three. In the rest of the peals the tierce tuning is not at all consistent, with nothing like the control shown of hums and primes in the later peals. In all except St Paul's, there is a general drift towards flatter tierces in the smaller bells, which suggests some transition of design from front to back. In the last peal (Henfield, in 1913) the slope from front to back is beginning to level off and the scatter is less. Even in this peal, Taylors were not tuning the tierce to the theoretical value.

The 1928 bell at Newcastle (bell 4 on the light blue 1892 line) has a tierce very close to the target value of 900 cents below the nominal, as do the front two bells at Bromham (cast in 1931). Analysis of other Taylor bells of these dates suggests that they made a further change to their bell designs in the 1920s, and from the mid/late 20s were consistently producing minor third tierces.

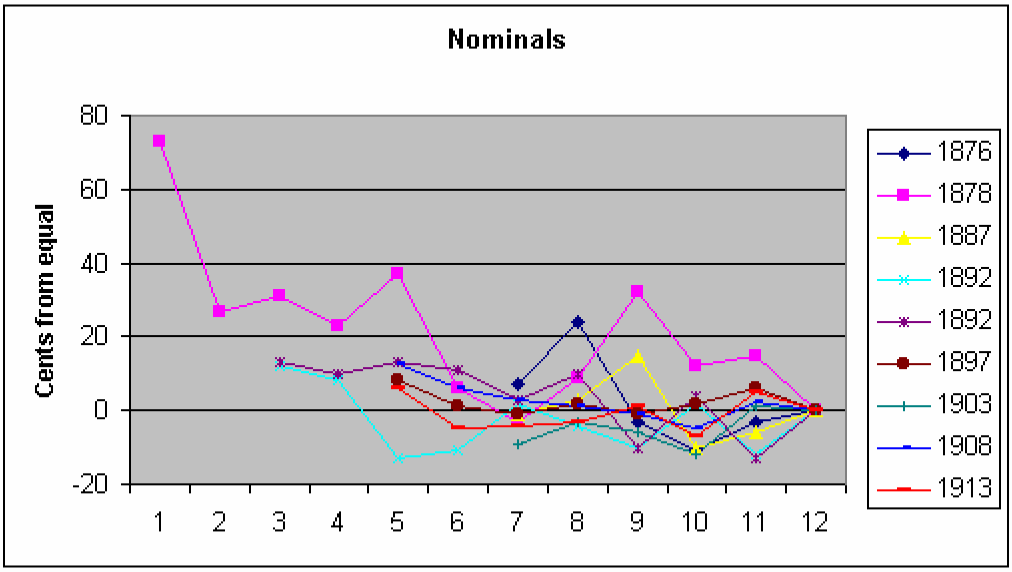
The last plot shows quint tuning. The quint is usually not an important partial, as it is very quiet in most bells. In the recordings I analysed for this study, it could not be detected in a number of the bells. In most of the peals examined here, there is even less pattern in the quints than in the tierces. But strikingly, in the last peal (Henfield, 1913) the quints are tuned almost exactly to the theoretical value. The two previous peals, of 1903 and 1908, show this partial slowly coming under control. The two front bells at Bromham, of 1931, also have quints tuned exactly to the theoretical value.

In summary, there was a dramatic change in Taylor's tuning of hums and primes beginning perhaps in the early 1890s, and complete by 1897. By 1897, Taylors had mastered true harmonic tuning. But it wasn't until just before the first world war that they discovered how to tune the quint, and it took until the 1920s and a further change in bell design for them to finally master the tierce. The irony is that they mastered the quint first even though it is the tierce, much the louder partial, which has the greater effect on the timbre of a true harmonic bell. With this basic roadmap through the transition, there are clear pointers as to the dates of peals to be analysed to confirm the sequence of events.

Outer tuning of Taylor peals

It is not possible to show the transitions in outer tuning in such a simple way. This is because there is not one right way to tune the nominals of a peal of bells. They can be tuned in equal temperament (in which all semitones are the same size), just tuning (which is the easiest to calculate by hand), or in many other temperaments. In addition, bells are from time to time tuned with stretch, so that the trebles are sharper than one would expect. For some background on this see the section of this website on nominal tuning.

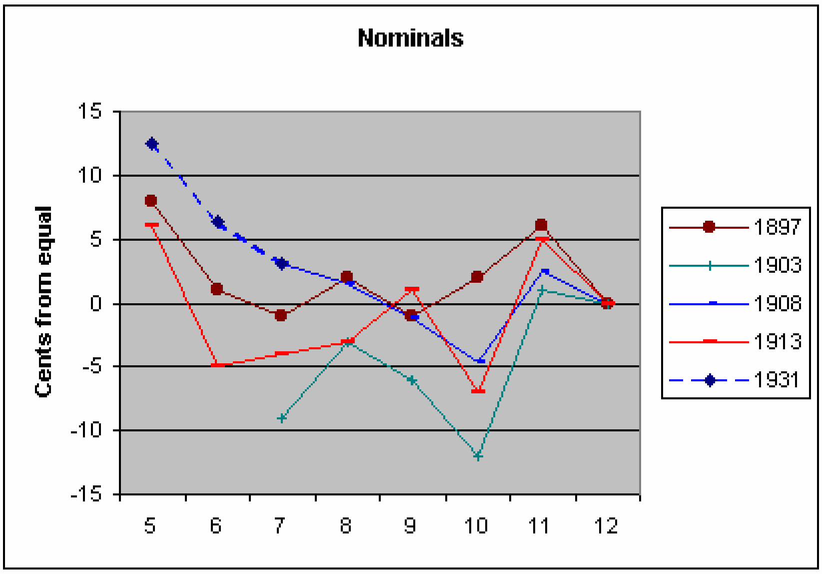
The plot that follows shows the tuning of all the nominals of the peals. The vertical axis shows the cents deviation of each bell's nominal from equal temperament:



The three oldest peals on the plot - Hovingham in 1876, St Paul's in 1878 and Poynton in 1887 - show nominals which are pretty scattered. The St Paul's bells also have considerable stretch. This might be present in the two sixes also but it would be speculative to assume this across such a small number of bells. Clearly, in these peals Taylors were not tuning their nominals at all accurately and so it is not practical to say what temperament or tuning style was being used. Research into Taylor's records would be necessary to see on what principles they tuned their nominals.

In the next two peals - Newcastle and Imperial College, both cast in 1892 - a pattern is beginning to emerge. Apart from the three bells labelled 5, 6 and 8, all the bells have nominals which are very close indeed. (The 4th at Newcastle is a replacement bell.) However, the two peals have slightly different pitches, so were not tuned against each other. Both peals are stretched, a little. Neither conform to any particular temperament.

The plot below shows the nominals of the later peals alone on a larger scale, for clarity. The bells added to the 1908 Bromham peal in 1931 are shown with a dotted line.



By 1897, when the Towcester bells were produced, the nominals were being tuned much more accurately. The last three peals - Lahore in 1903, Bromham in 1908 and Henfield in 1913 - are tuned in something close to Just - shown by the sharpness of the bell labelled 11, and the flatness of the bells labelled 6, 7 and 10. The Henfield and the (1931) Bromham trebles are sharp, giving some stretch across the peal, and giving the relative flatness of the bells labelled 6 and 7 required for Just tuning. The Towcester bells follow a similar shape, but the bell labelled 10 is rather sharp. The tuning of all the nominals in the last four peals is much closer than anything in the earlier ones.

There is clearly something interesting happening here with the nominal tuning, but analysis of further peals is needed to be certain what Taylor's intentions were. The tuning books of the time give intended frequencies as well as actual achieved on the tuning machine. Research in these records would probably shed a lot of light on what principles guided the choice of nominals.

The open question

One fascinating question is the influence which Simpson had over Taylor's true-harmonic tuning. Did he start the process with his letter in 1894 or were Taylors already experimenting? The evidence from this analysis is inconclusive and more work is needed!

Appendix 7 – Ringing World article on the strike pitch of bells

This article was published in *The Ringing World* of 20 June 2003 (page 589 et seq.). It documents some very early results from the research reported in this thesis.

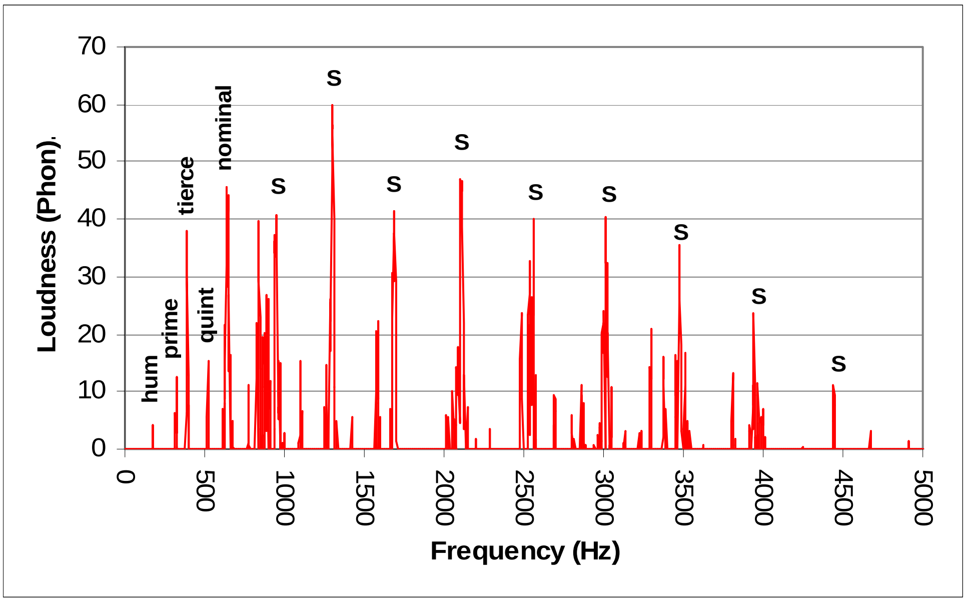
The Strike Note of Bells – an old mystery solved

What determines the pitch of a bell – the note we assign it – has been one of the continuing puzzles of campanology. It was Lord Rayleigh in 1890 who first established scientifically that the pitch of a bell was about an octave below its nominal partial, and that the pitch need not correspond to any actual mode of vibration of the bell. No explanation for this was attempted, and bell-founders have been content to tune the nominals of a peal of bells and assume that this would ensure they sounded in tune with one another. However, recent research has shown that a bell's pitch is determined by a set of partials, not just the nominal, and that tuning the nominal of a bell to the other nominals in a peal can result in it sounding noticeably out of tune!

In this article I give the theoretical explanation of this remarkable effect, followed by two real examples.

The Theory

It is the ear, not any instrument, that ultimately determines what we hear when a bell is struck. The ear does not respond equally to sounds of different frequency. The plot shows the partials of a bell, the tenor at Ranmore in Surrey, with a vertical scale of actual loudness in the ear. (The scale is in phons, a logarithmic scale, weighted according to the Fletcher-Munson equal loudness curves.) The low five partials (hum, prime, tierce, quint and nominal) are marked. It is not obvious why the nominal should play such an important part in setting the pitch of the bell.



What is very noticeable is the regular pattern of partials - starting with the nominal and including those labelled with an 'S' - roughly equally spaced and almost always the loudest in a particular frequency range. This regular pattern is seen in every bell I have analysed with an identifiable pitch. These partials are those most stimulated by the impact of the clapper on the soundbow and for simplicity I call them the strike partials below.

Regularly spaced partials like this in a sound generate an effect in the ear called virtual pitch or the missing fundamental, giving rise to a very strong pitch sensation at a frequency roughly equal to the spacing between the partials. The origin of this effect is not certain, other than that it is generated somewhere within the ear, auditory nerve or brain, but its existence has been researched in many different sounds and is a matter of scientific fact. That the virtual pitch sensation is very strong, and can dominate any sensation of the actual partials in the sound, can be intuitively understood because it is the result of so many strong partials. In a bell of normal shape, the spacing of the lower strike partials gives a virtual pitch of roughly the half nominal, just as we experience in practice.

This explanation of the pitch of a bell was first proposed in the 1960s, and investigated and demonstrated by Eggen and Houtsma in Holland, and Terhardt in Germany, in the 1980s. As further confirmation of the effect, bells with a poor tone or ill-defined pitch typically do not have the clearly defined pattern of strike partials visible in the example I have given. Although it is known qualitatively that a bell's pitch goes up when the strike partials get further apart, and goes down when they get closer, the exact relationship between partial spacing and virtual pitch has not yet been established.

The Effect in Practice

Now for some practical examples, which show why this effect needs to be taken into account when tuning bells. The spacing between the strike partials depends a lot on the thickness of the bell, i.e. its weight relative to the note we hear. In ringing peals, it is traditional to cast the trebles to a heavier scale than the tenors to make then easier to ring together. I was first alerted to the effects I describe when I began to notice that the trebles of peals of eight could sound flat, even though measurements of their nominals showed them to be exactly in tune. In particular, I noticed when ringing at Southwold in Suffolk on a ringing trip a couple of years ago that the trebles sounded quite flat to me even though they were retuned about ten years before.

I recently had the opportunity to go back to Southwold and take a confirmatory set of recordings, and I am very grateful to the ringers there for allowing me to use them as a demonstration. The back six bells at Southwold, though they are from a range of founders dating from the 15th to the 19th centuries, all have strike partials of comparable and wide spacing. The two trebles, both cast by Moore, Holmes and Mackenzie in 1881, have strike partials unusually close together, which by the theory should flatten the pitch.

I carried out experiments, sharpening the trebles by adjusting the recordings, and discovered that for the bells to sound in tune I had to sharpen the treble by a huge 35 cents or over 1/3 of a semitone, and the second by 20 cents or 1/5 of a semitone. These results have since been confirmed by an acquaintance, a carillon player, with a good musical ear. There is no problem with the tuning of the bells; they were tuned by Whitechapel to their usual exact standards and have nominals correct to within a cent or so.

As another example, I have recently investigated the trebles of the twelve at Kidderminster, two true-harmonic Gillett and Johnston bells of 1935. These bells are soon to be replaced with a new peal, and are to be re-deployed by the Keltek Trust. As is usual in the trebles of twelve, these bells are thick and heavy for their note, and have correspondingly flat and closely-spaced strike partials. As part of a discussion on a potential new home for these bells, I put recordings of them together with six old-style bells to form an eight. The six tenors had wider spaced strike partials than the Kidderminster bells, though not as wide as the back six at Southwold. Even so, I had to put the treble nominal sharp by 20 or 25 cents to get it to sound in tune.

This effect of a lowered pitch in bells with a thicker profile provides an elegant and convincing explanation of the use of stretch tuning. In many old-style rings of eight or more, and in higher-numbered rings produced both by Whitechapel and Taylors in the mid-20th century, the trebles were tuned sharp. We can now understand that this would be necessary, in a peal tuned by ear rather than with forks, to make the trebles sound in tune. The effect does not just apply to trebles, it is an issue for any bell in a ring with upper partials spaced differently than the rest.

This flattening of the strike note is best demonstrated with bells rung together in changes. When bells are rung singly some listeners, especially professional musicians, hear individual partials rather than virtual pitch, and would describe trebles tuned with stretch to be sharp. As with everything in bell tuning, there is no single way to please everyone.

Though the virtual pitch effect is well proven scientifically, there is more research to do – in particular, to establish the relationship in bells between the strike partial spacing and virtual pitch. However, the effect is so remarkable and unexpected that I thought it worth writing this preliminary report on my investigations.

Bill Hibbert
Great Bookham, Surrey

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Lord Rayleigh; On Bells; The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, Fifth Series, Vol. 29, No. 176, January 1890.

E. Terhardt and M. Seewann; Auditive und objektive Bestimmung der Schlagtonhöhe von historischen Kirchenglocken; Acustica 1984 Vol. 54 pp. 129-144.

J. H. Eggen and A. J. M. Houtsma; The pitch perception of bell sounds; Institut voor Perceptie Onderzoek, Annual Progress Report 21, 1986 pp. 15-23.

Appendix 8 – English translation of Terhardt's 1984 paper

Aural and algorithmic determination of the strike note of historical church bells

This paper was originally published as "Auditive und objective Bestimmung der Schlagtonhöhe von Historischen Kirchenglocken" in Acustica 1984 Vol. 54 pp. 129-144. I translated it from the original German into English in autumn 2003.

One of the objectives of the work described in the paper was to test an algorithmic method of pitch determination. However, the paper also includes important information on the strike pitches of bells and how they are generated.

Contents of the paper are as follows:

Aural and algorithmic determination of the strike note of historical church bells

by E. Terhardt and M. Seewann
Chair for Electroacoustics, Technical University of Munich

Dedicated to the 60th birthday of Professor E. Zwicker, Doctor of Engineering.

Summary

If a bell sound is capable of evoking a predominant pitch sensation, one assigns a so-called strike note to it. Depending on various characteristics of the sound, the strike note can be the same pitch as a partial tone or can be a virtual pitch. In two different experiments with 15 and 11 test subjects the strike notes were aurally determined for 17 and 137 historical church-bells. With 3% of the bells, no significant strike note could be determined. With 69%, there was a dominant strike-note which was unambiguous with respect to musical note name, though not necessarily with respect to octave position. To the extent that the partial tones can be arranged in today's usual pattern (Hum, Prime, Tierce, Quint, Nominal, Tenth, Twelfth, Octave Nominal), the octave position of the strike note is often ambiguous as to the Hum vs. Prime position. Fine tuning of the strike note on the average is sharp relative to the Prime, and flat relative to the Nominal. Algorithmic determination of the strike notes was carried out with the procedure of Terhardt et al. [18]. With 79% of the 137 bells, the musical note-name of the strike notes was correctly

identified by the procedure. If the strike note of a bell is sufficiently pronounced, it can be determined algorithmically with almost perfect performance. As a criterion of pitch strength, the calculated pitch weight is used. If it exceeds the average value of the historical bells investigated (0.62), this is sufficient for a successful determination. It is shown that the algorithmic procedure also meets realistic expectations for determination of the octave position and fine tuning of the strike note.

1. Introduction

The strike note heard when a bell is struck is still considered to be an unsolved mystery of acoustics. By strike note is meant the dominant musical pitch of the bell sound. Depending upon the nature and sound quality of a bell the strike note can be more or less pronounced or unique. It is because of the strike note that a musical note can be ascribed to a bell when it is struck. Experience shows that the strike note phenomenon is closely associated with the sound of the bell during the first second after the clapper impact [4, 6].

The term “strike note” is in certain respects unfounded and misleading. We therefore intend to describe what is heard as the “strike pitch”. On the one hand this preserves the connection to the original designation, and on the other hand we can maintain the distinction between tone and pitch: A tone is in the language of acoustics a special sound, i.e. a physical thing. In contrast to this the pitch is a characteristic of a tone, i.e. its felt height, and is a sensory (subjective) thing. The solution of the "strike note mystery" lies not in finding a tone, but in explaining how the described pitch sensation arises from the physical sound. This formulation of the problem makes it clear that it does not cover only an isolated, bell-specific mystery, but rather a special case of a general psycho-acoustical problem, that of explaining the pitch perception of any waveform.

Bell sounds represent a special challenge to any pitch theory. While one can find directly identifiable features of the pitch with most tonal waveforms of daily life either in the time function or in the amplitude spectrum (or in both), this is not generally the case with bell sounds. In general there is neither a periodicity of the sound pressure time function at the strike pitch, nor a partial tone extractable by spectrographic analysis, whose magnitude is sufficient to account for the strike pitch. Nor are strike pitches explicable as aural difference tones ("internal partial tones" created by non-linearities in the ear). Indeed it has long been proved by psycho-acoustical experiments that neither a partial tone is required in the spectrum, where a pitch is perceived [12, 9], nor need the waveform have a periodicity at the appropriate frequency [9]. Psycho-acoustic theory shows that instead, pitch perception is generally to be understood as a process of spectral shape perception [15, 9]. This explanation extends to the strike pitches of bells, and the theory can provide a basic scientific explanation of observed effects.

However, so far only limited experience is available regarding the extent to which existing pitch theories give reliable and exact determination of strike pitch. The work documented here makes a particular contribution to this question. We report on the determination of the strike pitch for a total of 137 historical church bells from the Lower Franconia area dating from 1249 to 1845. Understandably older bells often only have a moderate sound quality and the clarity of their strike pitch often leaves much to be desired. Such bell sounds therefore provide a particularly stringent test of algorithmic strike pitch determination procedures. Tape recordings of the sounds of the individual bells [footnote - We thank Dr. M. Nitz (Bavarian National Office for the Care of Monuments, Munich) who made the tape recordings as part of the art-historical documentation of the bells] were aurally judged by 15 test subjects, and were also subjected to a pitch analysis based on virtual pitch theory [15]. The procedure was recently described in detail [18]. The second type of analysis is called algorithmic strike pitch determination, because it uses a fixed, specified algorithm. Particularly detailed aural pitch measurements using the comparison method were carried out by a sub-group on 17 bells, as has already been briefly reported [13]. The data obtained has been further analysed in this study and compared with the appropriate algorithmic results. First, summarised below as a starting point for the actual investigations, are the significant physical and aural sound features of the bells.

2. Partial tone structure and pitch analysis

Some of the pitch sensations caused by bell sounds are of "primary" type, i.e., they correspond directly to certain partial tones in the sound; these are called spectral pitches. A second type of pitch sensation results from holistic perception of the sound (aural effect); these are called virtual pitches [15]. The dominant strike pitch under various conditions can be either a spectral or a virtual pitch. Both spectral pitches and virtual pitches depend on the partial tone spectrum. The nature of this dependency is described by the detailed pitch computation method already referred to [18]. In the description below the results and criteria of the underlying psycho-acoustical pitch theory are applied to bell sounds. The most important result of this work is validation of the procedure for algorithmic pitch analysis suggested by us; in addition, a contribution is also made to the general understanding of perception of bell sounds. Various different results of the investigations are explained and illustrated.

2.1. Natural modes of vibration and partial tones

A bell represents a vibratory system, which possesses a multiplicity of natural modes of vibration. The frequencies and attenuation factors of these modes depend only on the form and the material of the bell, if one ignores non-linear effects (dependency of the natural frequencies on the amplitude) and the effect of temperature (the measurements of Jones [5] and Stueber and Kallenbach [14] show that the temperature coefficient of the natural frequencies amounts to approximately 1.5 x 10-4 per deg. C with bronze bells). The natural frequencies, when stimulated by the blow of a clapper, have a large initial oscillation of very short duration with a large bandwidth. The initial amplitude of each natural oscillation depends on the strength and the place of the clapper impact. This set of natural oscillations radiated as airborne sound produces a waveform, which the sound of the bell is called in the environment. The sinusoidal spectral components of the sound, which can be found by spectrographic analysis, are called partial tones. Their frequencies correspond with good approximation to those of the appropriate natural modes of vibration of the bell. To the amplitudes this does not apply; that is, the partial tone amplitudes measured in the sound signal depend not only on the amplitudes of the natural oscillations, but additionally on the sound radiation and propagation conditions. Nevertheless, the sound signal arriving in the ear is the authoritative source of the audible sound impression, so that it is realistic to describe bell sounds by the partial tone spectra of the sound signals in the environment instead of giving merely the partial frequencies. The partial frequencies determine the sound impression to a considerable degree, and are in some sense "invariant parameters" of a bell, so that they rightfully play a large role in discussions of a bell's acoustic characteristics.

In the procedure for algorithmic pitch analysis the partial frequencies and amplitudes are determined from a digitally calculated Fourier spectrum (FFT) of the sound pressure time function (with time window 80 ms; bandwidth 5 kHz), by selecting the most prominent amplitude maxima of the quasi-continuous spectral function [18]. The spectrographic analyses on which all the algorithmic determinations were performed, were made with a delay of 40 ms after the clapper blow, in order on the one hand to capture the sound essentially at the beginning, while on the other hand avoiding the initial clapper impact noise. The sound signals were recorded in the bell chamber from approximately 2 m. distance on tape. The bell was rung by hand with its own clapper.

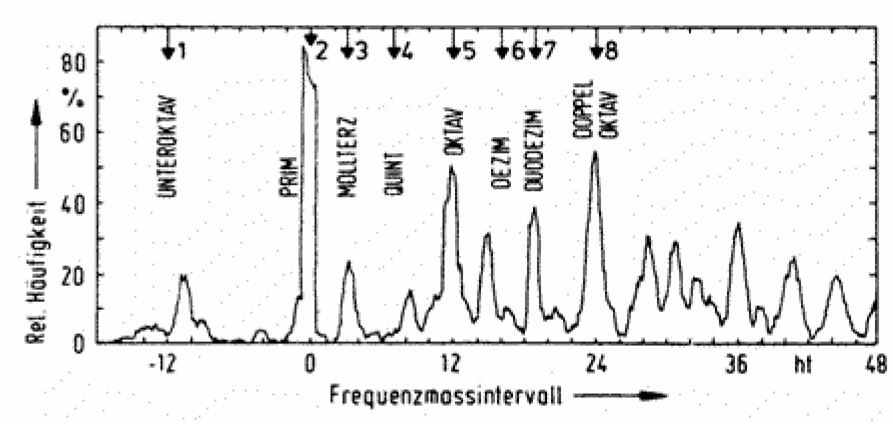

Fig. 1. Frequency of occurrence of audible partial frequencies in 137 bells in each semitone interval, plotted against the centre frequency of the "window". The frequency in semitones along the abscissa is defined by the frequency measure interval of 12 x log2 (f/fs), whereby the frequency is related to the calculated strike pitch frequency; the latter corresponds to the zero point of the abscissa. The ordinate shows the proportion of the total bells with a partial at that frequency. The numbered arrows at the top margin mark the ideal position of the partial tones.

The relationship between the partial frequencies does not follow a simple, natural pattern, and depends on the shape and cross section of the bell. In modern church bells the first eight partial frequencies are required to be in the ratios 1 : 2 : 2.4 : 3 : 4 : 5 : 6 : 8 and these ratios are achieved with good approximation [2, 11]. Older church-bells can deviate substantially from these ratios. Fig. 1 illustrates the frequency ratios observed by us in 137 historical bells. The calculated strike pitch was taken as the frequency reference, and the deviation of the frequency of each observed partial tone from it was indicated by the frequency measure interval of 12 x log2 (fP / fS) semitones. (fP = partial tone frequency; fS = appropriate strike pitch frequency). The ordinate shows the frequency of occurrence of individual partial frequencies within a semitone-wide "window"; the window was continuously shifted along the abscissa. Only those partial tones, which are separately audible in the sound (see section 2.2.) were considered. The ideal partial frequencies are shown in the top margin by perpendicular arrows and the usual designations of the partial tones by numbers or names are indicated.

The diagram shows that the partial tone structure of older bells in many individual cases deviates considerably from the ideal, although it is clear that first eight partial tones tend at least to approximate to it. The frequency of the first partial tone deviates substantially from an octave below the second (or the strike pitch) and tends to be either a seventh or a ninth (see [2, 11, 19]).

If with all the examined bells the pitch of the second partial tone were identical to the strike pitch, a vertical rectangle of width 1 semitone and height 100% would appear, positioned symmetrically about the zero point in the diagram. The actual position, shape and height of the distribution show that there is indeed a high measure of agreement between the pitch of the second partial tone and the strike pitch. However it is clear that the second partial tone, if in this area, is usually below rather than above the strike pitch.

The two octave tones (Nos. 5 and 8) were audible at the ideal frequencies in approximately half of the cases, as can be inferred from Fig. 1. The frequency maxima belonging to the Tierce, Quint, Tenth and Twelfth are markedly smaller. Nonetheless there are unique maxima for the Tierce and Twelfth in a position corresponding well with the desired values. Clear deviations from the ideal can be seen for the Quint (4th partial tone) and the Tenth (6th partial tone). The 4th partial tone shows a pronounced tendency towards a sixth (8 - 9 semitones), and the 6th Partial tone tends to a minor tenth (15 semitones) in place of the major tenth preferred nowadays (16 semitones). Above 24 semitones the frequency maxima show similar musical intervals. They clearly tend to form a major chord based on the respective partial tone (major third, fifth, octave).

2.2. Primary pitch perceptions (spectral pitches)

Each of the partial tones can cause a consciously perceptible pitch (a spectral pitch), if it is not masked by neighbouring partial tones or noise. In the pitch computation method the degree of audibility of a spectral pitch is described by the so-called level surplus. This level surplus is the difference between the volume of the partial tone concerned and that volume, which corresponds to the masking effect of all remaining partial tones or noise [18, 16]. If the level surplus is less than zero, then the partial tone concerned is regarded as inaudible in the sense that it does not cause a perceptible pitch. In earlier experiments it was shown that the procedure supplies in this regard quite reliable forecasts of the audibility of partial tones [16]. In the case of the 137 bells we examined, the average number of audible partial tones found by the computation method was 8.1. These usually included the eight partial tones required by modern tuning standards.

The pitch sensation is thus always ambiguous, because individual spectral pitches can stand out more or less strongly according to their level surplus. One generally hears a sound, which is a musical chord assembled from pure tones with "percussion-like" amplitude envelopes. This chord is made up of intervals which musical theory would say were dissonant because of the combination of major third and Tierce. The fact that bell sounds in this sense have the character of a musical chord, explains in part the attractive effect of these sounds and justifies the term "bell music" used by Griesbacher [3].

The weight, with which a certain partial tone contributes to the sound effect, depends not only on the degree of audibility (that is, the level surplus), but also on the absolute partial frequency. Hearing perception does not give the same weight to all audible partial tones, but prefers those spectral pitches situated in a dominant frequency range [8, 10]. This range extends approximately from 500 cycles per second to 1500 cycles per second [15]. In the pitch computation method this is taken into account, as the level surplus of the individual partial tones is weighted, so that the importance of a partial tone depends on both its level surplus and its partial frequency.

The phenomenon of spectral dominance has great importance for the tonal evaluation of bells. For example, if a bell has a strike pitch in the one-primed octave [translator's note: the octave from middle C upwards] (which is often the case for church bells), then the first partial tone falls below the dominant frequency range. The first partial then plays a reduced role in sound perception even if it is actually well audible. In particular its tuning is not then critical. Then at the same time the Nominal, Tenth and Twelfth will lie more or less completely in the dominant range, so that they are of large importance. They then play a significant role in forming the virtual pitches of the bell. On the other hand, if the strike pitch is at the centre of the dominant range or higher, then the partial tones above the second fall in importance. They play a diminishing role in determining the strike pitch the higher they are above the critical region (that is, over 1500 cycles per second). The strike pitch is then formed from the 2nd or 1st partial tone alone and is almost certainly a spectral pitch (for this see [1, 3]).

In large agreement with the experiences of the bell specialists one can therefore state, due to the described psycho-acoustical factors, that substantial aspects of the sound perception of a bell are not only determined by relative frequencies of the partial tones, but to a considerable degree also by their absolute frequencies (the general tone position).

For the evaluation of the fine tuning of spectral pitches, the phenomenon of pitch shift is of importance. This effect arises because the spectral pitch of a partial tone of fixed frequency to some extent depends on its intensity of sound and particularly on the strength of other neighbouring partial tones. This circumstance can be expressed also in such a way that the equivalent pitch is not identical in all cases with the partial frequency. Rather both can differ by a few parts per thousand or per hundred from each other, and this difference is called pitch shift [15]. Pitch shifts are fundamentally the same for all persons with normal hearing, but can clearly differ in amount from person to person. This means that pitch measurements by comparison with a tone (for example a tuning fork) can result in systematically different results depending on which person takes the measurement, without one being able to say that one measurement result is more "correct" than another. It follows that an algorithmic pitch determination procedure which considers pitch shifts can only supply a representative average value and indicate the shift direction. We only have limited experience so far of the accuracy and reliability of the procedure we use. This means that one must allow a considerable error tolerance to the relevant algorithmic forecasts. However, since pitch shifts are a “second order effect”, this is acceptable, particularly since our experience is that the results of the procedure are more exact than they would be if the pitch shifts were not allowed for at all.

2.3. Virtual pitches

If by a complex tone one understands a sound which creates several pitch sensations at the same time, in contrast to a single note, then one can say that bell sounds usually represent a border line case; they are both complex tone and single note at once. The nature of the pitch perception of individual partial tones described above essentially constitutes the complex tone character. The only exception is bells with a high pitch, for their first partial tone alone determines the pitch if its intensity and frequency position permits. There is however another pitch perception mechanism, which is characterized by its integrating effect, in such a manner that a "note" is heard under certain circumstances from a complex partial tone spectrum, with a single dominant pitch, without any one partial tone prevailing. This perception mode is not the exception but the rule in acoustic communication with language and music. It is for example responsible for the fact that the complex partial tone spectrum of the human voice has a uniform pitch sensation. This type of pitch sensation is called virtual pitch. Since the partial tone spectrum of the bells indicates a certain similarity with the inharmonious structured spectra of periodic acoustic vibrations (for example of the voice and other musical tones), it is understandable that a bell sound can also cause more or less pronounced virtual pitches. Work by Schouten [12] already referred to demonstrates this.

Using our pitch computation method the virtual pitches are found by first determining the audible and spectrally weighted spectral pitches of the partial tones, as described in the preceding section. Next, that basic pitch is identified, which corresponds to the hypothetical lowest partial tone of a harmonic series, such that actual partial frequencies could be regarded as a coincidental selection of frequencies from the harmonic series. The procedure used to find this basic pitch is called "sub-harmonic coincidence detection" [18]. The procedure supplies a number of virtual pitches, in agreement with numerous psycho-acoustical measurement results, with a weight number specifying their relative importance. This weighting depends on the weight of the spectral pitches involved, as determined previously, as well as on their "harmonicity", i.e. on the accuracy with which they fit into the harmonic series mentioned above.

The fine tuning of a certain virtual pitch is directly linked with the spectral pitch of the determining partial tone. For example if the second partial tone is near A' (440 cycles per second) or below, then the succeeding partial tones fall more or less completely in the dominant spectral region. That is, they are candidates for the development of virtual pitches, if they have a sufficiently large level surplus. The fifth and sixth partial tones generate a virtual pitch in the region of the second, because they are in a harmonic series with a fundamental close to this partial. If the fifth partial tone possesses the most pronounced spectral pitch, which because of its favourable physical prerequisites (strength, and position within the dominant spectral region) is probable, then its spectral pitch determines the fine tuning of the virtual pitch an octave below. This directly explains the frequently-made observation that the strike pitch corresponds better in its fine tuning with that of the fifth partial tone rather than the second, in whose proximity it is situated [1, 2, 4, 6].

In addition, something similar applies to the fine tuning of virtual pitches as with spectral pitch: there are pitch shifts, which arise because the pitch-equivalent frequencies are not all exact integral divisors of the determining partial tone frequency. The algorithmic computation method includes a calculation to cater for this effect.

2.4. The nature of the strike pitch

The aural effect of tone caused by a bell sound can be ascribed both to spectral pitches and to virtual pitches. The first represent the "actually present" partial tones, and the latter the "tonal meaning" of them. A fundamentally false conclusion would be to be assume that the spectral pitches possess a higher degree of reality than the virtual pitches, or that the latter are a "psychological illusion" (for this see [7]). Although the connection between a spectral pitch and the accompanying partial tone frequency is formally essentially easier than that between a virtual pitch and the partial tones generating it, however, with both pitch kinds there is the basic difference between stimulus and effect, that is of the world of physical occurrences and that of the sensations. A spectral pitch is by no means less “psychological” than a virtual pitch.

In the perceived bell sound the various spectral and virtual pitches compete over which prevails at an instant in the conscious experience. Following our calculation procedure, the pitch which shows the biggest calculated weight is regarded as dominant and is therefore the strike pitch, whether it is a spectral pitch or a virtual pitch. Experience shows that pitches are virtual pitches in the area of the one-primed octave [translator's note: the octave from middle C upwards] and below, while higher strike pitches are as a rule spectral pitches. Therefore, one can distinguish the following three cases:

1. The strike pitch is identical in nature to the spectral pitch of a partial tone (this can often be the second partial, or the first in bells of higher pitch); the strike pitch is then naturally also identical in ‘pitch height’ to the spectral pitch concerned. The quantity of such cases explains the considerable extent, as demonstrated in Fig. 1, to which the frequency of the second partial tone corresponds exactly to the strike pitch.

2. The strike pitch is a virtual pitch and is situated more or less near to the spectral pitch of a partial tone (with church-bells, with a typical partial tone pattern, this is the second or first partial tone). The quantity of these cases explains the extent to which, in Fig. 1, the strike pitch is close to the second partial tone but not co-incident with it.

3. The strike pitch is a virtual pitch and is not situated near any partial tone. These cases account for the deficit by which the number of cases against the second partial tone falls below 100% in Fig. 1.

For the 137 historical church bells studied, 12% fell into case 1, 77% into case 2 and 11% into case 3.

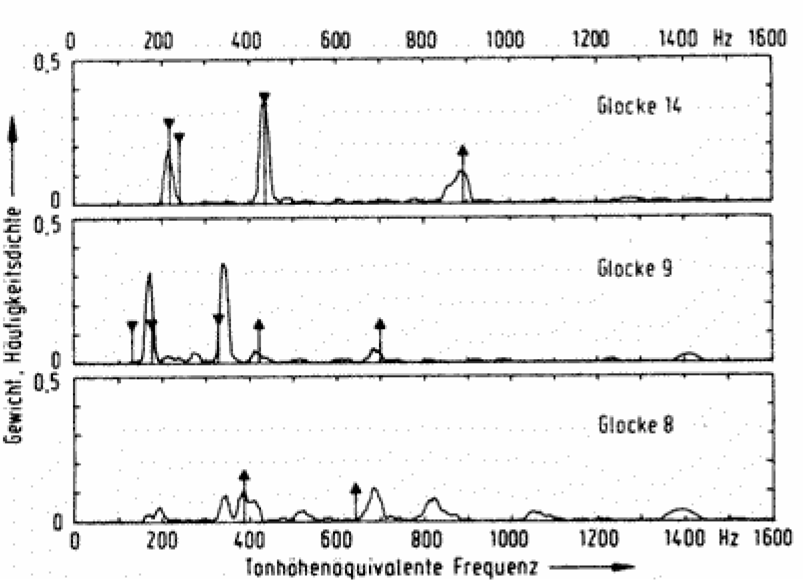

Fig. 2. Strike pitches determined aurally by an adjustment test (the smooth curve), or algorithmically (vertical lines with triangle). The frequency density (ordinate) refers to the results from the hearing comparison and gives the relative number of values which were found in a window 0.2 frequency groups wide, whose centre frequency was continuously shifted (abscissa). The reference value of the frequency density is the total number of settings (N = 147). The algorithmically calculated strike pitches indicated by the points of the triangles mark the pitch-equivalent frequency (abscissa) or calculated weight (ordinate). The weight specifications on the ordinate should be multiplied by 2 to get the actual calculated values. Only those calculated pitches are shown whose weight amounts to at least half of the maximum value, or at least a value of 0.2.

Fig. 2 shows three examples of pitch determination by the computation method. The points of the vertical triangles show for the significant calculated pitches the frequency (abscissa) and weight (ordinate). Triangles pointing upward mark the spectral pitches, and those downward the virtual pitches. Pitches of a bell are taken as significant if their weight is at least half the maximum weight occurring for the bell, and also at least 0.2 in value (note that the weights in Fig. 2 are shown at half their true values). The strike pitches in these examples which have the largest weight, correspond to frequencies of 442 cycles per second with bell 14, 425 cycles per second with bell 9 and 388 cycles per second with bell 8.

In a carillon or chime the strike pitches of the individual bells are usually used for melodic purposes. In the sound of an individual bell, as well as in a church tower ring, strike pitches can also be the tonics of musical chords. As explained elsewhere [17], musical chord tonics can explain the Rameau "fundamental bass" of music theory as virtual pitches and be psycho-acoustically justified. This ambivalence in the nature of the strike pitch gives a further explanation of the attractive effect of bell sounds on the hearing.

3. Aural determination of the strike pitches

Some methodical problems arise from the large number of bells to be examined, the tonal ambiguity of the sounds, and also the fact that the relative amplitude of the partial tones changes as the sound dies away. The pitch ambiguity means that all the possible strike pitches must be encompassed. The changes over time of the partial tone spectrum means that, in addition, clarification was needed of the extent to which pitch allocation for different time segments of the sound was different. Because of the large amount of time needed to clarify these questions, a group of 17 bells was selected with the criterion that they cover the whole quality range from melodious to dissonant. The pitch sensations caused by these bells were measured using the comparison method [20] and displayed in histograms. The experience gained from this test led to a simpler and less time-consuming method for examining the majority of the bells, i.e. the "vocal reproduction" approach. The two methods are described below.

3.1. Comparison method (adjustment test)

After listening to a bell sound, the test subject adjusts the frequency of a pure tone so that the pitch impression of the comparison tone corresponds to the pitch of the preceding test sound, which was spontaneously heard and then kept uppermost in the mind. The frequency of the pure tone then serves as a direct measure of the strike pitch of the test sound. In our experiments the test sound and comparison tone were repeated, until the test subject was content with the alignment and indicated this to the test manager, who then noted the comparison audio frequency. The test subject adjusted the frequency setting using a helipot with no scale. Sufficient repetitions of the adjustment procedure were performed to provide an indication of the statistical distribution of the adjusted comparison audio frequencies and thus the assigned pitches.

In order to decide the influence of different time periods of the bell sound, the adjustment test was executed in three parts:

- on the whole sound, i.e. a full three seconds of time beginning at the impact;
- on the initial sound, i.e. a time segment 100 ms long beginning at the impact; and
- on the tail of the sound, i.e. a time segment 100 ms long beginning 600 ms after the impact.

The test sounds were played with a peak volume level of 70 phons, the comparison tone at a volume of 60 dB. The sounds were presented to each test subject separately with DT48 binaural free field equalised headphones in a sound-isolated measurement room. For each bell, three tests were done on the initial sound and the tail of the sound, and five tests on the whole sound. The sounds were presented in random order. Eighteen test subjects were involved altogether.

An intermediate analysis of the results showed that 3 test subjects were not capable of delivering sufficiently systematic pitch judgments. Their results were therefore excluded from those brought together for the final analysis. Also it became clear that the pitch distributions of the three different types of sound sample were not significantly different from one another (for this see [13]). Therefore the results were amalgamated, in order to increase the statistical significance of the distributions. Each pitch distribution is therefore based on 147 individual comparisons.

Fig. 2 shows as examples the pitch distributions of bells 14, 9 and 8. Bell 14 has a partial tone structure quite close to the ideal arrangement. Bell 9 is also melodious, though it deviates considerably from the ideal because the fourth, sixth and seventh partial tones are absent. The partial tones of bell 8 were not arranged according to any meaningful pattern; their strike pitch proved to be particularly ambiguous. The relative frequency density (the ordinate in Fig. 2) is the proportion of the total number of results (N=147) that fall into a particular frequency interval 0.2 frequency groups wide (the “integration interval") [20]. The abscissa is the centre frequency of the integration interval. The frequency density functions arise as a result of the continuous shift of the integration interval along the abscissa. The width of the frequency groups and thus the integration interval are periodic in that the integration interval covers a fixed number of approximately ten evenly perceptible pitch differences [20]. As a result, almost co-incident frequency settings are aggregated, whereas significantly different pitches are shown separately.

The significance of the frequency densities was checked with a Chi-squared test by comparing with a random distribution of answers ranging across 15 frequency groups. The test gave a frequency density of 0.05 for a confidence level of 5% with N = 147. There is a high probability that those frequency density values which exceed this amount represent perception of an actual pitch, and the probability of mistake with higher frequency densities decreases rapidly. Therefore bell 14 has three, bell 9 at least four, and bell 8 at least six significant strike pitches (Fig. 2). Note that the distributions do not necessarily show all the audible pitches; this is not to be expected given the question posed to the test subjects and was also not the purpose of the experiment.

Since in general several maxima of different height occur in the distributions, all of which are significant in the above sense, the question arises as to what degree the frequency differences of the maxima concerned are significant. This was also estimated by means of a Chi-squared test; the significance of the differences between pairs of maxima was compared in each case against the proposition that if the total number of values allocated to them were constant, whether their heights would be the same in any population. It was inferred from this that the distributions of pitch values of the individual bells which are not significantly different from each other in the frequency density maxima, must in this sense be treated as equal. This means that, for example, the two biggest peaks for bell 14 are significantly different, so that one can speak of this bell as having a significantly dominant strike pitch (Fig. 2). On the other hand, for bell 9 the two highest maxima do not have a significantly different height. Therefore it is not possible to judge the order of significance of these two strike pitches; they must be regarded as equal. With bell 8 six strike pitches altogether appear equivalent in this sense.

The strike pitch can be quantified as the average value of all adjusted frequencies, indicated by a local maximum of the distribution, which fall into the window belonging to the maximum frequency density.

3.2. Vocal reproduction (reproduction experiment)

A method of post vocalisation was applied to the main group of 120 bells and as a check additionally on five bells of the aforementioned sub-group. The test subject was in an sound-isolated measurement room and could play a tape recording of the test sound by pushing a button (the whole sound, that is a sound segment consisting of the first three seconds after the impact; peak value of the volume was 70 dB; playback used DT48 binaural free field equalized headphones). The test subject was asked to reproduce the spontaneously perceived strike pitch of the test sound immediately afterwards by humming or singing it. Repeating the test sound was not allowed, in order to encourage a spontaneous impression of the pitch and to avoid analytic hearing if possible. Then the next test sound stored in random order on the test tape could be recalled, and so on. The tones reproduced by the test subjects were recorded with a microphone on another tape and analysed later by the test manager. The analysis involved the aural determination of the pitch-equivalent frequency of a pure tone by the test manager. This procedure was preferred to the direct "objective" measurement of the basic frequency of the hummed tones, because the accuracy and reliability possible for basic frequency measurement of the natural voice is rather less than the performance of an experienced listener. Additionally it emerged from the experiments that some test subjects did not immediately hit the exact pitch desired, but hummed a sliding tone which reached the final value desired only after a little time. The test manager could easily take this into account in the aural determination of the pitch-equivalent frequency and therefore get reliable results without too much work. The main advantage of the reproduction method which was crucial for these investigations is its saving of time as well as in the fact that it facilitates spontaneous, realistic pitch specification by the test subjects. A disadvantage is that it relies on the ability of the test subjects to hum pitches, and that for determination of the pitch-equivalent frequency two successive hearing comparisons are necessary (one by the test subject, the other by the test manager), in contrast to the comparison method. Finally, a further restriction arises as a result of the limited vocal range of the test subjects. These disadvantages seemed unavoidable within the scope of the given possibilities and were accepted.

The test subjects were trained musicians, and because of the results of the adjustment test it was assumed that most of the strike pitches would lie within the vocal range of the test subjects. Nevertheless, it was considered that the exactness or universal validity of strike pitches determined in this way was subject to certain constraints. Therefore from the outset the objective was only to determine the musical note name of the strike pitch in terms of the normal temperament (A' = 440 cycles per second). Consideration of the octave position was not attempted, i.e., all pitch judgments were transposed into the same octave. This is justified by the fact that for bells whose partial tone structure obeys the norm, the octave position in any case is usually in the region of the second partial tone (the Prime). However the results of the aural pitch determination show that octave ambiguity is almost the rule; the note name of the strike pitch is to a large extent unique, whereas there is a large uncertainty as to whether the octave position of the strike pitch is in the region of the Prime or the Hum. Eleven test subjects were involved. Nine of them did two pitch determinations for each bell, and the remaining two test subjects only one, so that for each bell altogether 20 pitch values were obtained. The results, i.e., the pitch-equivalent frequencies determined by the test manager, were represented in histograms in the same way as the results of the adjustment test.

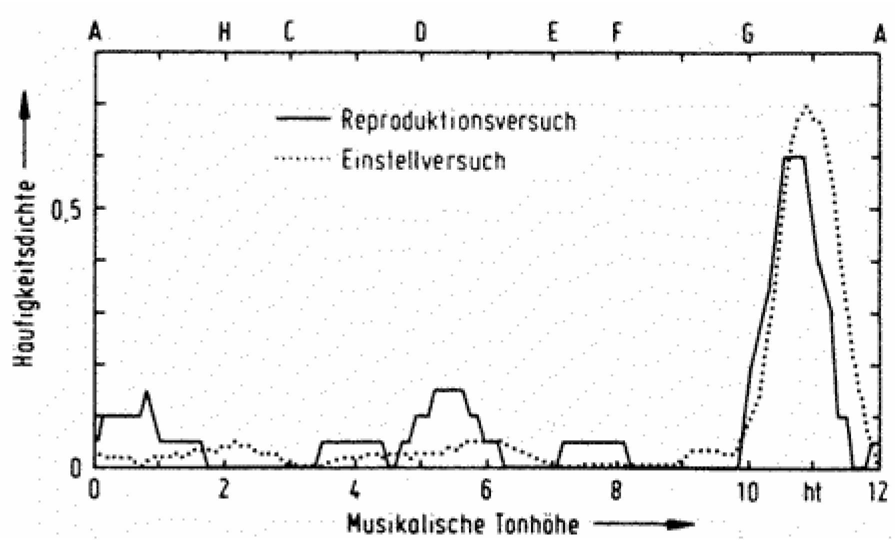

Fig. 3. Octave-normalised strike pitch distribution for bell 1. Ordinate: Frequency density as in Fig. 2, however refined with a window a semitone wide. Abscissa: Musical pitch, expressed by frequency measure interval, related to the frequency 2n 440Hz (n completely arbitrary). Increment of the half-tone window 0.1 semitones. Continuous curve: Reproduction experiment; dotted: Adjustment test.

Fig. 3 shows as an example the frequency density distribution of the octave normalized results for vocally reproduced strike pitches for bell 1 (the solid curve). The frequency density (ordinate) has a similar meaning to Fig. 2, but with an integration interval of 1 semitone = 100 cents in place of 0.2 frequency groups. To produce the distribution, the integration interval was shifted in steps of 10 cents along the abscissa. The musical pitch within the single-height octave is the frequency measure interval, related to the note representing A in normal temperament. In the same sense as discussed with the adjustment test (comparison method), it is to be assumed that a maximum in the frequency density distribution forms the basis of perception of an appropriate pitch, if the height of the maximum exceeds a certain value. With the same method as before, but taking into account the test conditions (12 value classes; 20 judgments), for a probability of mistake of 1% a minimum value of the frequency density was found of 30%. A strike pitch at the appropriate value from the abscissa is inferred from each maximum which approximates to this value or exceeds it. The latter was objectively determined in the same way as explained at the end of section 3.1. Thus the results from the histogram in Fig. 3 for bell 1 show only one significant strike pitch with the value 10.71 semitones, or G# - 29 cents.

3.3. Comparison of the two methods

The reliability of the reproduction method was checked, because the strike pitches of the bells 1, 7, 9, 12 and 14 were determined by both methods. For comparison, all individual values of the adjustment test were converted into the same octave and displayed by means of the same integration interval as with the reproduction experiment (1 semitone) as a frequency density distribution. The dotted curve in Fig. 3 shows the result for bell 1. From the position of the major peak the strike pitch is determined as G# -13 cents. The largest difference between the strike pitches determined by both experiments had the value of 64 cents (for bell 14). The largest difference of the frequency densities of the dominant strike pitches was 13% (for bell 9). We conclude from this that it is indeed possible to determine the strike pitch categories reliably with the reproduction method.

4. Results

As the examples of pitch distributions in Fig. 2 and Fig. 3 make clear, it is not easy to present the results of either the aural or the algorithmic strike pitch determinations both concisely and without ambiguity. We try to solve this problem below, by concentrating on the overall question, which is the extent to which the dominant strike pitches determined by aural and algorithmic methods agree.

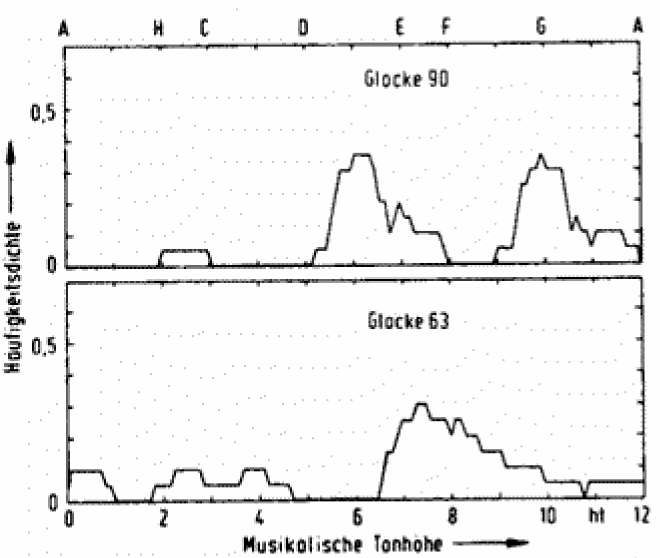

Fig. 4. Octave normalised strike pitch distributions from the reproduction experiment, displayed as Fig. 3. Bell 90 is typical of one with an ambiguous strike pitch, bell 63 has a strike pitch which is on the boundary of significance.

4.1. Aurally measured strike pitches

From the pitch distributions already presented for both the adjustment and the reproduction experiments it appears that the width of the maxima of the determined pitches is of the order of magnitude of a semitone. Fig. 4 shows two further distributions from the reproduction experiment, which are meant as examples of ambiguous strike pitches (bell 90), or of the significance boundary for a strike pitch (bell 63). We conclude from this data that it is not sensible to try to give reliable and representative values for the strike pitch to an accuracy of a few cents. The identification of the musical note name (that is, the "coarse tuning" with semitone accuracy) and the trend of possible divergence from normal temperament (that is, the "fine tuning") is the best that is attainable. This does not exclude an individual experienced observer obtaining a higher accuracy of reproducible pitch determinations. Because of the considerable differences in pitch sensation between different individuals, such measurements are only of limited importance.

A summary of the results of the reproduction experiment with all 137 bells (including the results of those 12 bells, which were taken only in the adjustment test, correspondingly converted and included) is given by the following figures – note that all pitch values taken in this experiment are octave normalised:

- 97% of the bells have at least one significant strike pitch
- 69% of the bells have a single dominant strike pitch.
- across all 137 bells no significant preference for any strike pitch shows up within the normalized octave; the strike pitches are evenly distributed.

Table I. Aurally measured, or algorithmically calculated pitch-equivalent frequencies (in cycles per second) of 17 bells (adjustment test; 15 test subjects; 147 adjustments), in each case in the reverse order of their frequency density or its calculated weight. The strike pitches listed are, for the aural measurements strike pitches, only those with equal high importance, and for the algorithmically determined, only the most significant ones (see text).

Bell No.	Strike pitch frequency	
	Aural	Algorithmic
1	205 / 414	203 / 99 / 245 / 846 / 419
2	880 / 1782	894 / 444 / 218 / 293 / 1176 / 1805 / 144
3	1107 / 864 / 914 / 226 / 1734	1102 / 500 / 1855 / 362 / 246 / 2809 / 163 / 551
4	1119 / 273 / 549 / 2251 / 808 / 853 / 1740 / 890 / 653 / 928 / 1397	153 / 1119 / 476 / 234 / 75
5	886	182 / 130 / 449 / 369 / 96 / 905 / 400 / 221 / 745 / 76
6	586	243 / 745 / 602 / 191 / 340 / 1212 / 166 / 144 / 289 / 110
7	511	512 / 1101 / 1609 / 259 / 1872 / 167
8	688 / 387 / 344 / 822 / 412 / 813	388 / 639
9	340 / 172	423 / 704 / 332 / 181 / 137
10	544 / 277 / 1105	272 / 1115 / 674 / 333 / 220 / 525 / 164 / 130 / 92 / 553
11	443 / 881 / 1730 / 1042	218 / 457 / 1050 / 814 / 344 / 1756 / 256 / 149 / 111 / 225
12	366 / 181	365 / 180 / 94 / 146 / 738
13	937 / 1893	561 / 277 / 924 / 1199 / 183 / 1966 / 233 / 394 / 303 / 108
14	435	442 / 217 / 236 / 891
15	635 / 313	318 / 156
16	347 / 609 / 1220	348 / 245 / 170 / 613 / 370 / 748 / 182 / 1233 / 301 / 112
17	1338 / 690	717 / 1333 / 234 / 355 / 139

Further conclusions are available from the results of the adjustment test. Table I shows the equivalent frequencies (in cycles per second) of the aurally measured strike pitch for each of the 17 bells, in the reverse order of the appropriate frequency densities, i.e., in the order of rank of their importance. Only the most significant pitches in the sense of the definitions indicated in paragraph 3.1 are listed. It is clear the indicated rank orders are not very reliable. Only with bells 5, 6, 7 and 14 one can speak of a significantly dominant strike pitch. Bells 1, 2, 9, 10, 12, 13, 15 and 17 either have two strike pitches or have an ambiguous octave position. Octave ambiguity must obviously be regarded as a more or less typical characteristic of the strike pitch. Taking this into account, one can then say that 12 of the 17 bells, i.e. 71%, possess only one significantly dominant octave normalised strike pitch. The good agreement of this value with the appropriate percentage of all 137 bells given previously suggests that this sub-group of 17 bells is largely typical of the whole. The proportion of historical bells, whose strike pitches are ambiguous regarding their musical note name, can therefore reliably be stated as approximately 30%.

Bearing in mind the limits described to measurement accuracy and octave ambiguity, it is appropriate to classify the strike pitches according to three criteria: note name, fine tuning and octave position, particularly since this corresponds to a large extent to normal practice. In this way each of these three aspects can be separately discussed. In Table II an appropriate representation method is used. As far as possible, the partial tones of the 17 bells were arranged as in Fig. 1, where the Nominal frequency served as reference value. The tuning of the remaining partial tones is described by their deviation in cents from the ideal value referred to the Nominal. So, for example, if the Prime tone had exactly half the frequency of the Nominal, it would appear in Table II with a deviation of 0 cents. The absolute height of the Nominal is indicated by its note name, octave position and fine tuning related to normal temperament (A' = 440 cycles per second). For example the Nominal indicated for bell 1 is gis2 + 7 (7 cents positive deviation from the two-primed G#), which is a frequency of 834 cycles per second. For those bells whose partial tones could not be conveniently fitted into the pattern, the partial frequencies are listed for those partial tones which were reasonable audible, i.e. those with positive level surplus. Also listed in Table II are those strike pitches from Table I ranked first in order of significance (either aurally measured, or calculated), again related to normal temperament. For instance, the designation gis0 - 22 for bell 1 in Table II corresponds to the frequency of 205 hertz in Table 1.

The figures show that the tuning of the partial tones throughout is quite inaccurate compared with the ideal. Furthermore, they suggest the tendency already mentioned that the fine tuning of the strike pitches corresponds better with that of the Nominals than with the Prime. In order to verify this quantitatively, those bells from Table II are considered for which the note name of the strike pitch corresponds with that of the Nominals. The average value of the deviations between the fine tuning of the aurally determined strike pitch and the Nominals amounts in these bells to 25 cents, whereas it is 40 cents compared with the Prime. Additionally, clear trends show up concerning the direction of the deviations. They can be described by the arithmetic average values of the appropriate tuning differences. These same amount to – 24 cents compared with the Nominals and + 30 cents compared with the Prime. That is, the strike pitch is on the average somewhat flatter than the octave-displaced Nominal, and sharper than the Prime. The letter tendency was already clear from the discussion in section 2 about Fig. 1.

As regards the octave position of the strike pitch, it appears that with those bells whose partial tones follow the standard pattern, there is a pronounced tendency of the strike pitch to fall in the area of the Prime; often, however, equivalent strike pitches occur in the region of the Hum. This is remarkable, when with bells with standard partial tone structure, strike pitches in the area of the Hum are usually not considered.

Table II.
Summary of frequencies or pitches with note name, octave position and fine tuning, for the sub-group of 17 bells (as in Table 1). All fine intonation values in cents in columns 2 to 5 and 7 to 9: deviations of the partial frequencies from the norm; reference value is frequency of the Nominals. Column 6: absolute tuning of the Nominal, relative to normal temperament. Columns 10 to 12: aurally or algorithmically determined strike pitch (relative to normal temperament) and weights of the latter. The strike pitches in each case are those listed first in Table I. If the partial tones are not in the normal arrangement, their frequencies are given.

Bell No.	Hum	Prime	Tierce	Quint	Nominal	Tenth	Twelfth	Octave Nom.	Strike pitch		Calc. weight
									aural	calc.	
1	-38	-128	-26	-156	gis^{2} +7	-	+10	+70	gis^{0} -22	gis^{0} -39	0.74
2	-66	+7	+44	-219	a^{3} +13	+21	-	+120	a^{2} +0	a2 +27	0.39
3	Partial tones: 147 / 497 / 1101 / 1842 / 2751 / 3763 / 4872 Hz								cis^{3} -3	cis^{3} -10	0.37
4	Partial tones: 312 / 467 / 1113 / 1309 / 1399 / 2135 Hz								cis^{3} +16	dis^{0} -29	0.40
5	+154	+21	+29	+31	fis^{3} -27	-33	+101	+75	a^{2} +12	fis^{0} -28	0.44
6	+210	-73	+29	+124	d^{3} +43	+39	-26	+21	d^{2} -4	h^{0} -28	0.43
7	Partial tones: 512 / 1098 / 1588 / 1872 Hz								c^{2} -41	c^{2} -38	0.43
8	Partial tones: 171 / 388 / 622 / 2062 Hz								f^{2} -26	g^{1} -18	0.36
9	+84	-84	-9	-	f^{2} -14	-	-	-33	f^{1} -46	gis^{1} +32	0.31
10	+51	-104	0	-16	cis^{3} -3	-49	-9	+61	cis^{2} -33	cis^{1} -33	0.44
11	Partial tones: 213 / 545 / 808 / 1042 / 1388 / 1746 / 2591 / 3538 Hz								a^{1} +12	a^{0} -16	0.61
12	+96	+9	+19	+36	fis^{2} -24	+48	-22	+16	fis^{1} -19	fis^{1} -24	0.74
13	Partial tones: 561 / 917 / 1186 / 1649 / 1947 / 2887 / 3964 Hz								ais^{2} +9	cis^{2} +21	0.49
14	+116	-63	+2	+66	a^{2} +14	+7	-10	+31	a^{1} -20	a^{1} +8	0.72
15	-34	+7	-29	+94	dis^{3} +45	+66	+127	-58	dis^{2} +35	dis^{1} +38	1.00
16	+219	-104	+12	+76	dis^{3} -26	-	-24	+12	f^{1} -11	f^{1} -6	0.62
17	Partial tones: 717 / 1330 / 1696 / 1909 / 2801 / 4962 Hz								e^{3} +25	f^{2} -21	0.44

4.2. Algorithmically determined strike pitches

In Table I the calculated equivalent frequencies of the strike pitches in the third column are specified in the reverse order of their calculated weight, i.e. in the order of rank of their importance. Only the significant pitches were considered, i.e., those whose weight is at least half of the maximum weight or a value of 0.2 (this limit is arbitrary; it was chosen for clarity).

In comparing the algorithmically calculated and aurally measured strike pitches, again three aspects were considered; note name (coarse tuning with semi-tone accuracy), fine tuning (deviation from normal temperament within the note name) and octave position. In this way we can more easily take into account the basic statistical character of both the algorithmic and the aural pitch determinations. An analysis of the data from the adjustment test in Table I, performed in this way, results in the following:

- the first calculated strike pitch agrees both in octave position and note name with the first aurally measured pitch in seven bells, i.e. 41 %.
- the first calculated strike pitch agrees both in octave position and note name with any of the equivalent aurally measured pitches in eleven bells, i.e. 65%.
- the first two calculated strike pitches agree both in octave position and note name with two of the equivalently aurally measured pitches in 13 bells, i.e. 76%.
- the first calculated strike pitch agrees with the note name (but not necessarily the octave position) of the first aurally measured pitch in 10 bells, i.e. 59%, (see Table II).
- the first calculated strike pitch agrees in note name only with any of the equivalent aurally measured pitches in 12 bells, i.e. 71%.
- in the last group specified the octave position but not the note name also corresponds in 11 of 12 cases (92%).

In order to measure the statistical significance of the algorithmic note name determination even more reliably, the results of the reproduction experiment can be considered. This results in the following:

- the first calculated strike pitch agrees with the note name with the first aurally measured pitch in 79% of the bells.
- for 84 of 95 bells (88%) with only one dominant octave-normalised strike pitch, this pitch agrees in note name with the first calculated pitch.

- the first-ranked aurally measured strike pitch agrees in note name with one of the first two, three, or four calculated strike pitches in 86%, 97%, or 99% of the bells.

In view of the fact that a substantial number of the historical church-bells had only moderate tone quality and almost indefinable strike pitches, one would not expect the close relationship between algorithmic and aurally determined strike pitches demonstrated by the average values above. On the other hand, for bells with well-pronounced strike pitches a higher measure of agreement could be expected. The weight of the calculated strike pitch is a suitable basis for discrimination. The last column in Table II gives examples of this. Compared with the pitch weight of "natural" complex tones the values are quite small, which shows how weakly pronounced the strike pitches of the bells concerned are. For example, a harmonic complex tone with 10 harmonics and a fundamental frequency of 300 cycles per second has a weight for the basic pitch of 2.7. It can be assumed that above a certain weight the strike pitch is unique and so pronounced that an almost certain prediction (at least as regards note name) can be made. A better determination of this limit value can be made from the average value of the weights of the correctly calculated strike pitches. For example, the average value of those weights in Table II for note names for which algorithmically and aurally determined values agree, is 0.61. The similar value from the reproduction experiment with 137 bells is 0.62. It is therefore a good assumption that strike pitches with calculated weights over approximately 0.6 are reasonably predictable. Indeed, one finds in Table II that for those six bells whose calculated pitch weight is larger than 0.6, the calculated and aurally determined note names correspond without exception. From all 137 bells the calculated pitch weight is larger than 0.6 in 64 cases (47%); again, for these bells the algorithmically and aurally determined note names correspond without exception.

It is a good assumption that the weights of the calculated strike pitches should be in close relationship with the frequency density determined in the aural test for the pitches concerned. If one correlates these two measures for all pitches from Table I, matching within a semitone, then indeed a relationship is found significant at the 99.9% level (r=0.597; N = 40). Nevertheless this relationship must be regarded as relatively loose, because for example the frequency densities concerned are relatively high for bells 5, 6 and 9 and the dominant (aural) strike pitches are to a large extent unique, while the appropriate calculated weights are situated below the average and the note names determined aurally or algorithmically do not correspond (see Tables I and II). A hearing calibration experiment showed that the significance of the strike pitches of these three bells is far smaller than is at first suggested by the height of the appropriate maxima in the pitch histogram. We infer from this result that from the clarity and frequency with which a strike pitch is dominant in the hearing experiment, cannot be related directly to its calculated significance, although a certain connection of the two measures exists. Looked at another way, the reliability with which a dominant strike pitch can be determined algorithmically is related to its significance, and the calculated weight seems to be a useful measure for this.

The following analysis will show conclusively to what extent, beyond the note name and the octave position, the tendencies of the fine tuning are also algorithmically assignable. In Table II are 10 bells for which the calculated and aurally determined strike pitches have corresponding note names.

The appropriate fine tuning does indeed show a correlation, which is significant at the 99%-level (r=0.764). The average value or the standard deviation of the differences between calculated and aural tuning differences amounts to 0.9 cents or 17.3 cents. With N = 10 this gives a confidence interval between -11.5 cents and + 13.3 cents. Therefore one can state the average accuracy of the algorithmically determined fine tuning as approximately plus/minus 15 cent. The findings are confirmed by consideration of the relationship between the fine tuning of the strike pitches and those of Nominal or Prime tone, as described in the preceding paragraph for the aurally determined strike pitches. The average values of the deviations between calculated strike pitches on the one hand, and Nominal or Prime tones on the other hand, are 15 cents or 38 cents. The algorithmically calculated strike pitch has therefore a closer relationship regarding its fine tuning with the Nominal as compared to the Prime tone, as was found before for the aurally measured strike pitch. Furthermore, the averages of the divergences also agree with the directions of the divergences established in the previous section: the arithmetic average value of the tuning discrepancy between calculated strike pitch and Nominal is -11 cent, while the same value relative to the Prime tone is +25 cent.

The results of the algorithmic strike pitch determination can be summarised as follows:

- For historical church bells of the type examined, the algorithmically determined note name ascertained from the weightiest calculated pitch agrees with the note name of the highest-rated aurally determined strike pitch in 79% of the cases. For bells which have a significantly dominant strike pitch, this agreement is observed in 88% of the cases. The algorithmic note name determination proves 100% reliable for those bells whose calculated pitch weight exceeds the value 0.6; the proportion of such bells was 47%.
- For those bells, whose strike pitch note name was correctly determined by the algorithmic procedure, the octave position of the strike pitch is correctly indicated by the weightiest calculated pitch with a probability of 90 to 100%. This means that where the aurally assessed octave position is clear, it is identical with the objectively ascertained ones, and if the aurally determined octave position is ambiguous, one of the two octave positions is given algorithmically.
- For those bells whose strike pitch note names are correctly determined by the algorithmic procedure, the fine tuning determined by the weightiest calculated pitch agrees reliably with the direction of the tuning deviation within the note name. The average accuracy of algorithmic determination of the fine tuning is approximately plus/minus 15 cents.

5. Final Conclusions

As described initially, the pitch sensation caused by a bell sound is basically ambiguous. From our aural pitch determinations it follows that, even if the test subjects are asked for the dominant strike pitch the responses never concentrate on only one pitch. Bells that possess a pronounced strike pitch can be assigned a musical note name which is unique, but the same cannot be said of their octave position. With many melodious bells it cannot be decided clearly whether the strike pitch is situated in the region of the Prime or the Hum.

The strike pitch determination rates regarding note name, octave position and fine tuning obtained with the algorithmic procedure, are so high that one could hardly expect a better agreement in view of the typically moderate sound quality of the examined bells and the limits of accuracy and significance attainable during the aural pitch determination. This does not preclude further improvements in the pitch computation method. However, we did not succeed in deriving criteria for improvements from the results described here. From the fact that the procedure and thus the psycho-acoustical concepts of pitch perception on which it was based worked to a considerable degree during the algorithmic strike pitch determination, we conclude the fact that - as initially stated - the "strike pitch mystery" can be regarded today as being to a large extent solved.

Concerning the practical solution of the problem, to determine the strike pitch of a bell algorithmically (which does not always give a clear determination), then we derive from the results described the following simple procedures.

One subjects the sound signal to the pitch analysis procedure of Terhardt et al. [18] and selects the pitch with the highest weight. If the weight exceeds the value 0.6, then it is practically certain to be the strike pitch. If the weight is below 0.6, then one can expect a high correlation with the aurally determined pitches, however a definite prediction of the strike pitch is not possible. In this case it is appropriate to take into consideration not just the pitch with greatest but also those with lesser weight. The complete set gives a good picture of the weaker strike pitches competing with each other. However, reliable statements about their order of rank are not possible.

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Appendix 9 Usage and calibration of the Wavanal software

All the bell sounds analysed for this research were investigated and the partial frequencies measured using the author’s ‘Wavanal’ software. This software, written some years ago, is not being presented as part of the research work. However, the reliance on the results obtained with Wavanal is such that presentation of calibration results for this software is appropriate.

The software is written in Microsoft Visual C++ and runs on any version of Microsoft Windows from Windows 98 upwards. It relies on the presence of Windows-compatible sound hardware for the recording and playback of sounds. Analysis of sound files is possible without any sound hardware being installed in the PC. Basic functions of the software as used in this research are as follows:

- Reading, writing and playback of sound files in .wav and .mp3 format
- Spectral analysis of sound files
- Visual display of sound files and of their spectra
- Identification of partials and display of their frequencies, notes, and intervals.

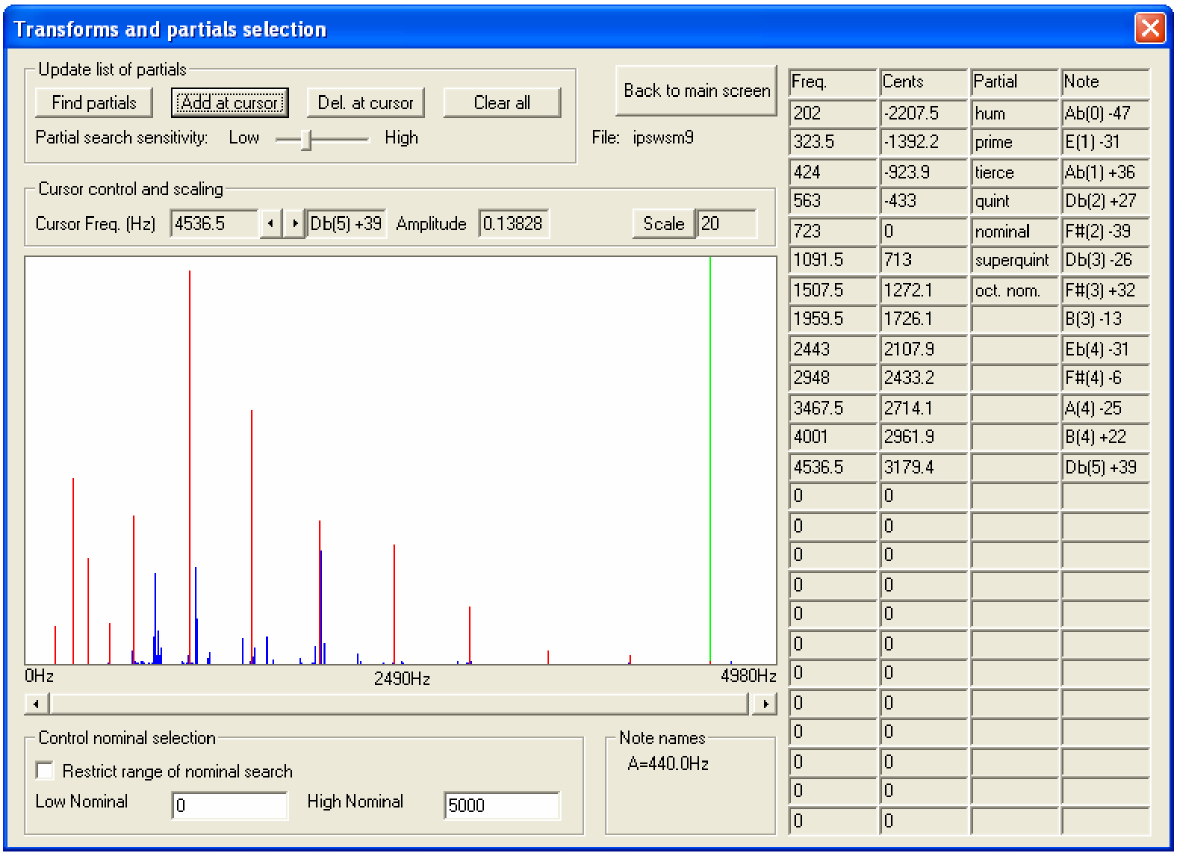
Spectral analysis is done using the Numerical Recipes in C FFT routine taken from (Press 2002), implemented with a Hamming window.

In principle any PC spectrum analysis package able to read .wav and .mp3 files could have been used for this work. Special features in Wavanal designed to assist with the analysis of bell sounds include:

- The ability to select specific portions of the sound file for analysis and select frequency bin widths based on recording length
- The author’s proprietary algorithm for locating spectral peaks
- Automatic identification and naming of major partials
- Display of intervals in cents between selected partials and the nominal partial
- Naming of notes using either the A=440Hz or A=435Hz pitch standards
- Saving of spectra and details of partials in comma-delimited format for further processing and charting.

The software has many additional facilities, but discussion of them is not relevant to the calibration process. The software and its documentation are available for free download from the author’s website http://www.hibberts.co.uk. The software has been used extensively for a number of years by bell founders and others for analysis of bell tuning.

A screenshot showing the screen which displays sound spectrum and partials information follows:



File: ipswsm9

Cursor Freq. (Hz) 4536.5 Db(5) +39 Amplitude 0.13828 Scale 20

☐ Restrict range of nominal search

Low Nominal 0 High Nominal 5000

A=440.0Hz

Freq.	Cents	Partial	Note
202	-2207.5	hum	Ab(0) -47
323.5	-1392.2	prime	E(1) -31
424	-923.9	tierce	Ab(1) +36
563	-433	quint	Db(2) +27
723	0	nominal	F#(2) -39
1091.5	713	superquint	Db(3) -26
1507.5	1272.1	oct. nom.	F#(3) +32
1959.5	1726.1		B(3) -13
2443	2107.9		Eb(4) -31
2948	2433.2		F#(4) -6
3467.5	2714.1		A(4) -25
4001	2961.9		B(4) +22
4536.5	3179.4		Db(5) +39
0	0		
0	0		
0	0		
0	0		
0	0		
0	0		
0	0		
0	0		
0	0		
0	0		
0	0		
0	0		

In the remainder of this appendix, calibration information for frequencies and amplitudes of spectral peaks is presented.

Frequency calibration

Identification of a partial frequency typically involves two steps:

- Digitisation of recorded sound using a PC sound-card, digital video camera or other input device
- Analysis of the resulting sound file using Wavanal.

For the first step, accuracy of digitisation depends critically on the accuracy and stability of the bit-clock in the input device. Analogue transformation of sound (e.g. capture via a microphone and subsequent analogue amplification) does not change partial frequencies, though it may introduce distortion which can appear as additional frequencies not present in the original sound.

The second step is a purely mathematical process on the data in the sound file and the purpose of calibration is to ensure that the software has been implemented correctly. Copying and transmission of sound files cannot affect frequencies (unless a change of format is involved, e.g. from .wav to .mp3).

The main factor affecting the precision with which frequencies can be measured is recording length. It is a feature of spectral analysis that, for a recording of length t seconds, the spectral energy appears in bins $1/t$ Hz wide. This in turn implies that frequencies in the spectrum can be measured to $\pm 1/2t$ Hz. To measure frequencies to ±0.1Hz requires a recording at least 5s in length.

The results of frequency analysis by Wavanal can be calibrated in three ways:

- Comparison, for a particular bell, with tuning figures measured by precise means
- Comparison with the NIST frequency standards available via telephone
- Comparison, for a particular bell, with the tuning figures quoted by the bell-founder.

In the first two cases, experience has shown that the critical issue is clock speed in the input device, as explained above. In the comparison with bell founder's figures, the critical issue is the ability of the founder to make accurate frequency measurements.

Frequency calibration – comparison with other precise measurements

An analysis of the bell studied by Perrin, Charnley and De Pont is presented in section 5.2. The author took a recording of this bell on 6 September 2006. The recording was taken using an AKG C1000S microphone, connected via an M-Audio Mobile Pre pre-amplifier across USB to a Toshiba Satellite Pro 4600 laptop. The digitisation took place in the M-Audio device, not the laptop. Care was taken while obtaining the recording to avoid distortion.

Table 5-1 shows an excellent correspondence between the figures measured by Perrin and his colleagues and those obtained via Wavanal, across a broad range of frequencies.

Frequency calibration – comparison with NIST standards

The National Institute of Standards and Technology (NIST) in the US provides a telephone time service. As well as transmission of UTC time signals, a regular sequence of 500Hz and 600Hz tones is provided. The tones last 45 seconds, giving adequate time for recording and analysis. A schedule of tone transmissions is repeated each hour, and most minutes in the hour contain at least one of the tones. The frequency of the tones is linked to the NIST time standard (quoted as correct to about 6×10^{-16}). The error in the tone frequencies is not quoted by NIST, and will no doubt be affected by digital transmission of telephone signals. However, the test tones are commonly used as a portable frequency standard. Details of the service are available at http://tf.nist.gov/timefreq/stations/sig.html. The service is accessed by dialling (from the UK) 001 303 499 7111.

As a calibration exercise, the NIST transmissions were recorded at about 10.00pm BST on 30 March 2008. Both the 500Hz and 600Hz tones were recorded over 45 seconds using the same equipment configuration used for the recording of the Perrin bell: AKG C1000S microphone, M-Audio Mobile Pre and Toshiba Satellite Pro. The resulting tones were analysed with Wavanal using 0.02Hz frequency bins (i.e. with the 45s recording zero-padded to 50s).

Wavanal reported the frequencies of the two test tones as 499.94 ± 0.01Hz and 599.94 ± 0.01Hz, i.e. in error to the extent of about 4×10^{-5}. The NIST tones have been used by the author for calibration of recording equipment on a number of occasions and these values are typical of those observed.

Frequency calibration – comparison with bell founder's tuning figures

The major factor introduced by this comparison is the accuracy of the equipment used by the bell founder for frequency measurements. Presented below are the tuning figures for the peal of 8 bells from Fairwarp in Sussex. These bells were cast by Gillett and Johnston of Croydon and tuned by them between 19 December 1935 and 11 Jan 1936. Gillett and Johnston would have used tuning forks to measure partial frequencies. The calibration process for these forks is unknown. Their measurement approach involved counting beats between the partial and the fork. The tuning figures were provided to me from the original tuning books by Alan Buswell. The bells were recorded by the author, again using the same equipment as for the previous two comparisons, on 13 April 2004.

Three sets of figures are presented in the table below:

- The author's frequency measurements taken in April 2004, from recordings of 5s duration (i.e. frequencies accurate to ± 0.1Hz)
- The figures from the Gillett and Johnston tuning books
- The April 2004 figures multiplied by a factor to minimise the sum of squared differences between the adjusted figures and the tuning book figures.

The factor which minimised the differences was 1.004, i.e. a 0.4% adjustment factor. This factor, which is commonly necessary in such comparisons, is accounted for by variations in digitisation speed, temperature and tuning fork calibration.

The Fairwarp bells were recorded while being rung 'full circle'. Splitting of the partials due to Doppler effects was evident, and to avoid this portions of the recordings with the bell stationary were selected for analysis. Where partials had doublets, that frequency of the pair was chosen which minimised the sum of squared differences.

Bell	**Hum**	**Prime**	**Tierce**	**Quint**	**Nominal**
April 2004 measurements					
8	182.4	365.0	438.0	546.8	729.0
7	205.2	410.0	494.6	615.8	819.4
6	228.0	456.6	546.6	684.6	912.8
5	243.6	487.0	579.0	728.6	971.4
4	273.6	546.6	659.2	818.2	1095.6
3	303.8	608.0	730.0	908.6	1214.6
2	341.6	684.0	813.2	1022.6	1367.0
1	364.8	727.4	871.8	1095.6	1459.8
G&J measurements from 1935 / 6					
8	183	366	439	548.5	732
7	206	412	494.5	618	824
6	229	458	549	687	916
5	244	488	581.5	732	976
4	274.5	549	661.5	823.5	1098
3	305	610	732.5	914.5	1220
2	343	686	818	1028	1372
1	366	732	878	1098	1464
April 2004 adjusted					
8	183.1	366.5	439.8	549.0	732.0
7	206.0	411.7	496.6	618.3	822.7
6	228.9	458.4	548.8	687.4	916.5
5	244.6	489.0	581.3	731.5	975.3
4	274.7	548.8	661.9	821.5	1100.0
3	305.0	610.5	733.0	912.3	1219.5
2	343.0	686.8	816.5	1026.7	1372.5
1	366.3	730.3	875.3	1100.0	1465.7

The differences between the bell founder's and the measured figures are typical of the variations seen in practice. Based on the good calibration reported in the previous two sections, it is surmised that the ability of the bell foundry to measure frequencies, plus any aging process in the bells themselves, accounts for the bulk of the variation. This comparison does demonstrate that bell tuning is stable over many decades.

Amplitude calibration

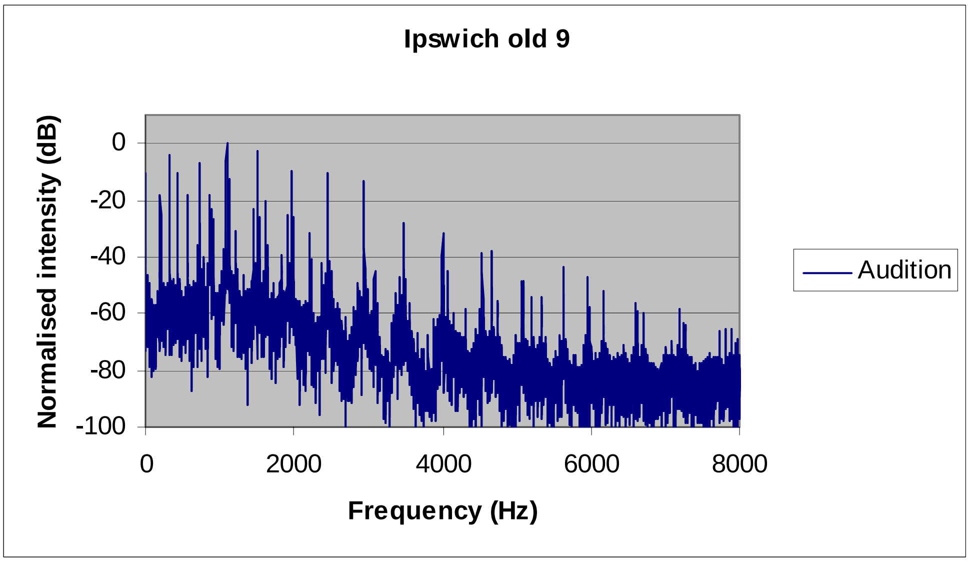
As explained in the literature survey in section 2.4, and as further demonstrated in the experiment in section 8.3, partial amplitude has a minor if any effect on virtual pitch. However, it is appropriate to carry out some verification of the amplitude figures measured in Wavanal to ensure confidence in the results.

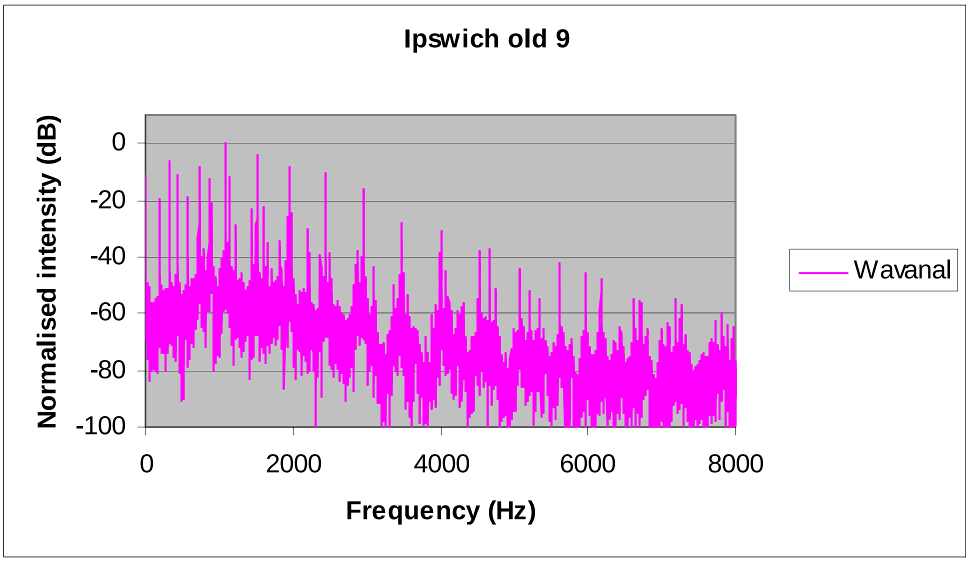
The author uses Adobe Audition 1.5 for general sound file editing and manipulation. This package can perform spectral analysis and export the spectrum into a spreadsheet for further analysis. Unlike Wavanal, which presents spectra in units of amplitude, Audition presents spectra in dB, but the formula $dB = 20\log_{10}\left({}^{a}\!/_{a_0} \right)$ can be used to convert between dB and amplitude. This allows the spectra from Wavanal and Audition to be plotted side by side for comparison. Recordings of three bells (one large, one medium and one small) were analysed with both packages to allow comparison at three points in the audible spectrum. The three bells were Petersglocke at Cologne (nominal 252Hz), the old 9th bell from Ipswich St Mary le Tower (nominal 723Hz) and the treble from St Martin in the Bullring, Birmingham (nominal 2363Hz). The Ipswich bell is examined in detail, and the other two in summary.

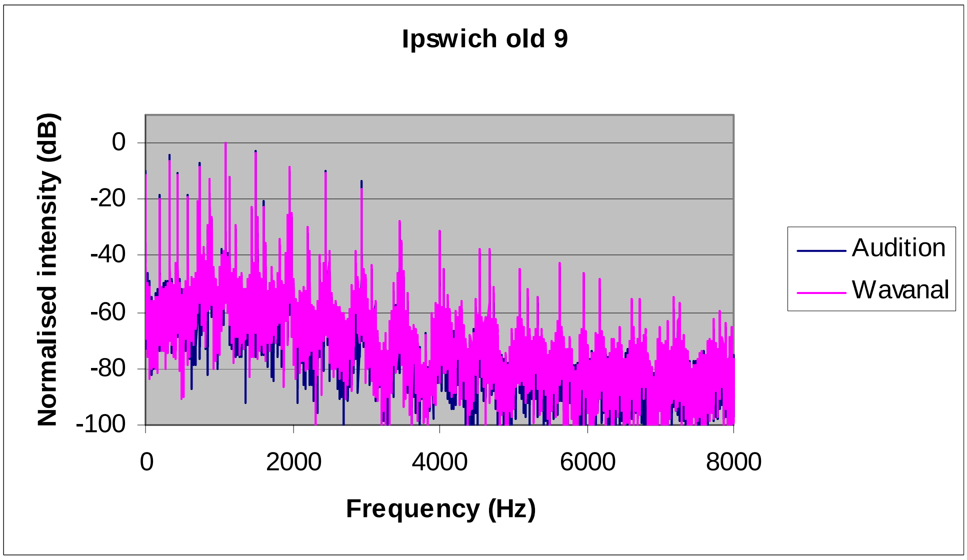
The same sound file was used in both Wavanal and Audition for each bell, with the same 2s of sound starting with the clapper strike selected in each case. Wavanal uses a Hamming window for the FFT, so this window type was also selected in Audition.

Amplitude comparison – Ipswich 9th

In the three charts below, spectra in dB from Audition and Wavanal are compared. They are each presented separately, and then overlaid on the same chart.





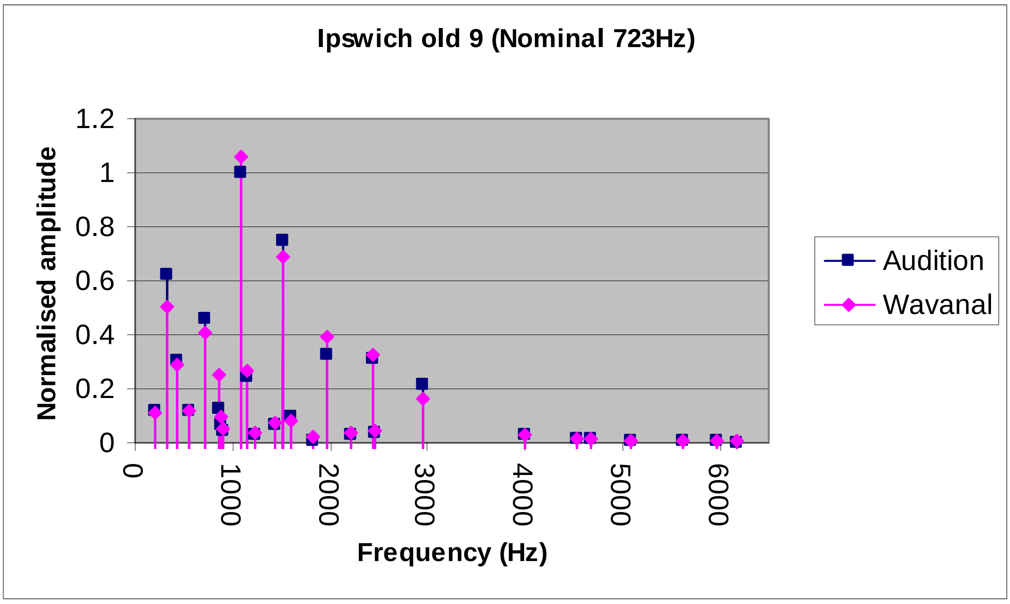


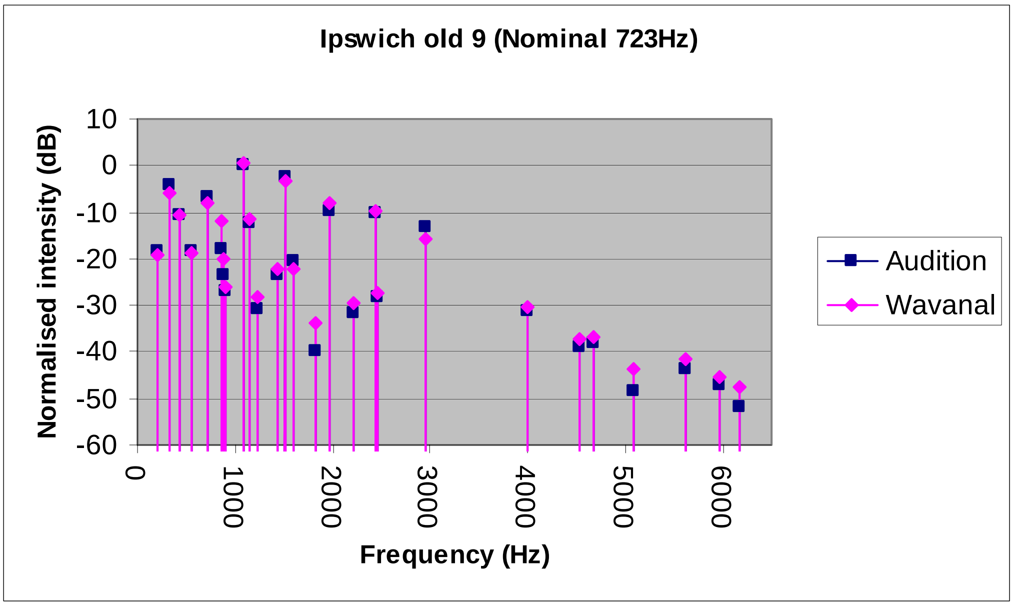
The two spectra are very similar indeed.

The frequencies and amplitudes of all the spectral peaks were obtained. These were obtained for Wavanal using the facilities of the software. For Audition they were obtained by manually identifying peak values in the raw spectrum data. The two sets of amplitude values were normalised, first by setting the maximum partial intensity measured by Audition to 0dB, and then by multiplying the Wavanal amplitudes by a factor which minimised the sum of squares of amplitude differences at each frequency. This method of normalisation was used so that larger peaks had more influence on the normalisation. The frequency values from Audition are accurate to ±0.336Hz (half the bin width in Audition). The frequency values from Wavanal are accurate to ±0.25Hz (half the bin width of 0.5Hz in Wavanal). Values measured from the two packages were as follows:

Audition			Wavanal		
Freq. (Hz)	Normalised dB	Normalised Amplitude	Freq. (Hz)	Normalised dB	Normalised Amplitude
201.9	-18.4	0.121	202	-19.3	0.109
323.7	-4.1	0.626	323.5	-5.9	0.505
423.9	-10.4	0.302	424	-10.7	0.291
562.6	-18.4	0.120	563	-18.7	0.116
722.7	-6.7	0.460	723	-7.9	0.405
862.0	-17.9	0.127	862	-12.0	0.250
882.2	-23.4	0.068	882.5	-20.2	0.097
902.4	-26.9	0.045	902.5	-25.9	0.051
1091.5	0.0	1.000	1091.5	0.5	1.061
1139.2	-12.3	0.244	1139.5	-11.5	0.265
1222.0	-30.8	0.029	1221.5	-28.4	0.038
1444.1	-23.3	0.068	1444.5	-22.3	0.077
1507.3	-2.5	0.748	1507.5	-3.2	0.689
1604.2	-20.4	0.095	1604	-22.1	0.078
1827	-39.6	0.010	1827.5	-33.6	0.021
1959.5	-9.8	0.325	1959.5	-8.1	0.394
2205.8	-31.5	0.026	2205.5	-29.5	0.034
2443.3	-10.2	0.311	2443	-9.7	0.326
2948.0	-13.3	0.215	2948	-15.9	0.161
2467.5	-28.2	0.039	3467.5	-27.3	0.043
4000.5	-31.4	0.027	4001	-30.5	0.030
4536.8	-39.0	0.011	4536.5	-37.3	0.014
4670.7	-38.2	0.012	4671	-36.7	0.015
5081.2	-48.3	0.004	5082	-43.8	0.006
5624.9	-43.8	0.006	5624.5	-41.7	0.008
5962.7	-47.1	0.004	5962.5	-45.3	0.005
6169.9	-51.7	0.003	6170	-47.6	0.004

The frequencies measured from the two packages agree exactly to within the bin width of the two FFTs. The amplitudes and intensities in dB for the partial frequencies listed in the above table compare graphically as follows:

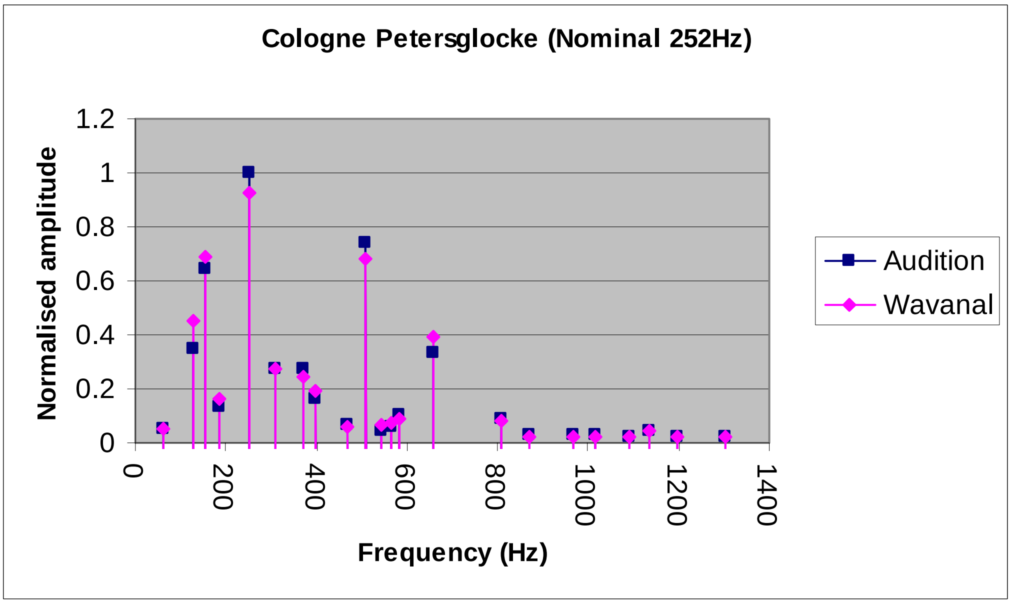


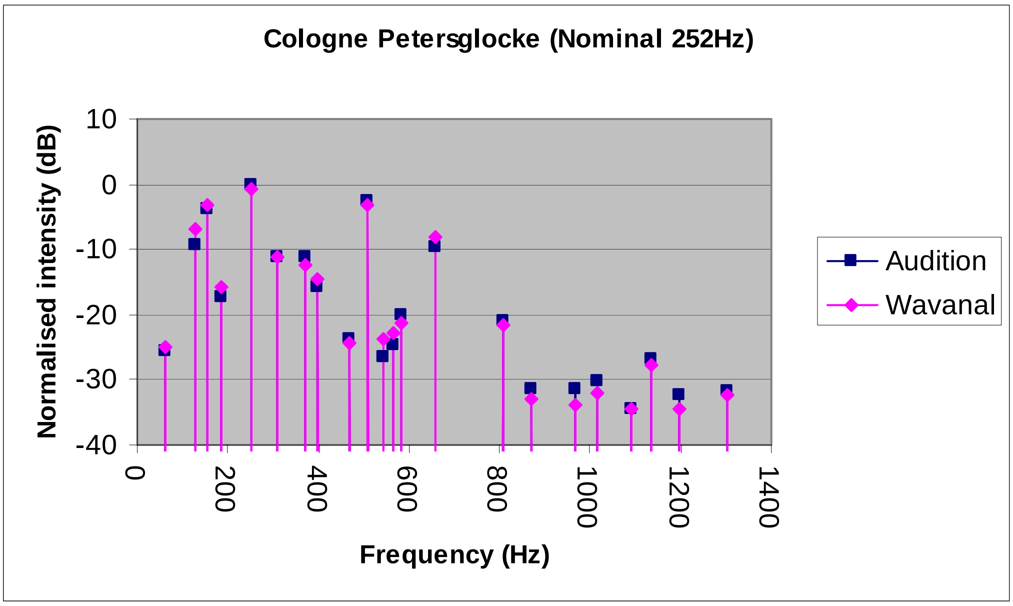


As can be seen, there is very good agreement between the output of the two packages.

Amplitude comparison – Cologne Petersglocke

The amplitude and intensity plots for the recording of this bell, produced and normalised in an identical fashion to those for the Ipswich 9 bell, are as follows:

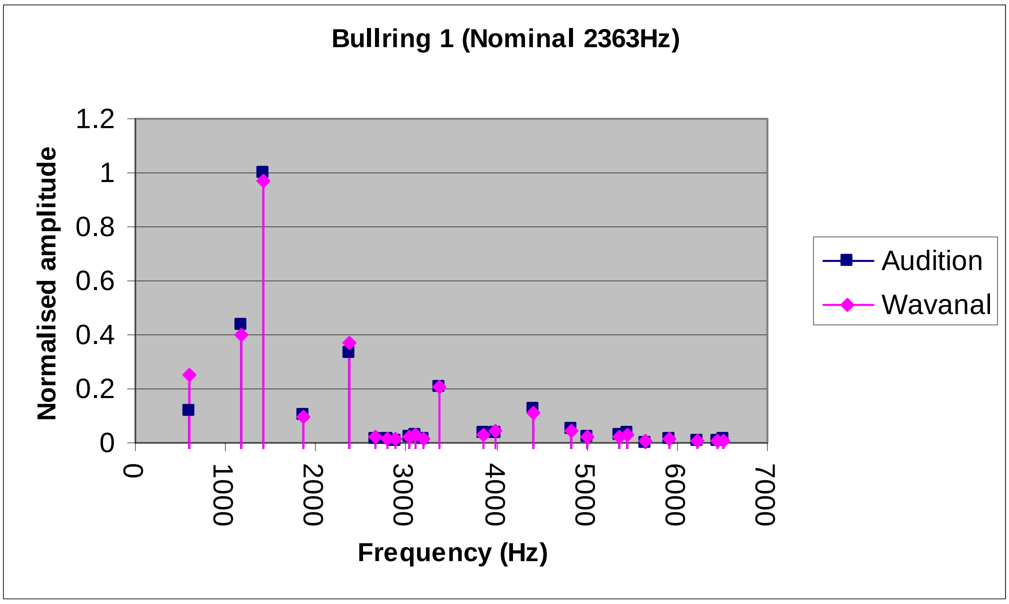


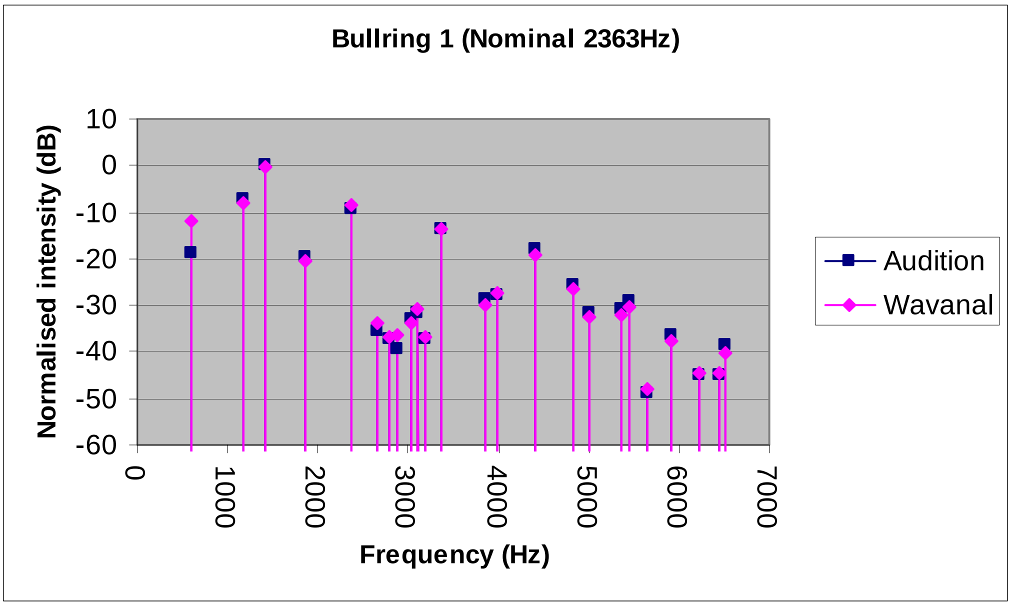


These plots show a good agreement between the two packages.

Amplitude comparison – Bullring no. 1 bell

The amplitude and intensity plots for the recording of this bell, produced in an identical fashion to those for the Ipswich 9 bell, are as follows:





These plots again show a good agreement between the two packages.

Wavanal calibration – Conclusions

The conclusion from the frequency calibration in the earlier part of this appendix is that the combination of Wavanal running on the hardware used by the author is capable of accurate frequency measurements, limited by the capability of the audio input equipment used to digitise sounds. The equipment used by the author for the most critical work in this thesis is capable of measuring frequency to within the limits imposed by the length of recordings, i.e. to ±0.1 Hz when recordings of 5s duration are used.

Partial amplitude measurements from Wavanal are very comparable to those from Adobe Audition. The experiment in section 8.3 showed that changes in partial amplitude of up to 50% had no significant effect on virtual pitch. The results of the amplitude comparison should therefore give no concern about the validity of the amplitude measurements in section 3.3, nor the interpretation of spectra in this thesis produced from Wavanal.

Appendix 10 – Parameter files for virtual pitch test and reference sounds

As described in section 6.9.1, a spreadsheet was used to calculate all the partial frequencies and other parameters for the virtual pitch tests. These files were then exported as comma-delimited files for input into the software which actually generated the sound files. The contents of these comma-delimited files are given in the tables below. Three sets of information are presented:

- The 144 test sounds and 144 reference sounds used in the virtual pitch tests reported in chapter 9
- The 16 test and 16 reference sounds used in the regression test in section 8.2 and the trial of the Terhardt model in section 10.2
- The 16 test and 16 reference sounds used in the amplitude test in section 8.3.

Explanations as to the column headings are as follows:

- The ‘duration’ column gives the length of the generated sound file in seconds
- Parameters for 15 partials are given. Most partial names are self-explanatory: Mp1, Mp2 and Mp3 are the three prominent partials between nominal and superquint
- For each partial, three pieces of information are given; the frequency in Hz; the intensity or amplitude in arbitrary units (this is a linear quantity, i.e. amplitude); and a decay parameter (the reciprocal of the time taken for the amplitude of the partial to decay to 60% of its initial value).

Parameter files for virtual pitch tests

Test file
3t

Duration	Hum_freq	Hum_int	Hum_dec	Prm_freq	Prm_int	Prm_dec	Trc_freq	Trc_int	Trc_dec	Qnt_freq	Qnt_int	Qnt_dec	Nom_freq	Nom_int	Nom_dec	Mp1_freq	Mp1_int	Mp1_dec	Mp2_freq	Mp2_int	Mp2_dec
0.25	78.74507	3.14	1	140.3078	4.02	5	187.2884	6.78	5	242.883	0.82	5	314.9803	10	5	374.5768	1.25	5	402.6226	2.43	5
0.25	78.74507	3.14	1	140.3078	4.02	5	187.2884	6.78	5	242.883	0.82	5	314.9803	10	5	374.5768	1.25	5	402.6226	2.43	5
0.25	78.74507	3.14	1	157.4901	4.02	5	187.2884	6.78	5	242.883	0.82	5	314.9803	10	5	374.5768	1.25	5	402.6226	2.43	5
0.25	78.74507	3.14	1	157.4901	4.02	5	187.2884	6.78	5	242.883	0.82	5	314.9803	10	5	374.5768	1.25	5	402.6226	2.43	5
0.25	78.74507	3.14	1	140.3078	4.02	5	198.4251	6.78	5	242.883	0.82	5	314.9803	10	5	374.5768	1.25	5	402.6226	2.43	5
0.25	78.74507	3.14	1	140.3078	4.02	5	198.4251	6.78	5	242.883	0.82	5	314.9803	10	5	374.5768	1.25	5	402.6226	2.43	5
0.25	78.74507	3.14	1	157.4901	4.02	5	198.4251	6.78	5	242.883	0.82	5	314.9803	10	5	374.5768	1.25	5	402.6226	2.43	5
0.25	78.74507	3.14	1	157.4901	4.02	5	198.4251	6.78	5	242.883	0.82	5	314.9803	10	5	374.5768	1.25	5	402.6226	2.43	5
0.25	78.74507	3.14	1	140.3078	4.02	5	187.2884	6.78	5	242.883	0.82	5	314.9803	10	5	374.5768	1.25	5	402.6226	2.43	5
0.25	78.74507	3.14	1	140.3078	4.02	5	187.2884	6.78	5	242.883	0.82	5	314.9803	10	5	374.5768	1.25	5	402.6226	2.43	5
0.25	78.74507	3.14	1	157.4901	4.02	5	187.2884	6.78	5	242.883	0.82	5	314.9803	10	5	374.5768	1.25	5	402.6226	2.43	5
0.25	78.74507	3.14	1	157.4901	4.02	5	187.2884	6.78	5	242.883	0.82	5	314.9803	10	5	374.5768	1.25	5	402.6226	2.43	5
0.25	78.74507	3.14	1	140.3078	4.02	5	198.4251	6.78	5	242.883	0.82	5	314.9803	10	5	374.5768	1.25	5	402.6226	2.43	5
0.25	78.74507	3.14	1	140.3078	4.02	5	198.4251	6.78	5	242.883	0.82	5	314.9803	10	5	374.5768	1.25	5	402.6226	2.43	5
0.25	78.74507	3.14	1	157.4901	4.02	5	198.4251	6.78	5	242.883	0.82	5	314.9803	10	5	374.5768	1.25	5	402.6226	2.43	5
0.25	78.74507	3.14	1	157.4901	4.02	5	198.4251	6.78	5	242.883	0.82	5	314.9803	10	5	374.5768	1.25	5	402.6226	2.43	5

Test file continued
3t

Mp3_freq	Mp3_int	Mp3_dec	SQ_freq	SQ_int	SQ_dec	ON_freq	ON_int	ON_dec	I-7_freq	I-7_int	I-7_dec	I-8_freq	I-8_int	I-8_dec	I-9_freq	I-9_int	I-9_dec	I-10_freq	I-10_int	I-10_dec	I-11_freq	I-11_int	I-11_dec
427.5505	1.83	5	455.4237	6.88	5	608.5018	5.15	5	770.1576	3.71	5	938.5394	2.39	5	1111.027	1.75	5	1290.729	1.13	5	1489.481	0.76	5
427.5505	1.83	5	455.4237	6.88	5	608.5018	5.15	5	770.1576	3.71	5	938.5394	2.39	5	1111.027	1.75	5	1290.729	1.13	5	1489.481	0.76	5
427.5505	1.83	5	455.4237	6.88	5	608.5018	5.15	5	770.1576	3.71	5	938.5394	2.39	5	1111.027	1.75	5	1290.729	1.13	5	1489.481	0.76	5
427.5505	1.83	5	455.4237	6.88	5	608.5018	5.15	5	770.1576	3.71	5	938.5394	2.39	5	1111.027	1.75	5	1290.729	1.13	5	1489.481	0.76	5
427.5505	1.83	5	455.4237	6.88	5	608.5018	5.15	5	770.1576	3.71	5	938.5394	2.39	5	1111.027	1.75	5	1290.729	1.13	5	1489.481	0.76	5
427.5505	1.83	5	455.4237	6.88	5	608.5018	5.15	5	770.1576	3.71	5	938.5394	2.39	5	1111.027	1.75	5	1290.729	1.13	5	1489.481	0.76	5
427.5505	1.83	5	455.4237	6.88	5	608.5018	5.15	5	770.1576	3.71	5	938.5394	2.39	5	1111.027	1.75	5	1290.729	1.13	5	1489.481	0.76	5
427.5505	1.83	5	455.4237	6.88	5	608.5018	5.15	5	770.1576	3.71	5	938.5394	2.39	5	1111.027	1.75	5	1290.729	1.13	5	1489.481	0.76	5
427.5505	1.83	5	475.6887	6.88	5	659.754	5.15	5	864.316	3.71	5	1083.497	2.39	5	1314.138	1.75	5	1556.243	1.13	5	1795.312	0.76	5
427.5505	1.83	5	475.6887	6.88	5	659.754	5.15	5	864.316	3.71	5	1083.497	2.39	5	1314.138	1.75	5	1556.243	1.13	5	1795.312	0.76	5
427.5505	1.83	5	475.6887	6.88	5	659.754	5.15	5	864.316	3.71	5	1083.497	2.39	5	1314.138	1.75	5	1556.243	1.13	5	1795.312	0.76	5
427.5505	1.83	5	475.6887	6.88	5	659.754	5.15	5	864.316	3.71	5	1083.497	2.39	5	1314.138	1.75	5	1556.243	1.13	5	1795.312	0.76	5
427.5505	1.83	5	475.6887	6.88	5	659.754	5.15	5	864.316	3.71	5	1083.497	2.39	5	1314.138	1.75	5	1556.243	1.13	5	1795.312	0.76	5
427.5505	1.83	5	475.6887	6.88	5	659.754	5.15	5	864.316	3.71	5	1083.497	2.39	5	1314.138	1.75	5	1556.243	1.13	5	1795.312	0.76	5
427.5505	1.83	5	475.6887	6.88	5	659.754	5.15	5	864.316	3.71	5	1083.497	2.39	5	1314.138	1.75	5	1556.243	1.13	5	1795.312	0.76	5
427.5505	1.83	5	475.6887	6.88	5	659.754	5.15	5	864.316	3.71	5	1083.497	2.39	5	1314.138	1.75	5	1556.243	1.13	5	1795.312	0.76	5

Reference file
3r

Duration	Hum_freq	Hum_int	Hum_dec	Prm_freq	Prm_int	Prm_dec	Trc_freq	Trc_int	Trc_dec	Qnt_freq	Qnt_int	Qnt_dec	Nom_freq	Nom_int	Nom_dec	Mp1_freq	Mp1_int	Mp1_dec	Mp2_freq	Mp2_int	Mp2_dec
0.25	73.04858	3.14	1	146.0972	4.02	5	175.7585	6.78	5	225.3126	0.82	5	292.1943	10	5	347.4796	1.25	5	373.4965	2.43	5
0.25	73.89736	3.14	1	147.7947	4.02	5	177.8008	6.78	5	227.9306	0.82	5	295.5894	10	5	351.5171	1.25	5	377.8363	2.43	5
0.25	74.75601	3.14	1	149.512	4.02	5	179.8667	6.78	5	230.579	0.82	5	299.024	10	5	355.6015	1.25	5	382.2266	2.43	5
0.25	75.62463	3.14	1	151.2493	4.02	5	181.9566	6.78	5	233.2582	0.82	5	302.4985	10	5	359.7334	1.25	5	386.6678	2.43	5
0.25	76.50335	3.14	1	153.0067	4.02	5	184.0709	6.78	5	235.9686	0.82	5	306.0134	10	5	363.9133	1.25	5	391.1607	2.43	5
0.25	77.39227	3.14	1	154.7845	4.02	5	186.2097	6.78	5	238.7104	0.82	5	309.5691	10	5	368.1418	1.25	5	395.7058	2.43	5
0.25	78.29153	3.14	1	156.5831	4.02	5	188.3733	6.78	5	241.4841	0.82	5	313.1661	10	5	372.4194	1.25	5	400.3036	2.43	5
0.25	79.20123	3.14	1	158.4025	4.02	5	190.5621	6.78	5	244.29	0.82	5	316.8049	10	5	376.7467	1.25	5	404.955	2.43	5
0.25	80.1215	3.14	1	160.243	4.02	5	192.7764	6.78	5	247.1285	0.82	5	320.486	10	5	381.1243	1.25	5	409.6603	2.43	5
0.25	81.05247	3.14	1	162.1049	4.02	5	195.0163	6.78	5	250	0.82	5	324.2099	10	5	385.5527	1.25	5	414.4203	2.43	5
0.25	81.99426	3.14	1	163.9885	4.02	5	197.2823	6.78	5	252.9049	0.82	5	327.977	10	5	390.0326	1.25	5	419.2357	2.43	5
0.25	82.94698	3.14	1	165.894	4.02	5	199.5746	6.78	5	255.8435	0.82	5	331.7879	10	5	394.5646	1.25	5	424.1069	2.43	5
0.25	83.91078	3.14	1	167.8216	4.02	5	201.8935	6.78	5	258.8162	0.82	5	335.6431	10	5	399.1492	1.25	5	429.0348	2.43	5
0.25	84.88578	3.14	1	169.7716	4.02	5	204.2394	6.78	5	261.8235	0.82	5	339.5431	10	5	403.7871	1.25	5	434.02	2.43	5
0.25	85.8721	3.14	1	171.7442	4.02	5	206.6126	6.78	5	264.8658	0.82	5	343.4884	10	5	408.4789	1.25	5	439.063	2.43	5
0.25	86.86989	3.14	1	173.7398	4.02	5	209.0133	6.78	5	267.9434	0.82	5	347.4796	10	5	413.2252	1.25	5	444.1647	2.43	5

Reference file continued
3r

Mp3_freq	Mp3_int	Mp3_dec	SQ_freq	SQ_int	SQ_dec	ON_freq	ON_int	ON_dec	I-7_freq	I-7_int	I-7_dec	I-8_freq	I-8_int	I-8_dec	I-9_freq	I-9_int	I-9_dec	I-10_freq	I-10_int	I-10_dec	I-11_freq	I-11_int	I-11_dec
396.6211	1.83	5	431.7752	6.88	5	587.774	5.15	5	756.8581	3.71	5	935.4673	2.39	5	1120.911	1.75	5	1314.754	1.13	5	1516.966	0.76	5
401.2296	1.83	5	436.7921	6.88	5	594.6036	5.15	5	765.6524	3.71	5	946.3369	2.39	5	1133.936	1.75	5	1330.031	1.13	5	1534.593	0.76	5
405.8917	1.83	5	441.8674	6.88	5	601.5125	5.15	5	774.5489	3.71	5	957.3328	2.39	5	1147.111	1.75	5	1345.485	1.13	5	1552.424	0.76	5
410.6079	1.83	5	447.0017	6.88	5	608.5018	5.15	5	783.5487	3.71	5	968.4565	2.39	5	1160.44	1.75	5	1361.119	1.13	5	1570.462	0.76	5
415.3789	1.83	5	452.1956	6.88	5	615.5722	5.15	5	792.6531	3.71	5	979.7094	2.39	5	1173.924	1.75	5	1376.934	1.13	5	1588.71	0.76	5
420.2054	1.83	5	457.4498	6.88	5	622.7248	5.15	5	801.8633	3.71	5	991.0931	2.39	5	1187.564	1.75	5	1392.934	1.13	5	1607.17	0.76	5
425.088	1.83	5	462.7652	6.88	5	629.9605	5.15	5	811.1805	3.71	5	1002.609	2.39	5	1201.363	1.75	5	1409.119	1.13	5	1625.844	0.76	5
430.0273	1.83	5	468.1422	6.88	5	637.2803	5.15	5	820.6059	3.71	5	1014.259	2.39	5	1215.322	1.75	5	1425.492	1.13	5	1644.736	0.76	5
435.0239	1.83	5	473.5818	6.88	5	644.6852	5.15	5	830.1409	3.71	5	1026.044	2.39	5	1229.443	1.75	5	1442.055	1.13	5	1663.846	0.76	5
440.0787	1.83	5	479.0845	6.88	5	652.176	5.15	5	839.7867	3.71	5	1037.966	2.39	5	1243.729	1.75	5	1458.811	1.13	5	1683.179	0.76	5
445.1921	1.83	5	484.6512	6.88	5	659.754	5.15	5	849.5445	3.71	5	1050.027	2.39	5	1258.18	1.75	5	1475.762	1.13	5	1702.737	0.76	5
450.365	1.83	5	490.2826	6.88	5	667.4199	5.15	5	859.4158	3.71	5	1062.227	2.39	5	1272.8	1.75	5	1492.909	1.13	5	1722.522	0.76	5
455.598	1.83	5	495.9794	6.88	5	675.175	5.15	5	869.4017	3.71	5	1074.57	2.39	5	1287.589	1.75	5	1510.256	1.13	5	1742.537	0.76	5
460.8918	1.83	5	501.7424	6.88	5	683.0201	5.15	5	879.5037	3.71	5	1087.056	2.39	5	1302.55	1.75	5	1527.804	1.13	5	1762.784	0.76	5
466.2471	1.83	5	507.5724	6.88	5	690.9564	5.15	5	889.723	3.71	5	1099.687	2.39	5	1317.685	1.75	5	1545.557	1.13	5	1783.267	0.76	5
471.6646	1.83	5	513.4701	6.88	5	698.985	5.15	5	900.0611	3.71	5	1112.464	2.39	5	1332.996	1.75	5	1563.515	1.13	5	1803.987	0.76	5

Test file

4t

Duration	Hum_freq	Hum_int	Hum_dec	Prm_freq	Prm_int	Prm_dec	Trc_freq	Trc_int	Trc_dec	Qnt_freq	Qnt_int	Qnt_dec	Nom_freq	Nom_int	Nom_dec	Mp1_freq	Mp1_int	Mp1_dec	Mp2_freq	Mp2_int	Mp2_dec
0.25	99.21257	3.14	1	176.7767	4.02	5	235.9686	6.78	5	280.6155	0.82	5	396.8503	10	5	471.9372	1.25	5	507.2727	2.43	5
0.25	99.21257	3.14	1	176.7767	4.02	5	235.9686	6.78	5	333.71	0.82	5	396.8503	10	5	471.9372	1.25	5	507.2727	2.43	5
0.25	99.21257	3.14	1	198.4251	4.02	5	235.9686	6.78	5	280.6155	0.82	5	396.8503	10	5	471.9372	1.25	5	507.2727	2.43	5
0.25	99.21257	3.14	1	198.4251	4.02	5	235.9686	6.78	5	333.71	0.82	5	396.8503	10	5	471.9372	1.25	5	507.2727	2.43	5
0.25	99.21257	3.14	1	176.7767	4.02	5	250	6.78	5	280.6155	0.82	5	396.8503	10	5	471.9372	1.25	5	507.2727	2.43	5
0.25	99.21257	3.14	1	176.7767	4.02	5	250	6.78	5	333.71	0.82	5	396.8503	10	5	471.9372	1.25	5	507.2727	2.43	5
0.25	99.21257	3.14	1	198.4251	4.02	5	250	6.78	5	280.6155	0.82	5	396.8503	10	5	471.9372	1.25	5	507.2727	2.43	5
0.25	99.21257	3.14	1	198.4251	4.02	5	250	6.78	5	333.71	0.82	5	396.8503	10	5	471.9372	1.25	5	507.2727	2.43	5
0.25	99.21257	3.14	1	176.7767	4.02	5	235.9686	6.78	5	280.6155	0.82	5	396.8503	10	5	471.9372	1.25	5	507.2727	2.43	5
0.25	99.21257	3.14	1	176.7767	4.02	5	235.9686	6.78	5	333.71	0.82	5	396.8503	10	5	471.9372	1.25	5	507.2727	2.43	5
0.25	99.21257	3.14	1	198.4251	4.02	5	235.9686	6.78	5	280.6155	0.82	5	396.8503	10	5	471.9372	1.25	5	507.2727	2.43	5
0.25	99.21257	3.14	1	198.4251	4.02	5	235.9686	6.78	5	333.71	0.82	5	396.8503	10	5	471.9372	1.25	5	507.2727	2.43	5
0.25	99.21257	3.14	1	176.7767	4.02	5	250	6.78	5	280.6155	0.82	5	396.8503	10	5	471.9372	1.25	5	507.2727	2.43	5
0.25	99.21257	3.14	1	176.7767	4.02	5	250	6.78	5	333.71	0.82	5	396.8503	10	5	471.9372	1.25	5	507.2727	2.43	5
0.25	99.21257	3.14	1	198.4251	4.02	5	250	6.78	5	280.6155	0.82	5	396.8503	10	5	471.9372	1.25	5	507.2727	2.43	5
0.25	99.21257	3.14	1	198.4251	4.02	5	250	6.78	5	333.71	0.82	5	396.8503	10	5	471.9372	1.25	5	507.2727	2.43	5

Test file continued

4t

Mp3_freq	Mp3_int	Mp3_dec	SQ_freq	SQ_int	SQ_dec	ON_freq	ON_int	ON_dec	I-7_freq	I-7_int	I-7_dec	I-8_freq	I-8_int	I-8_dec	I-9_freq	I-9_int	I-9_dec	I-10_freq	I-10_int	I-10_dec	I-11_freq	I-11_int	I-11_dec
538.6798	1.83	5	572.8041	6.88	5	766.6642	5.15	5	973.8297	3.71	5	1190.17	2.39	5	1413.445	1.75	5	1640.549	1.13	5	1871.049	0.76	5
538.6798	1.83	5	572.8041	6.88	5	766.6642	5.15	5	973.8297	3.71	5	1190.17	2.39	5	1413.445	1.75	5	1640.549	1.13	5	1871.049	0.76	5
538.6798	1.83	5	572.8041	6.88	5	766.6642	5.15	5	973.8297	3.71	5	1190.17	2.39	5	1413.445	1.75	5	1640.549	1.13	5	1871.049	0.76	5
538.6798	1.83	5	572.8041	6.88	5	766.6642	5.15	5	973.8297	3.71	5	1190.17	2.39	5	1413.445	1.75	5	1640.549	1.13	5	1871.049	0.76	5
538.6798	1.83	5	572.8041	6.88	5	766.6642	5.15	5	973.8297	3.71	5	1190.17	2.39	5	1413.445	1.75	5	1640.549	1.13	5	1871.049	0.76	5
538.6798	1.83	5	572.8041	6.88	5	766.6642	5.15	5	973.8297	3.71	5	1190.17	2.39	5	1413.445	1.75	5	1640.549	1.13	5	1871.049	0.76	5
538.6798	1.83	5	572.8041	6.88	5	766.6642	5.15	5	973.8297	3.71	5	1190.17	2.39	5	1413.445	1.75	5	1640.549	1.13	5	1871.049	0.76	5
538.6798	1.83	5	572.8041	6.88	5	766.6642	5.15	5	973.8297	3.71	5	1190.17	2.39	5	1413.445	1.75	5	1640.549	1.13	5	1871.049	0.76	5
538.6798	1.83	5	599.2109	6.88	5	831.2379	5.15	5	1087.296	3.71	5	1361.984	2.39	5	1652.049	1.75	5	1953.19	1.13	5	2265.19	0.76	5
538.6798	1.83	5	599.2109	6.88	5	831.2379	5.15	5	1087.296	3.71	5	1361.984	2.39	5	1652.049	1.75	5	1953.19	1.13	5	2265.19	0.76	5
538.6798	1.83	5	599.2109	6.88	5	831.2379	5.15	5	1087.296	3.71	5	1361.984	2.39	5	1652.049	1.75	5	1953.19	1.13	5	2265.19	0.76	5
538.6798	1.83	5	599.2109	6.88	5	831.2379	5.15	5	1087.296	3.71	5	1361.984	2.39	5	1652.049	1.75	5	1953.19	1.13	5	2265.19	0.76	5
538.6798	1.83	5	599.2109	6.88	5	831.2379	5.15	5	1087.296	3.71	5	1361.984	2.39	5	1652.049	1.75	5	1953.19	1.13	5	2265.19	0.76	5
538.6798	1.83	5	599.2109	6.88	5	831.2379	5.15	5	1087.296	3.71	5	1361.984	2.39	5	1652.049	1.75	5	1953.19	1.13	5	2265.19	0.76	5
538.6798	1.83	5	599.2109	6.88	5	831.2379	5.15	5	1087.296	3.71	5	1361.984	2.39	5	1652.049	1.75	5	1953.19	1.13	5	2265.19	0.76	5
538.6798	1.83	5	599.2109	6.88	5	831.2379	5.15	5	1087.296	3.71	5	1361.984	2.39	5	1652.049	1.75	5	1953.19	1.13	5	2265.19	0.76	5

Reference file

4r

Duration	Hum_freq	Hum_int	Hum_dec	Prm_freq	Prm_int	Prm_dec	Trc_freq	Trc_int	Trc_dec	Qnt_freq	Qnt_int	Qnt_dec	Nom_freq	Nom_int	Nom_dec	Mp1_freq	Mp1_int	Mp1_dec	Mp2_freq	Mp2_int	Mp2_dec
0.25	94.66391	3.14	1	189.3278	4.02	5	227.7661	6.78	5	291.9834	0.82	5	378.6556	10	5	450.3	1.25	5	484.0154	2.43	5
0.25	95.34988	3.14	1	190.6998	4.02	5	229.4166	6.78	5	294.0992	0.82	5	381.3995	10	5	453.563	1.25	5	487.5228	2.43	5
0.25	96.04083	3.14	1	192.0817	4.02	5	231.079	6.78	5	296.2304	0.82	5	384.1633	10	5	456.8497	1.25	5	491.0556	2.43	5
0.25	96.73678	3.14	1	193.4736	4.02	5	232.7535	6.78	5	298.377	0.82	5	386.9471	10	5	460.1603	1.25	5	494.614	2.43	5
0.25	97.43777	3.14	1	194.8755	4.02	5	234.4402	6.78	5	300.5392	0.82	5	389.7511	10	5	463.4948	1.25	5	498.1982	2.43	5
0.25	98.14385	3.14	1	196.2877	4.02	5	236.139	6.78	5	302.717	0.82	5	392.5754	10	5	466.8535	1.25	5	501.8083	2.43	5
0.25	98.85504	3.14	1	197.7101	4.02	5	237.8502	6.78	5	304.9106	0.82	5	395.4202	10	5	470.2365	1.25	5	505.4446	2.43	5
0.25	99.57138	3.14	1	199.1428	4.02	5	239.5737	6.78	5	307.1201	0.82	5	398.2855	10	5	473.644	1.25	5	509.1073	2.43	5
0.25	100.2929	3.14	1	200.5858	4.02	5	241.3098	6.78	5	309.3457	0.82	5	401.1717	10	5	477.0762	1.25	5	512.7965	2.43	5
0.25	101.0197	3.14	1	202.0394	4.02	5	243.0584	6.78	5	311.5873	0.82	5	404.0787	10	5	480.5333	1.25	5	516.5124	2.43	5
0.25	101.7517	3.14	1	203.5034	4.02	5	244.8197	6.78	5	313.8452	0.82	5	407.0069	10	5	484.0154	1.25	5	520.2553	2.43	5
0.25	102.489	3.14	1	204.9781	4.02	5	246.5938	6.78	5	316.1194	0.82	5	409.9562	10	5	487.5228	1.25	5	524.0253	2.43	5
0.25	103.2317	3.14	1	206.4635	4.02	5	248.3807	6.78	5	318.4102	0.82	5	412.9269	10	5	491.0556	1.25	5	527.8226	2.43	5
0.25	103.9798	3.14	1	207.9596	4.02	5	250.1806	6.78	5	320.7175	0.82	5	415.9191	10	5	494.614	1.25	5	531.6474	2.43	5
0.25	104.7333	3.14	1	209.4665	4.02	5	251.9935	6.78	5	323.0416	0.82	5	418.9331	10	5	498.1982	1.25	5	535.4999	2.43	5
0.25	105.4922	3.14	1	210.9844	4.02	5	253.8195	6.78	5	325.3824	0.82	5	421.9688	10	5	501.8083	1.25	5	539.3804	2.43	5

Reference file continued

4r

Mp3_freq	Mp3_int	Mp3_dec	SQ_freq	SQ_int	SQ_dec	ON_freq	ON_int	ON_dec	I-7_freq	I-7_int	I-7_dec	I-8_freq	I-8_int	I-8_dec	I-9_freq	I-9_int	I-9_dec	I-10_freq	I-10_int	I-10_dec	I-11_freq	I-11_int	I-11_dec
513.9827	1.83	5	558.9986	6.88	5	761.6983	5.15	5	981.8229	3.71	5	1214.81	2.39	5	1458.038	1.75	5	1707.987	1.13	5	1964.322	0.76	5
517.7072	1.83	5	563.0493	6.88	5	767.2179	5.15	5	988.9376	3.71	5	1223.613	2.39	5	1468.603	1.75	5	1720.364	1.13	5	1978.556	0.76	5
521.4587	1.83	5	567.1294	6.88	5	772.7775	5.15	5	996.1039	3.71	5	1232.48	2.39	5	1479.245	1.75	5	1732.83	1.13	5	1992.893	0.76	5
525.2374	1.83	5	571.239	6.88	5	778.3774	5.15	5	1003.322	3.71	5	1241.411	2.39	5	1489.965	1.75	5	1745.387	1.13	5	2007.335	0.76	5
529.0435	1.83	5	575.3785	6.88	5	784.0178	5.15	5	1010.593	3.71	5	1250.406	2.39	5	1500.762	1.75	5	1758.035	1.13	5	2021.881	0.76	5
532.8772	1.83	5	579.5479	6.88	5	789.6991	5.15	5	1017.916	3.71	5	1259.467	2.39	5	1511.637	1.75	5	1770.774	1.13	5	2036.532	0.76	5
536.7386	1.83	5	583.7475	6.88	5	795.4216	5.15	5	1025.292	3.71	5	1268.594	2.39	5	1522.591	1.75	5	1783.606	1.13	5	2051.29	0.76	5
540.6281	1.83	5	587.9776	6.88	5	801.1856	5.15	5	1032.722	3.71	5	1277.787	2.39	5	1533.624	1.75	5	1796.531	1.13	5	2066.154	0.76	5
544.5457	1.83	5	592.2383	6.88	5	806.9913	5.15	5	1040.205	3.71	5	1287.046	2.39	5	1544.737	1.75	5	1809.549	1.13	5	2081.126	0.76	5
548.4917	1.83	5	596.5299	6.88	5	812.8391	5.15	5	1047.743	3.71	5	1296.373	2.39	5	1555.931	1.75	5	1822.662	1.13	5	2096.207	0.76	5
552.4663	1.83	5	600.8526	6.88	5	818.7292	5.15	5	1055.335	3.71	5	1305.767	2.39	5	1567.206	1.75	5	1835.87	1.13	5	2111.397	0.76	5
556.4697	1.83	5	605.2067	6.88	5	824.6621	5.15	5	1062.983	3.71	5	1315.229	2.39	5	1578.563	1.75	5	1849.173	1.13	5	2126.697	0.76	5
560.5021	1.83	5	609.5923	6.88	5	830.6379	5.15	5	1070.686	3.71	5	1324.76	2.39	5	1590.001	1.75	5	1862.573	1.13	5	2142.108	0.76	5
564.5637	1.83	5	614.0096	6.88	5	836.6571	5.15	5	1078.444	3.71	5	1334.359	2.39	5	1601.523	1.75	5	1876.07	1.13	5	2157.631	0.76	5
568.6548	1.83	5	618.459	6.88	5	842.7198	5.15	5	1086.259	3.71	5	1344.029	2.39	5	1613.129	1.75	5	1889.665	1.13	5	2173.266	0.76	5
572.7755	1.83	5	622.9406	6.88	5	848.8265	5.15	5	1094.13	3.71	5	1353.768	2.39	5	1624.818	1.75	5	1903.358	1.13	5	2189.014	0.76	5

Test file
5t

Duration	Hum_freq	Hum_int	Hum_dec	Prm_freq	Prm_int	Prm_dec	Trc_freq	Trc_int	Trc_dec	Qnt_freq	Qnt_int	Qnt_dec	Nom_freq	Nom_int	Nom_dec	Mp1_freq	Mp1_int	Mp1_dec	Mp2_freq	Mp2_int	Mp2_dec
0.25	125	3.14	1	222.7247	4.02	5	297.3018	6.78	5	385.5527	0.82	5	500	10	5	561.231	1.25	5	594.6036	2.43	5
0.25	125	3.14	1	222.7247	4.02	5	297.3018	6.78	5	385.5527	0.82	5	500	10	5	629.9605	1.25	5	686.9768	2.43	5
0.25	125	3.14	1	250	4.02	5	297.3018	6.78	5	385.5527	0.82	5	500	10	5	561.231	1.25	5	594.6036	2.43	5
0.25	125	3.14	1	250	4.02	5	297.3018	6.78	5	385.5527	0.82	5	500	10	5	629.9605	1.25	5	686.9768	2.43	5
0.25	125	3.14	1	222.7247	4.02	5	314.9803	6.78	5	385.5527	0.82	5	500	10	5	561.231	1.25	5	594.6036	2.43	5
0.25	125	3.14	1	222.7247	4.02	5	314.9803	6.78	5	385.5527	0.82	5	500	10	5	629.9605	1.25	5	686.9768	2.43	5
0.25	125	3.14	1	250	4.02	5	314.9803	6.78	5	385.5527	0.82	5	500	10	5	561.231	1.25	5	594.6036	2.43	5
0.25	125	3.14	1	250	4.02	5	314.9803	6.78	5	385.5527	0.82	5	500	10	5	629.9605	1.25	5	686.9768	2.43	5
0.25	125	3.14	1	222.7247	4.02	5	297.3018	6.78	5	385.5527	0.82	5	500	10	5	561.231	1.25	5	594.6036	2.43	5
0.25	125	3.14	1	222.7247	4.02	5	297.3018	6.78	5	385.5527	0.82	5	500	10	5	629.9605	1.25	5	686.9768	2.43	5
0.25	125	3.14	1	250	4.02	5	297.3018	6.78	5	385.5527	0.82	5	500	10	5	561.231	1.25	5	594.6036	2.43	5
0.25	125	3.14	1	250	4.02	5	297.3018	6.78	5	385.5527	0.82	5	500	10	5	629.9605	1.25	5	686.9768	2.43	5
0.25	125	3.14	1	222.7247	4.02	5	314.9803	6.78	5	385.5527	0.82	5	500	10	5	561.231	1.25	5	594.6036	2.43	5
0.25	125	3.14	1	222.7247	4.02	5	314.9803	6.78	5	385.5527	0.82	5	500	10	5	629.9605	1.25	5	686.9768	2.43	5
0.25	125	3.14	1	250	4.02	5	314.9803	6.78	5	385.5527	0.82	5	500	10	5	561.231	1.25	5	594.6036	2.43	5
0.25	125	3.14	1	250	4.02	5	314.9803	6.78	5	385.5527	0.82	5	500	10	5	629.9605	1.25	5	686.9768	2.43	5

Test file continued
5t

Mp3_freq	Mp3_int	Mp3_dec	SQ_freq	SQ_int	SQ_dec	ON_freq	ON_int	ON_dec	I-7_freq	I-7_int	I-7_dec	I-8_freq	I-8_int	I-8_dec	I-9_freq	I-9_int	I-9_dec	I-10_freq	I-10_int	I-10_dec	I-11_freq	I-11_int	I-11_dec
642.0837	1.83	5	721.688	6.88	5	965.9363	5.15	5	1226.949	3.71	5	1499.52	2.39	5	1780.829	1.75	5	2066.963	1.13	5	2357.374	0.76	5
717.3919	1.83	5	721.688	6.88	5	965.9363	5.15	5	1226.949	3.71	5	1499.52	2.39	5	1780.829	1.75	5	2066.963	1.13	5	2357.374	0.76	5
642.0837	1.83	5	721.688	6.88	5	965.9363	5.15	5	1226.949	3.71	5	1499.52	2.39	5	1780.829	1.75	5	2066.963	1.13	5	2357.374	0.76	5
717.3919	1.83	5	721.688	6.88	5	965.9363	5.15	5	1226.949	3.71	5	1499.52	2.39	5	1780.829	1.75	5	2066.963	1.13	5	2357.374	0.76	5
642.0837	1.83	5	721.688	6.88	5	965.9363	5.15	5	1226.949	3.71	5	1499.52	2.39	5	1780.829	1.75	5	2066.963	1.13	5	2357.374	0.76	5
717.3919	1.83	5	721.688	6.88	5	965.9363	5.15	5	1226.949	3.71	5	1499.52	2.39	5	1780.829	1.75	5	2066.963	1.13	5	2357.374	0.76	5
642.0837	1.83	5	721.688	6.88	5	965.9363	5.15	5	1226.949	3.71	5	1499.52	2.39	5	1780.829	1.75	5	2066.963	1.13	5	2357.374	0.76	5
717.3919	1.83	5	721.688	6.88	5	965.9363	5.15	5	1226.949	3.71	5	1499.52	2.39	5	1780.829	1.75	5	2066.963	1.13	5	2357.374	0.76	5
642.0837	1.83	5	754.9584	6.88	5	1047.294	5.15	5	1369.907	3.71	5	1715.992	2.39	5	2081.452	1.75	5	2460.865	1.13	5	2853.96	0.76	5
717.3919	1.83	5	754.9584	6.88	5	1047.294	5.15	5	1369.907	3.71	5	1715.992	2.39	5	2081.452	1.75	5	2460.865	1.13	5	2853.96	0.76	5
642.0837	1.83	5	754.9584	6.88	5	1047.294	5.15	5	1369.907	3.71	5	1715.992	2.39	5	2081.452	1.75	5	2460.865	1.13	5	2853.96	0.76	5
717.3919	1.83	5	754.9584	6.88	5	1047.294	5.15	5	1369.907	3.71	5	1715.992	2.39	5	2081.452	1.75	5	2460.865	1.13	5	2853.96	0.76	5
642.0837	1.83	5	754.9584	6.88	5	1047.294	5.15	5	1369.907	3.71	5	1715.992	2.39	5	2081.452	1.75	5	2460.865	1.13	5	2853.96	0.76	5
717.3919	1.83	5	754.9584	6.88	5	1047.294	5.15	5	1369.907	3.71	5	1715.992	2.39	5	2081.452	1.75	5	2460.865	1.13	5	2853.96	0.76	5
642.0837	1.83	5	754.9584	6.88	5	1047.294	5.15	5	1369.907	3.71	5	1715.992	2.39	5	2081.452	1.75	5	2460.865	1.13	5	2853.96	0.76	5
717.3919	1.83	5	754.9584	6.88	5	1047.294	5.15	5	1369.907	3.71	5	1715.992	2.39	5	2081.452	1.75	5	2460.865	1.13	5	2853.96	0.76	5

Reference file

5r

Duration	Hum_freq	Hum_int	Hum_dec	Prm_freq	Prm_int	Prm_dec	Trc_freq	Trc_int	Trc_dec	Qnt_freq	Qnt_int	Qnt_dec	Nom_freq	Nom_int	Nom_dec	Mp1_freq	Mp1_int	Mp1_dec	Mp2_freq	Mp2_int	Mp2_dec
0.25	119.7004	3.14	1	239.4008	4.02	5	288.0052	6.78	5	369.2065	0.82	5	478.8016	10	5	569.3943	1.25	5	612.0268	2.43	5
0.25	120.3938	3.14	1	240.7877	4.02	5	289.6736	6.78	5	371.3453	0.82	5	481.5753	10	5	572.6928	1.25	5	615.5722	2.43	5
0.25	121.0913	3.14	1	242.1825	4.02	5	291.3516	6.78	5	373.4965	0.82	5	484.365	10	5	576.0104	1.25	5	619.1382	2.43	5
0.25	121.7927	3.14	1	243.5855	4.02	5	293.0394	6.78	5	375.6602	0.82	5	487.1709	10	5	579.3472	1.25	5	622.7248	2.43	5
0.25	122.4983	3.14	1	244.9965	4.02	5	294.737	6.78	5	377.8363	0.82	5	489.9931	10	5	582.7033	1.25	5	626.3322	2.43	5
0.25	123.2079	3.14	1	246.4158	4.02	5	296.4444	6.78	5	380.0251	0.82	5	492.8316	10	5	586.0788	1.25	5	629.9605	2.43	5
0.25	123.9216	3.14	1	247.8433	4.02	5	298.1617	6.78	5	382.2266	0.82	5	495.6865	10	5	589.474	1.25	5	633.6098	2.43	5
0.25	124.6395	3.14	1	249.279	4.02	5	299.8889	6.78	5	384.4408	0.82	5	498.558	10	5	592.8888	1.25	5	637.2803	2.43	5
0.25	125.3615	3.14	1	250.7231	4.02	5	301.6261	6.78	5	386.6678	0.82	5	501.4461	10	5	596.3233	1.25	5	640.972	2.43	5
0.25	126.0877	3.14	1	252.1755	4.02	5	303.3734	6.78	5	388.9078	0.82	5	504.351	10	5	599.7778	1.25	5	644.6852	2.43	5
0.25	126.8182	3.14	1	253.6363	4.02	5	305.1309	6.78	5	391.1607	0.82	5	507.2727	10	5	603.2523	1.25	5	648.4198	2.43	5
0.25	127.5528	3.14	1	255.1056	4.02	5	306.8985	6.78	5	393.4267	0.82	5	510.2113	10	5	606.7469	1.25	5	652.176	2.43	5
0.25	128.2917	3.14	1	256.5834	4.02	5	308.6763	6.78	5	395.7058	0.82	5	513.1669	10	5	610.2617	1.25	5	655.9541	2.43	5
0.25	129.0349	3.14	1	258.0698	4.02	5	310.4645	6.78	5	397.9981	0.82	5	516.1396	10	5	613.7969	1.25	5	659.754	2.43	5
0.25	129.7824	3.14	1	259.5648	4.02	5	312.263	6.78	5	400.3036	0.82	5	519.1296	10	5	617.3526	1.25	5	663.5759	2.43	5
0.25	130.5342	3.14	1	261.0684	4.02	5	314.0719	6.78	5	402.6226	0.82	5	522.1369	10	5	620.9289	1.25	5	667.4199	2.43	5

Reference file continued

5r

Mp3_freq	Mp3_int	Mp3_dec	SQ_freq	SQ_int	SQ_dec	ON_freq	ON_int	ON_dec	I-7_freq	I-7_int	I-7_dec	I-8_freq	I-8_int	I-8_dec	I-9_freq	I-9_int	I-9_dec	I-10_freq	I-10_int	I-10_dec	I-11_freq	I-11_int	I-11_dec
649.9197	1.83	5	706.8412	6.88	5	963.1506	5.15	5	1241.493	3.71	5	1536.1	2.39	5	1843.656	1.75	5	2159.712	1.13	5	2483.841	0.76	5
653.6846	1.83	5	710.9359	6.88	5	968.7301	5.15	5	1248.685	3.71	5	1544.999	2.39	5	1854.337	1.75	5	2172.223	1.13	5	2498.23	0.76	5
657.4714	1.83	5	715.0543	6.88	5	974.3419	5.15	5	1255.919	3.71	5	1553.949	2.39	5	1865.079	1.75	5	2184.806	1.13	5	2512.702	0.76	5
661.2801	1.83	5	719.1966	6.88	5	979.9862	5.15	5	1263.194	3.71	5	1562.951	2.39	5	1875.883	1.75	5	2197.463	1.13	5	2527.258	0.76	5
665.1108	1.83	5	723.3629	6.88	5	985.6632	5.15	5	1270.512	3.71	5	1572.005	2.39	5	1886.75	1.75	5	2210.193	1.13	5	2541.898	0.76	5
668.9638	1.83	5	727.5533	6.88	5	991.3731	5.15	5	1277.872	3.71	5	1581.111	2.39	5	1897.68	1.75	5	2222.996	1.13	5	2556.623	0.76	5
672.839	1.83	5	731.768	6.88	5	997.1161	5.15	5	1285.274	3.71	5	1590.27	2.39	5	1908.673	1.75	5	2235.874	1.13	5	2571.434	0.76	5
676.7368	1.83	5	736.007	6.88	5	1002.892	5.15	5	1292.72	3.71	5	1599.483	2.39	5	1919.73	1.75	5	2248.826	1.13	5	2586.33	0.76	5
680.6571	1.83	5	740.2707	6.88	5	1008.702	5.15	5	1300.209	3.71	5	1608.749	2.39	5	1930.85	1.75	5	2261.853	1.13	5	2601.312	0.76	5
684.6001	1.83	5	744.559	6.88	5	1014.545	5.15	5	1307.741	3.71	5	1618.068	2.39	5	1942.036	1.75	5	2274.956	1.13	5	2616.381	0.76	5
688.5659	1.83	5	748.8722	6.88	5	1020.423	5.15	5	1315.316	3.71	5	1627.441	2.39	5	1953.286	1.75	5	2288.135	1.13	5	2631.538	0.76	5
692.5547	1.83	5	753.2104	6.88	5	1026.334	5.15	5	1322.936	3.71	5	1636.869	2.39	5	1964.601	1.75	5	2301.39	1.13	5	2646.782	0.76	5
696.5667	1.83	5	757.5737	6.88	5	1032.279	5.15	5	1330.6	3.71	5	1646.351	2.39	5	1975.982	1.75	5	2314.722	1.13	5	2662.115	0.76	5
700.6018	1.83	5	761.9623	6.88	5	1038.259	5.15	5	1338.308	3.71	5	1655.888	2.39	5	1987.429	1.75	5	2328.131	1.13	5	2677.536	0.76	5
704.6604	1.83	5	766.3763	6.88	5	1044.274	5.15	5	1346.06	3.71	5	1665.481	2.39	5	1998.942	1.75	5	2341.617	1.13	5	2693.047	0.76	5
708.7424	1.83	5	770.8158	6.88	5	1050.323	5.15	5	1353.858	3.71	5	1675.129	2.39	5	2010.521	1.75	5	2355.182	1.13	5	2708.648	0.76	5

Test file
6t

Duration	Hum_freq	Hum_int	Hum_dec	Prm_freq	Prm_int	Prm_dec	Trc_freq	Trc_int	Trc_dec	Qnt_freq	Qnt_int	Qnt_dec	Nom_freq	Nom_int	Nom_dec	Mp1_freq	Mp1_int	Mp1_dec	Mp2_freq	Mp2_int	Mp2_dec
0.25	157.4901	3.14	1	280.6155	4.02	5	374.5768	6.78	5	445.4494	0.82	5	629.9605	10	5	749.1535	1.25	5	805.2452	2.43	5
0.25	157.4901	3.14	1	280.6155	4.02	5	374.5768	6.78	5	529.7315	0.82	5	629.9605	10	5	749.1535	1.25	5	805.2452	2.43	5
0.25	157.4901	3.14	1	314.9803	4.02	5	374.5768	6.78	5	445.4494	0.82	5	629.9605	10	5	749.1535	1.25	5	805.2452	2.43	5
0.25	157.4901	3.14	1	314.9803	4.02	5	374.5768	6.78	5	529.7315	0.82	5	629.9605	10	5	749.1535	1.25	5	805.2452	2.43	5
0.25	157.4901	3.14	1	280.6155	4.02	5	396.8503	6.78	5	445.4494	0.82	5	629.9605	10	5	749.1535	1.25	5	805.2452	2.43	5
0.25	157.4901	3.14	1	280.6155	4.02	5	396.8503	6.78	5	529.7315	0.82	5	629.9605	10	5	749.1535	1.25	5	805.2452	2.43	5
0.25	157.4901	3.14	1	314.9803	4.02	5	396.8503	6.78	5	445.4494	0.82	5	629.9605	10	5	749.1535	1.25	5	805.2452	2.43	5
0.25	157.4901	3.14	1	314.9803	4.02	5	396.8503	6.78	5	529.7315	0.82	5	629.9605	10	5	749.1535	1.25	5	805.2452	2.43	5
0.25	157.4901	3.14	1	280.6155	4.02	5	374.5768	6.78	5	445.4494	0.82	5	629.9605	10	5	749.1535	1.25	5	805.2452	2.43	5
0.25	157.4901	3.14	1	280.6155	4.02	5	374.5768	6.78	5	529.7315	0.82	5	629.9605	10	5	749.1535	1.25	5	805.2452	2.43	5
0.25	157.4901	3.14	1	314.9803	4.02	5	374.5768	6.78	5	445.4494	0.82	5	629.9605	10	5	749.1535	1.25	5	805.2452	2.43	5
0.25	157.4901	3.14	1	314.9803	4.02	5	374.5768	6.78	5	529.7315	0.82	5	629.9605	10	5	749.1535	1.25	5	805.2452	2.43	5
0.25	157.4901	3.14	1	280.6155	4.02	5	396.8503	6.78	5	445.4494	0.82	5	629.9605	10	5	749.1535	1.25	5	805.2452	2.43	5
0.25	157.4901	3.14	1	280.6155	4.02	5	396.8503	6.78	5	529.7315	0.82	5	629.9605	10	5	749.1535	1.25	5	805.2452	2.43	5
0.25	157.4901	3.14	1	314.9803	4.02	5	396.8503	6.78	5	445.4494	0.82	5	629.9605	10	5	749.1535	1.25	5	805.2452	2.43	5
0.25	157.4901	3.14	1	314.9803	4.02	5	396.8503	6.78	5	529.7315	0.82	5	629.9605	10	5	749.1535	1.25	5	805.2452	2.43	5

Test file continued
6t

Mp3_freq	Mp3_int	Mp3_dec	SQ_freq	SQ_int	SQ_dec	ON_freq	ON_int	ON_dec	I-7_freq	I-7_int	I-7_dec	I-8_freq	I-8_int	I-8_dec	I-9_freq	I-9_int	I-9_dec	I-10_freq	I-10_int	I-10_dec	I-11_freq	I-11_int	I-11_dec
855.101	1.83	5	909.2699	6.88	5	1217.004	5.15	5	1545.858	3.71	5	1889.277	2.39	5	2243.704	1.75	5	2604.21	1.13	5	2970.105	0.76	5
855.101	1.83	5	909.2699	6.88	5	1217.004	5.15	5	1545.858	3.71	5	1889.277	2.39	5	2243.704	1.75	5	2604.21	1.13	5	2970.105	0.76	5
855.101	1.83	5	909.2699	6.88	5	1217.004	5.15	5	1545.858	3.71	5	1889.277	2.39	5	2243.704	1.75	5	2604.21	1.13	5	2970.105	0.76	5
855.101	1.83	5	909.2699	6.88	5	1217.004	5.15	5	1545.858	3.71	5	1889.277	2.39	5	2243.704	1.75	5	2604.21	1.13	5	2970.105	0.76	5
855.101	1.83	5	909.2699	6.88	5	1217.004	5.15	5	1545.858	3.71	5	1889.277	2.39	5	2243.704	1.75	5	2604.21	1.13	5	2970.105	0.76	5
855.101	1.83	5	909.2699	6.88	5	1217.004	5.15	5	1545.858	3.71	5	1889.277	2.39	5	2243.704	1.75	5	2604.21	1.13	5	2970.105	0.76	5
855.101	1.83	5	909.2699	6.88	5	1217.004	5.15	5	1545.858	3.71	5	1889.277	2.39	5	2243.704	1.75	5	2604.21	1.13	5	2970.105	0.76	5
855.101	1.83	5	909.2699	6.88	5	1217.004	5.15	5	1545.858	3.71	5	1889.277	2.39	5	2243.704	1.75	5	2604.21	1.13	5	2970.105	0.76	5
855.101	1.83	5	951.188	6.88	5	1319.508	5.15	5	1725.975	3.71	5	2162.015	2.39	5	2622.465	1.75	5	3100.495	1.13	5	3595.764	0.76	5
855.101	1.83	5	951.188	6.88	5	1319.508	5.15	5	1725.975	3.71	5	2162.015	2.39	5	2622.465	1.75	5	3100.495	1.13	5	3595.764	0.76	5
855.101	1.83	5	951.188	6.88	5	1319.508	5.15	5	1725.975	3.71	5	2162.015	2.39	5	2622.465	1.75	5	3100.495	1.13	5	3595.764	0.76	5
855.101	1.83	5	951.188	6.88	5	1319.508	5.15	5	1725.975	3.71	5	2162.015	2.39	5	2622.465	1.75	5	3100.495	1.13	5	3595.764	0.76	5
855.101	1.83	5	951.188	6.88	5	1319.508	5.15	5	1725.975	3.71	5	2162.015	2.39	5	2622.465	1.75	5	3100.495	1.13	5	3595.764	0.76	5
855.101	1.83	5	951.188	6.88	5	1319.508	5.15	5	1725.975	3.71	5	2162.015	2.39	5	2622.465	1.75	5	3100.495	1.13	5	3595.764	0.76	5
855.101	1.83	5	951.188	6.88	5	1319.508	5.15	5	1725.975	3.71	5	2162.015	2.39	5	2622.465	1.75	5	3100.495	1.13	5	3595.764	0.76	5
855.101	1.83	5	951.188	6.88	5	1319.508	5.15	5	1725.975	3.71	5	2162.015	2.39	5	2622.465	1.75	5	3100.495	1.13	5	3595.764	0.76	5

Reference file 6r

Duration	Hum_freq	Hum_int	Hum_dec	Prm_freq	Prm_int	Prm_dec	Trc_freq	Trc_int	Trc_dec	Qnt_freq	Qnt_int	Qnt_dec	Nom_freq	Nom_int	Nom_dec	Mp1_freq	Mp1_int	Mp1_dec	Mp2_freq	Mp2_int	Mp2_dec
0.25	152.4553	3.14	1	304.9106	4.02	5	366.8151	6.78	5	470.2365	0.82	5	609.8213	10	5	725.2038	1.25	5	779.5022	2.43	5
0.25	153.1172	3.14	1	306.2344	4.02	5	368.4077	6.78	5	472.278	0.82	5	612.4688	10	5	728.3523	1.25	5	782.8865	2.43	5
0.25	153.782	3.14	1	307.564	4.02	5	370.0071	6.78	5	474.3285	0.82	5	615.1279	10	5	731.5145	1.25	5	786.2854	2.43	5
0.25	154.4496	3.14	1	308.8993	4.02	5	371.6135	6.78	5	476.3878	0.82	5	617.7985	10	5	734.6904	1.25	5	789.6991	2.43	5
0.25	155.1202	3.14	1	310.2404	4.02	5	373.2269	6.78	5	478.4561	0.82	5	620.4807	10	5	737.8801	1.25	5	793.1277	2.43	5
0.25	155.7936	3.14	1	311.5873	4.02	5	374.8473	6.78	5	480.5333	0.82	5	623.1746	10	5	741.0837	1.25	5	796.5711	2.43	5
0.25	156.47	3.14	1	312.9401	4.02	5	376.4747	6.78	5	482.6196	0.82	5	625.8802	10	5	744.3011	1.25	5	800.0294	2.43	5
0.25	157.1494	3.14	1	314.2987	4.02	5	378.1092	6.78	5	484.7149	0.82	5	628.5975	10	5	747.5326	1.25	5	803.5028	2.43	5
0.25	157.8316	3.14	1	315.6633	4.02	5	379.7508	6.78	5	486.8193	0.82	5	631.3266	10	5	750.778	1.25	5	806.9913	2.43	5
0.25	158.5169	3.14	1	317.0337	4.02	5	381.3995	6.78	5	488.9329	0.82	5	634.0675	10	5	754.0376	1.25	5	810.4949	2.43	5
0.25	159.2051	3.14	1	318.4102	4.02	5	383.0554	6.78	5	491.0556	0.82	5	636.8203	10	5	757.3113	1.25	5	814.0137	2.43	5
0.25	159.8963	3.14	1	319.7926	4.02	5	384.7185	6.78	5	493.1876	0.82	5	639.5851	10	5	760.5992	1.25	5	817.5478	2.43	5
0.25	160.5905	3.14	1	321.181	4.02	5	386.3888	6.78	5	495.3288	0.82	5	642.3619	10	5	763.9014	1.25	5	821.0972	2.43	5
0.25	161.2877	3.14	1	322.5754	4.02	5	388.0663	6.78	5	497.4793	0.82	5	645.1508	10	5	767.2179	1.25	5	824.6621	2.43	5
0.25	161.9879	3.14	1	323.9759	4.02	5	389.7511	6.78	5	499.6391	0.82	5	647.9518	10	5	770.5489	1.25	5	828.2424	2.43	5
0.25	162.6912	3.14	1	325.3824	4.02	5	391.4432	6.78	5	501.8083	0.82	5	650.7649	10	5	773.8942	1.25	5	831.8383	2.43	5

Reference file continued 6r

Mp3_freq	Mp3_int	Mp3_dec	SQ_freq	SQ_int	SQ_dec	ON_freq	ON_int	ON_dec	I-7_freq	I-7_int	I-7_dec	I-8_freq	I-8_int	I-8_dec	I-9_freq	I-9_int	I-9_dec	I-10_freq	I-10_int	I-10_dec	I-11_freq	I-11_int	I-11_dec
827.7641	1.83	5	900.2618	6.88	5	1226.708	5.15	5	1581.216	3.71	5	1956.44	2.39	5	2348.156	1.75	5	2750.697	1.13	5	3163.521	0.76	5
831.3579	1.83	5	904.1703	6.88	5	1232.034	5.15	5	1588.081	3.71	5	1964.934	2.39	5	2358.35	1.75	5	2762.639	1.13	5	3177.256	0.76	5
834.9673	1.83	5	908.0958	6.88	5	1237.383	5.15	5	1594.976	3.71	5	1973.464	2.39	5	2368.589	1.75	5	2774.633	1.13	5	3191.05	0.76	5
838.5924	1.83	5	912.0384	6.88	5	1242.755	5.15	5	1601.901	3.71	5	1982.032	2.39	5	2378.873	1.75	5	2786.68	1.13	5	3204.904	0.76	5
842.2332	1.83	5	915.9981	6.88	5	1248.15	5.15	5	1608.856	3.71	5	1990.637	2.39	5	2389.201	1.75	5	2798.778	1.13	5	3218.818	0.76	5
845.8898	1.83	5	919.9749	6.88	5	1253.569	5.15	5	1615.84	3.71	5	1999.28	2.39	5	2399.574	1.75	5	2810.929	1.13	5	3232.793	0.76	5
849.5623	1.83	5	923.9691	6.88	5	1259.012	5.15	5	1622.856	3.71	5	2007.96	2.39	5	2409.992	1.75	5	2823.133	1.13	5	3246.829	0.76	5
853.2507	1.83	5	927.9805	6.88	5	1264.478	5.15	5	1629.902	3.71	5	2016.678	2.39	5	2420.455	1.75	5	2835.39	1.13	5	3260.925	0.76	5
856.9552	1.83	5	932.0094	6.88	5	1269.968	5.15	5	1636.978	3.71	5	2025.433	2.39	5	2430.963	1.75	5	2847.7	1.13	5	3275.082	0.76	5
860.6757	1.83	5	936.0558	6.88	5	1275.481	5.15	5	1644.085	3.71	5	2034.227	2.39	5	2441.517	1.75	5	2860.063	1.13	5	3289.301	0.76	5
864.4124	1.83	5	940.1198	6.88	5	1281.019	5.15	5	1651.223	3.71	5	2043.058	2.39	5	2452.117	1.75	5	2872.48	1.13	5	3303.582	0.76	5
868.1653	1.83	5	944.2014	6.88	5	1286.58	5.15	5	1658.392	3.71	5	2051.929	2.39	5	2462.763	1.75	5	2884.952	1.13	5	3317.925	0.76	5
871.9345	1.83	5	948.3007	6.88	5	1292.166	5.15	5	1665.592	3.71	5	2060.837	2.39	5	2473.456	1.75	5	2897.477	1.13	5	3332.33	0.76	5
875.7201	1.83	5	952.4178	6.88	5	1297.776	5.15	5	1672.823	3.71	5	2069.784	2.39	5	2484.194	1.75	5	2910.056	1.13	5	3346.797	0.76	5
879.5221	1.83	5	956.5528	6.88	5	1303.411	5.15	5	1680.086	3.71	5	2078.771	2.39	5	2494.98	1.75	5	2922.691	1.13	5	3361.328	0.76	5
883.3406	1.83	5	960.7057	6.88	5	1309.069	5.15	5	1687.38	3.71	5	2087.796	2.39	5	2505.812	1.75	5	2935.38	1.13	5	3375.921	0.76	5

Test file
8t

Duration	Hum_freq	Hum_int	Hum_dec	Prm_freq	Prm_int	Prm_dec	Trc_freq	Trc_int	Trc_dec	Qnt_freq	Qnt_int	Qnt_dec	Nom_freq	Nom_int	Nom_dec	Mp1_freq	Mp1_int	Mp1_dec	Mp2_freq	Mp2_int	Mp2_dec
0.25	198.4251	3.14	1	353.5534	4.02	5	471.9372	6.78	5	612.0268	0.82	5	793.7005	10	5	890.8987	1.25	5	943.8743	2.43	5
0.25	198.4251	3.14	1	353.5534	4.02	5	471.9372	6.78	5	612.0268	0.82	5	793.7005	10	5	1000	1.25	5	1090.508	2.43	5
0.25	198.4251	3.14	1	396.8503	4.02	5	471.9372	6.78	5	612.0268	0.82	5	793.7005	10	5	890.8987	1.25	5	943.8743	2.43	5
0.25	198.4251	3.14	1	396.8503	4.02	5	471.9372	6.78	5	612.0268	0.82	5	793.7005	10	5	1000	1.25	5	1090.508	2.43	5
0.25	198.4251	3.14	1	353.5534	4.02	5	500	6.78	5	612.0268	0.82	5	793.7005	10	5	890.8987	1.25	5	943.8743	2.43	5
0.25	198.4251	3.14	1	353.5534	4.02	5	500	6.78	5	612.0268	0.82	5	793.7005	10	5	1000	1.25	5	1090.508	2.43	5
0.25	198.4251	3.14	1	396.8503	4.02	5	500	6.78	5	612.0268	0.82	5	793.7005	10	5	890.8987	1.25	5	943.8743	2.43	5
0.25	198.4251	3.14	1	396.8503	4.02	5	500	6.78	5	612.0268	0.82	5	793.7005	10	5	1000	1.25	5	1090.508	2.43	5
0.25	198.4251	3.14	1	353.5534	4.02	5	471.9372	6.78	5	612.0268	0.82	5	793.7005	10	5	890.8987	1.25	5	943.8743	2.43	5
0.25	198.4251	3.14	1	353.5534	4.02	5	471.9372	6.78	5	612.0268	0.82	5	793.7005	10	5	1000	1.25	5	1090.508	2.43	5
0.25	198.4251	3.14	1	396.8503	4.02	5	471.9372	6.78	5	612.0268	0.82	5	793.7005	10	5	890.8987	1.25	5	943.8743	2.43	5
0.25	198.4251	3.14	1	396.8503	4.02	5	471.9372	6.78	5	612.0268	0.82	5	793.7005	10	5	1000	1.25	5	1090.508	2.43	5
0.25	198.4251	3.14	1	353.5534	4.02	5	500	6.78	5	612.0268	0.82	5	793.7005	10	5	890.8987	1.25	5	943.8743	2.43	5
0.25	198.4251	3.14	1	353.5534	4.02	5	500	6.78	5	612.0268	0.82	5	793.7005	10	5	1000	1.25	5	1090.508	2.43	5
0.25	198.4251	3.14	1	396.8503	4.02	5	500	6.78	5	612.0268	0.82	5	793.7005	10	5	890.8987	1.25	5	943.8743	2.43	5
0.25	198.4251	3.14	1	396.8503	4.02	5	500	6.78	5	612.0268	0.82	5	793.7005	10	5	1000	1.25	5	1090.508	2.43	5

Test file continued
8t

Mp3_freq	Mp3_int	Mp3_dec	SQ_freq	SQ_int	SQ_dec	ON_freq	ON_int	ON_dec	I-7_freq	I-7_int	I-7_dec	I-8_freq	I-8_int	I-8_dec	I-9_freq	I-9_int	I-9_dec	I-10_freq	I-10_int	I-10_dec	I-11_freq	I-11_int	I-11_dec
1019.244	1.83	5	1145.608	6.88	5	1533.328	5.15	5	1947.659	3.71	5	2380.34	2.39	5	2826.89	1.75	5	3281.099	1.13	5	3742.098	0.76	5
1138.789	1.83	5	1145.608	6.88	5	1533.328	5.15	5	1947.659	3.71	5	2380.34	2.39	5	2826.89	1.75	5	3281.099	1.13	5	3742.098	0.76	5
1019.244	1.83	5	1145.608	6.88	5	1533.328	5.15	5	1947.659	3.71	5	2380.34	2.39	5	2826.89	1.75	5	3281.099	1.13	5	3742.098	0.76	5
1138.789	1.83	5	1145.608	6.88	5	1533.328	5.15	5	1947.659	3.71	5	2380.34	2.39	5	2826.89	1.75	5	3281.099	1.13	5	3742.098	0.76	5
1019.244	1.83	5	1145.608	6.88	5	1533.328	5.15	5	1947.659	3.71	5	2380.34	2.39	5	2826.89	1.75	5	3281.099	1.13	5	3742.098	0.76	5
1138.789	1.83	5	1145.608	6.88	5	1533.328	5.15	5	1947.659	3.71	5	2380.34	2.39	5	2826.89	1.75	5	3281.099	1.13	5	3742.098	0.76	5
1019.244	1.83	5	1145.608	6.88	5	1533.328	5.15	5	1947.659	3.71	5	2380.34	2.39	5	2826.89	1.75	5	3281.099	1.13	5	3742.098	0.76	5
1138.789	1.83	5	1145.608	6.88	5	1533.328	5.15	5	1947.659	3.71	5	2380.34	2.39	5	2826.89	1.75	5	3281.099	1.13	5	3742.098	0.76	5
1019.244	1.83	5	1198.422	6.88	5	1662.476	5.15	5	2174.592	3.71	5	2723.968	2.39	5	3304.099	1.75	5	3906.379	1.13	5	4530.379	0.76	5
1138.789	1.83	5	1198.422	6.88	5	1662.476	5.15	5	2174.592	3.71	5	2723.968	2.39	5	3304.099	1.75	5	3906.379	1.13	5	4530.379	0.76	5
1019.244	1.83	5	1198.422	6.88	5	1662.476	5.15	5	2174.592	3.71	5	2723.968	2.39	5	3304.099	1.75	5	3906.379	1.13	5	4530.379	0.76	5
1138.789	1.83	5	1198.422	6.88	5	1662.476	5.15	5	2174.592	3.71	5	2723.968	2.39	5	3304.099	1.75	5	3906.379	1.13	5	4530.379	0.76	5
1019.244	1.83	5	1198.422	6.88	5	1662.476	5.15	5	2174.592	3.71	5	2723.968	2.39	5	3304.099	1.75	5	3906.379	1.13	5	4530.379	0.76	5
1138.789	1.83	5	1198.422	6.88	5	1662.476	5.15	5	2174.592	3.71	5	2723.968	2.39	5	3304.099	1.75	5	3906.379	1.13	5	4530.379	0.76	5
1019.244	1.83	5	1198.422	6.88	5	1662.476	5.15	5	2174.592	3.71	5	2723.968	2.39	5	3304.099	1.75	5	3906.379	1.13	5	4530.379	0.76	5
1138.789	1.83	5	1198.422	6.88	5	1662.476	5.15	5	2174.592	3.71	5	2723.968	2.39	5	3304.099	1.75	5	3906.379	1.13	5	4530.379	0.76	5

Reference file
8r

Duration	Hum_freq	Hum_int	Hum_dec	Prm_freq	Prm_int	Prm_dec	Trc_freq	Trc_int	Trc_dec	Qnt_freq	Qnt_int	Qnt_dec	Nom_freq	Nom_int	Nom_dec	Mp1_freq	Mp1_int	Mp1_dec	Mp2_freq	Mp2_int	Mp2_dec
0.25	192.0817	3.14	1	384.1633	4.02	5	462.1581	6.78	5	592.4608	0.82	5	768.3266	10	5	913.6995	1.25	5	982.1112	2.43	5
0.25	192.9156	3.14	1	385.8312	4.02	5	464.1646	6.78	5	595.033	0.82	5	771.6624	10	5	917.6664	1.25	5	986.3751	2.43	5
0.25	193.7531	3.14	1	387.5063	4.02	5	466.1798	6.78	5	597.6164	0.82	5	775.0126	10	5	921.6505	1.25	5	990.6575	2.43	5
0.25	194.5943	3.14	1	389.1887	4.02	5	468.2037	6.78	5	600.211	0.82	5	778.3774	10	5	925.6519	1.25	5	994.9586	2.43	5
0.25	195.4392	3.14	1	390.8784	4.02	5	470.2365	6.78	5	602.8169	0.82	5	781.7567	10	5	929.6707	1.25	5	999.2782	2.43	5
0.25	196.2877	3.14	1	392.5754	4.02	5	472.278	6.78	5	605.434	0.82	5	785.1508	10	5	933.7069	1.25	5	1003.617	2.43	5
0.25	197.1399	3.14	1	394.2798	4.02	5	474.3285	6.78	5	608.0626	0.82	5	788.5596	10	5	937.7607	1.25	5	1007.974	2.43	5
0.25	197.9958	3.14	1	395.9916	4.02	5	476.3878	6.78	5	610.7025	0.82	5	791.9832	10	5	941.832	1.25	5	1012.35	2.43	5
0.25	198.8554	3.14	1	397.7108	4.02	5	478.4561	6.78	5	613.3539	0.82	5	795.4216	10	5	945.921	1.25	5	1016.745	2.43	5
0.25	199.7187	3.14	1	399.4375	4.02	5	480.5333	6.78	5	616.0168	0.82	5	798.875	10	5	950.0278	1.25	5	1021.16	2.43	5
0.25	200.5858	3.14	1	401.1717	4.02	5	482.6196	6.78	5	618.6913	0.82	5	802.3434	10	5	954.1524	1.25	5	1025.593	2.43	5
0.25	201.4567	3.14	1	402.9134	4.02	5	484.7149	6.78	5	621.3774	0.82	5	805.8268	10	5	958.2949	1.25	5	1030.046	2.43	5
0.25	202.3313	3.14	1	404.6627	4.02	5	486.8193	6.78	5	624.0751	0.82	5	809.3253	10	5	962.4554	1.25	5	1034.518	2.43	5
0.25	203.2098	3.14	1	406.4195	4.02	5	488.9329	6.78	5	626.7846	0.82	5	812.8391	10	5	966.634	1.25	5	1039.009	2.43	5
0.25	204.092	3.14	1	408.184	4.02	5	491.0556	6.78	5	629.5058	0.82	5	816.3681	10	5	970.8307	1.25	5	1043.52	2.43	5
0.25	204.9781	3.14	1	409.9562	4.02	5	493.1876	6.78	5	632.2389	0.82	5	819.9124	10	5	975.0456	1.25	5	1048.051	2.43	5

Reference file continued
8r

Mp3_freq	Mp3_int	Mp3_dec	SQ_freq	SQ_int	SQ_dec	ON_freq	ON_int	ON_dec	I-7_freq	I-7_int	I-7_dec	I-8_freq	I-8_int	I-8_dec	I-9_freq	I-9_int	I-9_dec	I-10_freq	I-10_int	I-10_dec	I-11_freq	I-11_int	I-11_dec
1042.917	1.83	5	1134.259	6.88	5	1545.555	5.15	5	1992.208	3.71	5	2464.959	2.39	5	2958.491	1.75	5	3465.661	1.13	5	3985.787	0.76	5
1047.445	1.83	5	1139.183	6.88	5	1552.265	5.15	5	2000.857	3.71	5	2475.661	2.39	5	2971.335	1.75	5	3480.707	1.13	5	4003.091	0.76	5
1051.993	1.83	5	1144.129	6.88	5	1559.004	5.15	5	2009.544	3.71	5	2486.409	2.39	5	2984.236	1.75	5	3495.819	1.13	5	4020.471	0.76	5
1056.56	1.83	5	1149.096	6.88	5	1565.773	5.15	5	2018.269	3.71	5	2497.204	2.39	5	2997.192	1.75	5	3510.996	1.13	5	4037.926	0.76	5
1061.147	1.83	5	1154.085	6.88	5	1572.571	5.15	5	2027.031	3.71	5	2508.046	2.39	5	3010.204	1.75	5	3526.239	1.13	5	4055.457	0.76	5
1065.754	1.83	5	1159.096	6.88	5	1579.398	5.15	5	2035.831	3.71	5	2518.935	2.39	5	3023.273	1.75	5	3541.549	1.13	5	4073.064	0.76	5
1070.381	1.83	5	1164.128	6.88	5	1586.255	5.15	5	2044.67	3.71	5	2529.871	2.39	5	3036.399	1.75	5	3556.925	1.13	5	4090.748	0.76	5
1075.029	1.83	5	1169.182	6.88	5	1593.142	5.15	5	2053.547	3.71	5	2540.855	2.39	5	3049.582	1.75	5	3572.367	1.13	5	4108.508	0.76	5
1079.696	1.83	5	1174.258	6.88	5	1600.059	5.15	5	2062.463	3.71	5	2551.886	2.39	5	3062.822	1.75	5	3587.877	1.13	5	4126.345	0.76	5
1084.383	1.83	5	1179.356	6.88	5	1607.006	5.15	5	2071.417	3.71	5	2562.965	2.39	5	3076.119	1.75	5	3603.454	1.13	5	4144.26	0.76	5
1089.091	1.83	5	1184.477	6.88	5	1613.983	5.15	5	2080.41	3.71	5	2574.092	2.39	5	3089.474	1.75	5	3619.099	1.13	5	4162.253	0.76	5
1093.82	1.83	5	1189.619	6.88	5	1620.99	5.15	5	2089.443	3.71	5	2585.268	2.39	5	3102.888	1.75	5	3634.811	1.13	5	4180.323	0.76	5
1098.569	1.83	5	1194.784	6.88	5	1628.027	5.15	5	2098.514	3.71	5	2596.492	2.39	5	3116.359	1.75	5	3650.592	1.13	5	4198.472	0.76	5
1103.338	1.83	5	1199.971	6.88	5	1635.096	5.15	5	2107.625	3.71	5	2607.765	2.39	5	3129.889	1.75	5	3666.441	1.13	5	4216.7	0.76	5
1108.128	1.83	5	1205.181	6.88	5	1642.194	5.15	5	2116.775	3.71	5	2619.087	2.39	5	3143.477	1.75	5	3682.359	1.13	5	4235.008	0.76	5
1112.939	1.83	5	1210.413	6.88	5	1649.324	5.15	5	2125.965	3.71	5	2630.458	2.39	5	3157.125	1.75	5	3698.347	1.13	5	4253.394	0.76	5

Test file

10t

Duration	Hum_freq	Hum_int	Hum_dec	Prm_freq	Prm_int	Prm_dec	Trc_freq	Trc_int	Trc_dec	Qnt_freq	Qnt_int	Qnt_dec	Nom_freq	Nom_int	Nom_dec	Mp1_freq	Mp1_int	Mp1_dec	Mp2_freq	Mp2_int	Mp2_dec
0.25	250	3.14	1	445.4494	4.02	5	594.6036	6.78	5	707.1068	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	445.4494	4.02	5	594.6036	6.78	5	840.8964	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	594.6036	6.78	5	707.1068	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	594.6036	6.78	5	840.8964	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	445.4494	4.02	5	629.9605	6.78	5	707.1068	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	445.4494	4.02	5	629.9605	6.78	5	840.8964	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	629.9605	6.78	5	707.1068	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	629.9605	6.78	5	840.8964	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	445.4494	4.02	5	594.6036	6.78	5	707.1068	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	445.4494	4.02	5	594.6036	6.78	5	840.8964	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	594.6036	6.78	5	707.1068	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	594.6036	6.78	5	840.8964	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	445.4494	4.02	5	629.9605	6.78	5	707.1068	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	445.4494	4.02	5	629.9605	6.78	5	840.8964	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	629.9605	6.78	5	707.1068	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	629.9605	6.78	5	840.8964	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5

Test file continued

10t

Mp3_freq	Mp3_int	Mp3_dec	SQ_freq	SQ_int	SQ_dec	ON_freq	ON_int	ON_dec	I-7_freq	I-7_int	I-7_dec	I-8_freq	I-8_int	I-8_dec	I-9_freq	I-9_int	I-9_dec	I-10_freq	I-10_int	I-10_dec	I-11_freq	I-11_int	I-11_dec
1357.388	1.83	5	1443.376	6.88	5	1931.873	5.15	5	2453.897	3.71	5	2999.041	2.39	5	3561.659	1.75	5	4133.925	1.13	5	4714.748	0.76	5
1357.388	1.83	5	1443.376	6.88	5	1931.873	5.15	5	2453.897	3.71	5	2999.041	2.39	5	3561.659	1.75	5	4133.925	1.13	5	4714.748	0.76	5
1357.388	1.83	5	1443.376	6.88	5	1931.873	5.15	5	2453.897	3.71	5	2999.041	2.39	5	3561.659	1.75	5	4133.925	1.13	5	4714.748	0.76	5
1357.388	1.83	5	1443.376	6.88	5	1931.873	5.15	5	2453.897	3.71	5	2999.041	2.39	5	3561.659	1.75	5	4133.925	1.13	5	4714.748	0.76	5
1357.388	1.83	5	1443.376	6.88	5	1931.873	5.15	5	2453.897	3.71	5	2999.041	2.39	5	3561.659	1.75	5	4133.925	1.13	5	4714.748	0.76	5
1357.388	1.83	5	1443.376	6.88	5	1931.873	5.15	5	2453.897	3.71	5	2999.041	2.39	5	3561.659	1.75	5	4133.925	1.13	5	4714.748	0.76	5
1357.388	1.83	5	1443.376	6.88	5	1931.873	5.15	5	2453.897	3.71	5	2999.041	2.39	5	3561.659	1.75	5	4133.925	1.13	5	4714.748	0.76	5
1357.388	1.83	5	1443.376	6.88	5	1931.873	5.15	5	2453.897	3.71	5	2999.041	2.39	5	3561.659	1.75	5	4133.925	1.13	5	4714.748	0.76	5
1357.388	1.83	5	1509.917	6.88	5	2094.588	5.15	5	2739.814	3.71	5	3431.985	2.39	5	4162.904	1.75	5	4921.729	1.13	5	5707.92	0.76	5
1357.388	1.83	5	1509.917	6.88	5	2094.588	5.15	5	2739.814	3.71	5	3431.985	2.39	5	4162.904	1.75	5	4921.729	1.13	5	5707.92	0.76	5
1357.388	1.83	5	1509.917	6.88	5	2094.588	5.15	5	2739.814	3.71	5	3431.985	2.39	5	4162.904	1.75	5	4921.729	1.13	5	5707.92	0.76	5
1357.388	1.83	5	1509.917	6.88	5	2094.588	5.15	5	2739.814	3.71	5	3431.985	2.39	5	4162.904	1.75	5	4921.729	1.13	5	5707.92	0.76	5
1357.388	1.83	5	1509.917	6.88	5	2094.588	5.15	5	2739.814	3.71	5	3431.985	2.39	5	4162.904	1.75	5	4921.729	1.13	5	5707.92	0.76	5
1357.388	1.83	5	1509.917	6.88	5	2094.588	5.15	5	2739.814	3.71	5	3431.985	2.39	5	4162.904	1.75	5	4921.729	1.13	5	5707.92	0.76	5
1357.388	1.83	5	1509.917	6.88	5	2094.588	5.15	5	2739.814	3.71	5	3431.985	2.39	5	4162.904	1.75	5	4921.729	1.13	5	5707.92	0.76	5
1357.388	1.83	5	1509.917	6.88	5	2094.588	5.15	5	2739.814	3.71	5	3431.985	2.39	5	4162.904	1.75	5	4921.729	1.13	5	5707.92	0.76	5

Reference file

10r

Duration	Hum_freq	Hum_int	Hum_dec	Prm_freq	Prm_int	Prm_dec	Trc_freq	Trc_int	Trc_dec	Qnt_freq	Qnt_int	Qnt_dec	Nom_freq	Nom_int	Nom_dec	Mp1_freq	Mp1_int	Mp1_dec	Mp2_freq	Mp2_int	Mp2_dec
0.25	242.0077	3.14	1	484.0154	4.02	5	582.2827	6.78	5	746.4539	0.82	5	968.0309	10	5	1151.189	1.25	5	1237.383	2.43	5
0.25	243.0584	3.14	1	486.1168	4.02	5	584.8107	6.78	5	749.6946	0.82	5	972.2337	10	5	1156.187	1.25	5	1242.755	2.43	5
0.25	244.1137	3.14	1	488.2273	4.02	5	587.3497	6.78	5	752.9495	0.82	5	976.4547	10	5	1161.207	1.25	5	1248.15	2.43	5
0.25	245.1735	3.14	1	490.347	4.02	5	589.8997	6.78	5	756.2185	0.82	5	980.694	10	5	1166.248	1.25	5	1253.569	2.43	5
0.25	246.2379	3.14	1	492.4759	4.02	5	592.4608	6.78	5	759.5016	0.82	5	984.9518	10	5	1171.312	1.25	5	1259.012	2.43	5
0.25	247.307	3.14	1	494.614	4.02	5	595.033	6.78	5	762.7991	0.82	5	989.228	10	5	1176.397	1.25	5	1264.478	2.43	5
0.25	248.3807	3.14	1	496.7614	4.02	5	597.6164	6.78	5	766.1108	0.82	5	993.5228	10	5	1181.504	1.25	5	1269.968	2.43	5
0.25	249.4591	3.14	1	498.9181	4.02	5	600.211	6.78	5	769.4369	0.82	5	997.8363	10	5	1186.634	1.25	5	1275.481	2.43	5
0.25	250.5421	3.14	1	501.0842	4.02	5	602.8169	6.78	5	772.7775	0.82	5	1002.168	10	5	1191.786	1.25	5	1281.019	2.43	5
0.25	251.6299	3.14	1	503.2597	4.02	5	605.434	6.78	5	776.1326	0.82	5	1006.519	10	5	1196.96	1.25	5	1286.58	2.43	5
0.25	252.7223	3.14	1	505.4446	4.02	5	608.0626	6.78	5	779.5022	0.82	5	1010.889	10	5	1202.157	1.25	5	1292.166	2.43	5
0.25	253.8195	3.14	1	507.6391	4.02	5	610.7025	6.78	5	782.8865	0.82	5	1015.278	10	5	1207.376	1.25	5	1297.776	2.43	5
0.25	254.9215	3.14	1	509.843	4.02	5	613.3539	6.78	5	786.2854	0.82	5	1019.686	10	5	1212.618	1.25	5	1303.411	2.43	5
0.25	256.0283	3.14	1	512.0565	4.02	5	616.0168	6.78	5	789.6991	0.82	5	1024.113	10	5	1217.883	1.25	5	1309.069	2.43	5
0.25	257.1398	3.14	1	514.2797	4.02	5	618.6913	6.78	5	793.1277	0.82	5	1028.559	10	5	1223.17	1.25	5	1314.753	2.43	5
0.25	258.2562	3.14	1	516.5124	4.02	5	621.3774	6.78	5	796.5711	0.82	5	1033.025	10	5	1228.481	1.25	5	1320.461	2.43	5

Reference file continued

10r

Mp3_freq	Mp3_int	Mp3_dec	SQ_freq	SQ_int	SQ_dec	ON_freq	ON_int	ON_dec	I-7_freq	I-7_int	I-7_dec	I-8_freq	I-8_int	I-8_dec	I-9_freq	I-9_int	I-9_dec	I-10_freq	I-10_int	I-10_dec	I-11_freq	I-11_int	I-11_dec
1313.994	1.83	5	1429.076	6.88	5	1947.277	5.15	5	2510.025	3.71	5	3105.654	2.39	5	3727.465	1.75	5	4366.459	1.13	5	5021.777	0.76	5
1319.698	1.83	5	1435.281	6.88	5	1955.732	5.15	5	2520.922	3.71	5	3119.138	2.39	5	3743.648	1.75	5	4385.416	1.13	5	5043.579	0.76	5
1325.428	1.83	5	1441.512	6.88	5	1964.222	5.15	5	2531.867	3.71	5	3132.679	2.39	5	3759.901	1.75	5	4404.456	1.13	5	5065.476	0.76	5
1331.182	1.83	5	1447.771	6.88	5	1972.75	5.15	5	2542.859	3.71	5	3146.28	2.39	5	3776.225	1.75	5	4423.578	1.13	5	5087.468	0.76	5
1336.962	1.83	5	1454.056	6.88	5	1981.315	5.15	5	2553.899	3.71	5	3159.94	2.39	5	3792.62	1.75	5	4442.783	1.13	5	5109.556	0.76	5
1342.766	1.83	5	1460.369	6.88	5	1989.917	5.15	5	2564.987	3.71	5	3173.659	2.39	5	3809.086	1.75	5	4462.072	1.13	5	5131.739	0.76	5
1348.596	1.83	5	1466.709	6.88	5	1998.556	5.15	5	2576.123	3.71	5	3187.438	2.39	5	3825.623	1.75	5	4481.444	1.13	5	5154.019	0.76	5
1354.451	1.83	5	1473.077	6.88	5	2007.233	5.15	5	2587.307	3.71	5	3201.276	2.39	5	3842.232	1.75	5	4500.901	1.13	5	5176.396	0.76	5
1360.332	1.83	5	1479.473	6.88	5	2015.948	5.15	5	2598.54	3.71	5	3215.175	2.39	5	3858.914	1.75	5	4520.442	1.13	5	5198.869	0.76	5
1366.238	1.83	5	1485.896	6.88	5	2024.7	5.15	5	2609.822	3.71	5	3229.134	2.39	5	3875.667	1.75	5	4540.068	1.13	5	5221.44	0.76	5
1372.169	1.83	5	1492.347	6.88	5	2033.491	5.15	5	2621.153	3.71	5	3243.153	2.39	5	3892.494	1.75	5	4559.779	1.13	5	5244.11	0.76	5
1378.127	1.83	5	1498.826	6.88	5	2042.319	5.15	5	2632.533	3.71	5	3257.234	2.39	5	3909.393	1.75	5	4579.575	1.13	5	5266.877	0.76	5
1384.11	1.83	5	1505.333	6.88	5	2051.186	5.15	5	2643.962	3.71	5	3271.375	2.39	5	3926.366	1.75	5	4599.458	1.13	5	5289.744	0.76	5
1390.119	1.83	5	1511.869	6.88	5	2060.091	5.15	5	2655.441	3.71	5	3285.578	2.39	5	3943.413	1.75	5	4619.427	1.13	5	5312.71	0.76	5
1396.154	1.83	5	1518.433	6.88	5	2069.035	5.15	5	2666.97	3.71	5	3299.843	2.39	5	3960.533	1.75	5	4639.482	1.13	5	5335.775	0.76	5
1402.216	1.83	5	1525.025	6.88	5	2078.018	5.15	5	2678.549	3.71	5	3314.169	2.39	5	3977.728	1.75	5	4659.625	1.13	5	5358.941	0.76	5

Test file 13t

Duration	Hum_freq	Hum_int	Hum_dec	Prm_freq	Prm_int	Prm_dec	Trc_freq	Trc_int	Trc_dec	Qnt_freq	Qnt_int	Qnt_dec	Nom_freq	Nom_int	Nom_dec	Mp1_freq	Mp1_int	Mp1_dec	Mp2_freq	Mp2_int	Mp2_dec
0.25	314.9803	3.14	1	561.231	4.02	5	749.1535	6.78	5	971.5319	0.82	5	1259.921	10	5	1414.214	1.25	5	1498.307	2.43	5
0.25	314.9803	3.14	1	561.231	4.02	5	749.1535	6.78	5	971.5319	0.82	5	1259.921	10	5	1587.401	1.25	5	1731.073	2.43	5
0.25	314.9803	3.14	1	629.9605	4.02	5	749.1535	6.78	5	971.5319	0.82	5	1259.921	10	5	1414.214	1.25	5	1498.307	2.43	5
0.25	314.9803	3.14	1	629.9605	4.02	5	749.1535	6.78	5	971.5319	0.82	5	1259.921	10	5	1587.401	1.25	5	1731.073	2.43	5
0.25	314.9803	3.14	1	561.231	4.02	5	793.7005	6.78	5	971.5319	0.82	5	1259.921	10	5	1414.214	1.25	5	1498.307	2.43	5
0.25	314.9803	3.14	1	561.231	4.02	5	793.7005	6.78	5	971.5319	0.82	5	1259.921	10	5	1587.401	1.25	5	1731.073	2.43	5
0.25	314.9803	3.14	1	629.9605	4.02	5	793.7005	6.78	5	971.5319	0.82	5	1259.921	10	5	1414.214	1.25	5	1498.307	2.43	5
0.25	314.9803	3.14	1	629.9605	4.02	5	793.7005	6.78	5	971.5319	0.82	5	1259.921	10	5	1587.401	1.25	5	1731.073	2.43	5
0.25	314.9803	3.14	1	561.231	4.02	5	749.1535	6.78	5	971.5319	0.82	5	1259.921	10	5	1414.214	1.25	5	1498.307	2.43	5
0.25	314.9803	3.14	1	561.231	4.02	5	749.1535	6.78	5	971.5319	0.82	5	1259.921	10	5	1587.401	1.25	5	1731.073	2.43	5
0.25	314.9803	3.14	1	629.9605	4.02	5	749.1535	6.78	5	971.5319	0.82	5	1259.921	10	5	1414.214	1.25	5	1498.307	2.43	5
0.25	314.9803	3.14	1	629.9605	4.02	5	749.1535	6.78	5	971.5319	0.82	5	1259.921	10	5	1587.401	1.25	5	1731.073	2.43	5
0.25	314.9803	3.14	1	561.231	4.02	5	793.7005	6.78	5	971.5319	0.82	5	1259.921	10	5	1414.214	1.25	5	1498.307	2.43	5
0.25	314.9803	3.14	1	561.231	4.02	5	793.7005	6.78	5	971.5319	0.82	5	1259.921	10	5	1587.401	1.25	5	1731.073	2.43	5
0.25	314.9803	3.14	1	629.9605	4.02	5	793.7005	6.78	5	971.5319	0.82	5	1259.921	10	5	1414.214	1.25	5	1498.307	2.43	5
0.25	314.9803	3.14	1	629.9605	4.02	5	793.7005	6.78	5	971.5319	0.82	5	1259.921	10	5	1587.401	1.25	5	1731.073	2.43	5

Test file continued 13t

Mp3_freq	Mp3_int	Mp3_dec	SQ_freq	SQ_int	SQ_dec	ON_freq	ON_int	ON_dec	I-7_freq	I-7_int	I-7_dec	I-8_freq	I-8_int	I-8_dec	I-9_freq	I-9_int	I-9_dec	I-10_freq	I-10_int	I-10_dec	I-11_freq	I-11_int	I-11_dec
1617.95	1.83	5	1818.54	6.88	5	2434.007	5.15	5	3091.717	3.71	5	3778.555	2.39	5	4487.409	1.75	5	5208.419	1.13	5	5940.21	0.76	5
1807.714	1.83	5	1818.54	6.88	5	2434.007	5.15	5	3091.717	3.71	5	3778.555	2.39	5	4487.409	1.75	5	5208.419	1.13	5	5940.21	0.76	5
1617.95	1.83	5	1818.54	6.88	5	2434.007	5.15	5	3091.717	3.71	5	3778.555	2.39	5	4487.409	1.75	5	5208.419	1.13	5	5940.21	0.76	5
1807.714	1.83	5	1818.54	6.88	5	2434.007	5.15	5	3091.717	3.71	5	3778.555	2.39	5	4487.409	1.75	5	5208.419	1.13	5	5940.21	0.76	5
1617.95	1.83	5	1818.54	6.88	5	2434.007	5.15	5	3091.717	3.71	5	3778.555	2.39	5	4487.409	1.75	5	5208.419	1.13	5	5940.21	0.76	5
1807.714	1.83	5	1818.54	6.88	5	2434.007	5.15	5	3091.717	3.71	5	3778.555	2.39	5	4487.409	1.75	5	5208.419	1.13	5	5940.21	0.76	5
1617.95	1.83	5	1818.54	6.88	5	2434.007	5.15	5	3091.717	3.71	5	3778.555	2.39	5	4487.409	1.75	5	5208.419	1.13	5	5940.21	0.76	5
1807.714	1.83	5	1818.54	6.88	5	2434.007	5.15	5	3091.717	3.71	5	3778.555	2.39	5	4487.409	1.75	5	5208.419	1.13	5	5940.21	0.76	5
1617.95	1.83	5	1902.376	6.88	5	2639.016	5.15	5	3451.95	3.71	5	4324.03	2.39	5	5244.93	1.75	5	6200.99	1.13	5	7191.529	0.76	5
1807.714	1.83	5	1902.376	6.88	5	2639.016	5.15	5	3451.95	3.71	5	4324.03	2.39	5	5244.93	1.75	5	6200.99	1.13	5	7191.529	0.76	5
1617.95	1.83	5	1902.376	6.88	5	2639.016	5.15	5	3451.95	3.71	5	4324.03	2.39	5	5244.93	1.75	5	6200.99	1.13	5	7191.529	0.76	5
1807.714	1.83	5	1902.376	6.88	5	2639.016	5.15	5	3451.95	3.71	5	4324.03	2.39	5	5244.93	1.75	5	6200.99	1.13	5	7191.529	0.76	5
1617.95	1.83	5	1902.376	6.88	5	2639.016	5.15	5	3451.95	3.71	5	4324.03	2.39	5	5244.93	1.75	5	6200.99	1.13	5	7191.529	0.76	5
1807.714	1.83	5	1902.376	6.88	5	2639.016	5.15	5	3451.95	3.71	5	4324.03	2.39	5	5244.93	1.75	5	6200.99	1.13	5	7191.529	0.76	5
1617.95	1.83	5	1902.376	6.88	5	2639.016	5.15	5	3451.95	3.71	5	4324.03	2.39	5	5244.93	1.75	5	6200.99	1.13	5	7191.529	0.76	5
1807.714	1.83	5	1902.376	6.88	5	2639.016	5.15	5	3451.95	3.71	5	4324.03	2.39	5	5244.93	1.75	5	6200.99	1.13	5	7191.529	0.76	5

Reference file
13r

Duration	Hum_freq	Hum_int	Hum_dec	Prm_freq	Prm_int	Prm_dec	Trc_freq	Trc_int	Trc_dec	Qnt_freq	Qnt_int	Qnt_dec	Nom_freq	Nom_int	Nom_dec	Mp1_freq	Mp1_int	Mp1_dec	Mp2_freq	Mp2_int	Mp2_dec
0.25	304.9106	3.14	1	609.8213	4.02	5	733.6302	6.78	5	940.4729	0.82	5	1219.643	10	5	1450.408	1.25	5	1559.004	2.43	5
0.25	306.2344	3.14	1	612.4688	4.02	5	736.8153	6.78	5	944.5561	0.82	5	1224.938	10	5	1456.705	1.25	5	1565.773	2.43	5
0.25	307.564	3.14	1	615.1279	4.02	5	740.0143	6.78	5	948.6569	0.82	5	1230.256	10	5	1463.029	1.25	5	1572.571	2.43	5
0.25	308.8993	3.14	1	617.7985	4.02	5	743.2271	6.78	5	952.7756	0.82	5	1235.597	10	5	1469.381	1.25	5	1579.398	2.43	5
0.25	310.2404	3.14	1	620.4807	4.02	5	746.4539	6.78	5	956.9121	0.82	5	1240.961	10	5	1475.76	1.25	5	1586.255	2.43	5
0.25	311.5873	3.14	1	623.1746	4.02	5	749.6946	6.78	5	961.0666	0.82	5	1246.349	10	5	1482.167	1.25	5	1593.142	2.43	5
0.25	312.9401	3.14	1	625.8802	4.02	5	752.9495	6.78	5	965.2391	0.82	5	1251.76	10	5	1488.602	1.25	5	1600.059	2.43	5
0.25	314.2987	3.14	1	628.5975	4.02	5	756.2185	6.78	5	969.4298	0.82	5	1257.195	10	5	1495.065	1.25	5	1607.006	2.43	5
0.25	315.6633	3.14	1	631.3266	4.02	5	759.5016	6.78	5	973.6386	0.82	5	1262.653	10	5	1501.556	1.25	5	1613.983	2.43	5
0.25	317.0337	3.14	1	634.0675	4.02	5	762.7991	6.78	5	977.8658	0.82	5	1268.135	10	5	1508.075	1.25	5	1620.99	2.43	5
0.25	318.4102	3.14	1	636.8203	4.02	5	766.1108	6.78	5	982.1112	0.82	5	1273.641	10	5	1514.623	1.25	5	1628.027	2.43	5
0.25	319.7926	3.14	1	639.5851	4.02	5	769.4369	6.78	5	986.3751	0.82	5	1279.17	10	5	1521.198	1.25	5	1635.096	2.43	5
0.25	321.181	3.14	1	642.3619	4.02	5	772.7775	6.78	5	990.6575	0.82	5	1284.724	10	5	1527.803	1.25	5	1642.194	2.43	5
0.25	322.5754	3.14	1	645.1508	4.02	5	776.1326	6.78	5	994.9586	0.82	5	1290.302	10	5	1534.436	1.25	5	1649.324	2.43	5
0.25	323.9759	3.14	1	647.9518	4.02	5	779.5022	6.78	5	999.2782	0.82	5	1295.904	10	5	1541.098	1.25	5	1656.485	2.43	5
0.25	325.3824	3.14	1	650.7649	4.02	5	782.8865	6.78	5	1003.617	0.82	5	1301.53	10	5	1547.788	1.25	5	1663.677	2.43	5

Reference file continued
13r

Mp3_freq	Mp3_int	Mp3_dec	SQ_freq	SQ_int	SQ_dec	ON_freq	ON_int	ON_dec	I-7_freq	I-7_int	I-7_dec	I-8_freq	I-8_int	I-8_dec	I-9_freq	I-9_int	I-9_dec	I-10_freq	I-10_int	I-10_dec	I-11_freq	I-11_int	I-11_dec
1655.528	1.83	5	1800.524	6.88	5	2453.416	5.15	5	3162.433	3.71	5	3912.879	2.39	5	4696.311	1.75	5	5501.393	1.13	5	6327.042	0.76	5
1662.716	1.83	5	1808.341	6.88	5	2464.067	5.15	5	3176.163	3.71	5	3929.867	2.39	5	4716.701	1.75	5	5525.278	1.13	5	6354.511	0.76	5
1669.935	1.83	5	1816.192	6.88	5	2474.765	5.15	5	3189.952	3.71	5	3946.929	2.39	5	4737.179	1.75	5	5549.267	1.13	5	6382.1	0.76	5
1677.185	1.83	5	1824.077	6.88	5	2485.51	5.15	5	3203.802	3.71	5	3964.065	2.39	5	4757.745	1.75	5	5573.359	1.13	5	6409.808	0.76	5
1684.466	1.83	5	1831.996	6.88	5	2496.301	5.15	5	3217.711	3.71	5	3981.275	2.39	5	4778.401	1.75	5	5597.556	1.13	5	6437.637	0.76	5
1691.78	1.83	5	1839.95	6.88	5	2507.138	5.15	5	3231.681	3.71	5	3998.56	2.39	5	4799.147	1.75	5	5621.858	1.13	5	6465.586	0.76	5
1699.125	1.83	5	1847.938	6.88	5	2518.023	5.15	5	3245.712	3.71	5	4015.92	2.39	5	4819.983	1.75	5	5646.266	1.13	5	6493.657	0.76	5
1706.501	1.83	5	1855.961	6.88	5	2528.956	5.15	5	3259.803	3.71	5	4033.355	2.39	5	4840.909	1.75	5	5670.78	1.13	5	6521.85	0.76	5
1713.91	1.83	5	1864.019	6.88	5	2539.935	5.15	5	3273.956	3.71	5	4050.866	2.39	5	4861.926	1.75	5	5695.4	1.13	5	6550.165	0.76	5
1721.351	1.83	5	1872.112	6.88	5	2550.962	5.15	5	3288.17	3.71	5	4068.454	2.39	5	4883.035	1.75	5	5720.127	1.13	5	6578.603	0.76	5
1728.825	1.83	5	1880.24	6.88	5	2562.038	5.15	5	3302.446	3.71	5	4086.117	2.39	5	4904.235	1.75	5	5744.961	1.13	5	6607.164	0.76	5
1736.331	1.83	5	1888.403	6.88	5	2573.161	5.15	5	3316.783	3.71	5	4103.857	2.39	5	4925.527	1.75	5	5769.903	1.13	5	6635.85	0.76	5
1743.869	1.83	5	1896.601	6.88	5	2584.332	5.15	5	3331.183	3.71	5	4121.674	2.39	5	4946.911	1.75	5	5794.954	1.13	5	6664.66	0.76	5
1751.44	1.83	5	1904.836	6.88	5	2595.552	5.15	5	3345.646	3.71	5	4139.569	2.39	5	4968.389	1.75	5	5820.113	1.13	5	6693.595	0.76	5
1759.044	1.83	5	1913.106	6.88	5	2606.821	5.15	5	3360.171	3.71	5	4157.541	2.39	5	4989.959	1.75	5	5845.381	1.13	5	6722.655	0.76	5
1766.681	1.83	5	1921.411	6.88	5	2618.139	5.15	5	3374.76	3.71	5	4175.591	2.39	5	5011.624	1.75	5	5870.759	1.13	5	6751.842	0.76	5

Test file
15t

Duration	Hum_freq	Hum_int	Hum_dec	Prm_freq	Prm_int	Prm_dec	Trc_freq	Trc_int	Trc_dec	Qnt_freq	Qnt_int	Qnt_dec	Nom_freq	Nom_int	Nom_dec	Mp1_freq	Mp1_int	Mp1_dec	Mp2_freq	Mp2_int	Mp2_dec
0.25	396.8503	3.14	1	707.1068	4.02	5	943.8743	6.78	5	1122.462	0.82	5	1587.401	10	5	1887.749	1.25	5	2029.091	2.43	5
0.25	396.8503	3.14	1	707.1068	4.02	5	943.8743	6.78	5	1334.84	0.82	5	1587.401	10	5	1887.749	1.25	5	2029.091	2.43	5
0.25	396.8503	3.14	1	793.7005	4.02	5	943.8743	6.78	5	1122.462	0.82	5	1587.401	10	5	1887.749	1.25	5	2029.091	2.43	5
0.25	396.8503	3.14	1	793.7005	4.02	5	943.8743	6.78	5	1334.84	0.82	5	1587.401	10	5	1887.749	1.25	5	2029.091	2.43	5
0.25	396.8503	3.14	1	707.1068	4.02	5	1000	6.78	5	1122.462	0.82	5	1587.401	10	5	1887.749	1.25	5	2029.091	2.43	5
0.25	396.8503	3.14	1	707.1068	4.02	5	1000	6.78	5	1334.84	0.82	5	1587.401	10	5	1887.749	1.25	5	2029.091	2.43	5
0.25	396.8503	3.14	1	793.7005	4.02	5	1000	6.78	5	1122.462	0.82	5	1587.401	10	5	1887.749	1.25	5	2029.091	2.43	5
0.25	396.8503	3.14	1	793.7005	4.02	5	1000	6.78	5	1334.84	0.82	5	1587.401	10	5	1887.749	1.25	5	2029.091	2.43	5
0.25	396.8503	3.14	1	707.1068	4.02	5	943.8743	6.78	5	1122.462	0.82	5	1587.401	10	5	1887.749	1.25	5	2029.091	2.43	5
0.25	396.8503	3.14	1	707.1068	4.02	5	943.8743	6.78	5	1334.84	0.82	5	1587.401	10	5	1887.749	1.25	5	2029.091	2.43	5
0.25	396.8503	3.14	1	793.7005	4.02	5	943.8743	6.78	5	1122.462	0.82	5	1587.401	10	5	1887.749	1.25	5	2029.091	2.43	5
0.25	396.8503	3.14	1	793.7005	4.02	5	943.8743	6.78	5	1334.84	0.82	5	1587.401	10	5	1887.749	1.25	5	2029.091	2.43	5
0.25	396.8503	3.14	1	707.1068	4.02	5	1000	6.78	5	1122.462	0.82	5	1587.401	10	5	1887.749	1.25	5	2029.091	2.43	5
0.25	396.8503	3.14	1	707.1068	4.02	5	1000	6.78	5	1334.84	0.82	5	1587.401	10	5	1887.749	1.25	5	2029.091	2.43	5
0.25	396.8503	3.14	1	793.7005	4.02	5	1000	6.78	5	1122.462	0.82	5	1587.401	10	5	1887.749	1.25	5	2029.091	2.43	5
0.25	396.8503	3.14	1	793.7005	4.02	5	1000	6.78	5	1334.84	0.82	5	1587.401	10	5	1887.749	1.25	5	2029.091	2.43	5

Test file continued
15t

Mp3_freq	Mp3_int	Mp3_dec	SQ_freq	SQ_int	SQ_dec	ON_freq	ON_int	ON_dec	I-7_freq	I-7_int	I-7_dec	I-8_freq	I-8_int	I-8_dec	I-9_freq	I-9_int	I-9_dec	I-10_freq	I-10_int	I-10_dec	I-11_freq	I-11_int	I-11_dec
2154.719	1.83	5	2291.217	6.88	5	3066.657	5.15	5	3895.319	3.71	5	4760.681	2.39	5	5653.781	1.75	5	6562.197	1.13	5	7484.196	0.76	5
2154.719	1.83	5	2291.217	6.88	5	3066.657	5.15	5	3895.319	3.71	5	4760.681	2.39	5	5653.781	1.75	5	6562.197	1.13	5	7484.196	0.76	5
2154.719	1.83	5	2291.217	6.88	5	3066.657	5.15	5	3895.319	3.71	5	4760.681	2.39	5	5653.781	1.75	5	6562.197	1.13	5	7484.196	0.76	5
2154.719	1.83	5	2291.217	6.88	5	3066.657	5.15	5	3895.319	3.71	5	4760.681	2.39	5	5653.781	1.75	5	6562.197	1.13	5	7484.196	0.76	5
2154.719	1.83	5	2291.217	6.88	5	3066.657	5.15	5	3895.319	3.71	5	4760.681	2.39	5	5653.781	1.75	5	6562.197	1.13	5	7484.196	0.76	5
2154.719	1.83	5	2291.217	6.88	5	3066.657	5.15	5	3895.319	3.71	5	4760.681	2.39	5	5653.781	1.75	5	6562.197	1.13	5	7484.196	0.76	5
2154.719	1.83	5	2291.217	6.88	5	3066.657	5.15	5	3895.319	3.71	5	4760.681	2.39	5	5653.781	1.75	5	6562.197	1.13	5	7484.196	0.76	5
2154.719	1.83	5	2291.217	6.88	5	3066.657	5.15	5	3895.319	3.71	5	4760.681	2.39	5	5653.781	1.75	5	6562.197	1.13	5	7484.196	0.76	5
2154.719	1.83	5	2396.844	6.88	5	3324.952	5.15	5	4349.184	3.71	5	5447.936	2.39	5	6608.198	1.75	5	7812.758	1.13	5	9060.758	0.76	5
2154.719	1.83	5	2396.844	6.88	5	3324.952	5.15	5	4349.184	3.71	5	5447.936	2.39	5	6608.198	1.75	5	7812.758	1.13	5	9060.758	0.76	5
2154.719	1.83	5	2396.844	6.88	5	3324.952	5.15	5	4349.184	3.71	5	5447.936	2.39	5	6608.198	1.75	5	7812.758	1.13	5	9060.758	0.76	5
2154.719	1.83	5	2396.844	6.88	5	3324.952	5.15	5	4349.184	3.71	5	5447.936	2.39	5	6608.198	1.75	5	7812.758	1.13	5	9060.758	0.76	5
2154.719	1.83	5	2396.844	6.88	5	3324.952	5.15	5	4349.184	3.71	5	5447.936	2.39	5	6608.198	1.75	5	7812.758	1.13	5	9060.758	0.76	5
2154.719	1.83	5	2396.844	6.88	5	3324.952	5.15	5	4349.184	3.71	5	5447.936	2.39	5	6608.198	1.75	5	7812.758	1.13	5	9060.758	0.76	5
2154.719	1.83	5	2396.844	6.88	5	3324.952	5.15	5	4349.184	3.71	5	5447.936	2.39	5	6608.198	1.75	5	7812.758	1.13	5	9060.758	0.76	5
2154.719	1.83	5	2396.844	6.88	5	3324.952	5.15	5	4349.184	3.71	5	5447.936	2.39	5	6608.198	1.75	5	7812.758	1.13	5	9060.758	0.76	5

Reference file
15r

Duration	Hum_freq	Hum_int	Hum_dec	Prm_freq	Prm_int	Prm_dec	Trc_freq	Trc_int	Trc_dec	Qnt_freq	Qnt_int	Qnt_dec	Nom_freq	Nom_int	Nom_dec	Mp1_freq	Mp1_int	Mp1_dec	Mp2_freq	Mp2_int	Mp2_dec
0.25	388.3466	3.14	1	776.6932	4.02	5	934.3813	6.78	5	1197.825	0.82	5	1553.386	10	5	1847.298	1.25	5	1985.611	2.43	5
0.25	389.4698	3.14	1	778.9396	4.02	5	937.0838	6.78	5	1201.289	0.82	5	1557.879	10	5	1852.641	1.25	5	1991.354	2.43	5
0.25	390.5962	3.14	1	781.1925	4.02	5	939.7941	6.78	5	1204.764	0.82	5	1562.385	10	5	1857.999	1.25	5	1997.114	2.43	5
0.25	391.726	3.14	1	783.4519	4.02	5	942.5123	6.78	5	1208.248	0.82	5	1566.904	10	5	1863.373	1.25	5	2002.89	2.43	5
0.25	392.8589	3.14	1	785.7179	4.02	5	945.2383	6.78	5	1211.743	0.82	5	1571.436	10	5	1868.763	1.25	5	2008.683	2.43	5
0.25	393.9952	3.14	1	787.9904	4.02	5	947.9722	6.78	5	1215.247	0.82	5	1575.981	10	5	1874.168	1.25	5	2014.493	2.43	5
0.25	395.1348	3.14	1	790.2695	4.02	5	950.714	6.78	5	1218.762	0.82	5	1580.539	10	5	1879.588	1.25	5	2020.319	2.43	5
0.25	396.2776	3.14	1	792.5552	4.02	5	953.4638	6.78	5	1222.287	0.82	5	1585.11	10	5	1885.025	1.25	5	2026.163	2.43	5
0.25	397.4238	3.14	1	794.8475	4.02	5	956.2214	6.78	5	1225.822	0.82	5	1589.695	10	5	1890.477	1.25	5	2032.023	2.43	5
0.25	398.5732	3.14	1	797.1464	4.02	5	958.9871	6.78	5	1229.368	0.82	5	1594.293	10	5	1895.944	1.25	5	2037.9	2.43	5
0.25	399.726	3.14	1	799.452	4.02	5	961.7608	6.78	5	1232.924	0.82	5	1598.904	10	5	1901.428	1.25	5	2043.794	2.43	5
0.25	400.8821	3.14	1	801.7643	4.02	5	964.5425	6.78	5	1236.49	0.82	5	1603.529	10	5	1906.928	1.25	5	2049.706	2.43	5
0.25	402.0416	3.14	1	804.0832	4.02	5	967.3322	6.78	5	1240.066	0.82	5	1608.166	10	5	1912.443	1.25	5	2055.634	2.43	5
0.25	403.2044	3.14	1	806.4088	4.02	5	970.13	6.78	5	1243.652	0.82	5	1612.818	10	5	1917.974	1.25	5	2061.579	2.43	5
0.25	404.3706	3.14	1	808.7412	4.02	5	972.9359	6.78	5	1247.249	0.82	5	1617.482	10	5	1923.522	1.25	5	2067.542	2.43	5
0.25	405.5402	3.14	1	811.0803	4.02	5	975.7499	6.78	5	1250.857	0.82	5	1622.161	10	5	1929.085	1.25	5	2073.522	2.43	5

Reference file
15r

Mp3_freq	Mp3_int	Mp3_dec	SQ_freq	SQ_int	SQ_dec	ON_freq	ON_int	ON_dec	I-7_freq	I-7_int	I-7_dec	I-8_freq	I-8_int	I-8_dec	I-9_freq	I-9_int	I-9_dec	I-10_freq	I-10_int	I-10_dec	I-11_freq	I-11_int	I-11_dec
2108.548	1.83	5	2293.22	6.88	5	3124.77	5.15	5	4027.803	3.71	5	4983.602	2.39	5	5981.413	1.75	5	7006.799	1.13	5	8058.378	0.76	5
2114.647	1.83	5	2299.853	6.88	5	3133.808	5.15	5	4039.453	3.71	5	4998.016	2.39	5	5998.713	1.75	5	7027.064	1.13	5	8081.685	0.76	5
2120.763	1.83	5	2306.505	6.88	5	3142.872	5.15	5	4051.136	3.71	5	5012.472	2.39	5	6016.063	1.75	5	7047.389	1.13	5	8105.06	0.76	5
2126.897	1.83	5	2313.176	6.88	5	3151.962	5.15	5	4062.853	3.71	5	5026.969	2.39	5	6033.463	1.75	5	7067.772	1.13	5	8128.502	0.76	5
2133.048	1.83	5	2319.866	6.88	5	3161.078	5.15	5	4074.604	3.71	5	5041.509	2.39	5	6050.914	1.75	5	7088.214	1.13	5	8152.012	0.76	5
2139.218	1.83	5	2326.576	6.88	5	3170.221	5.15	5	4086.389	3.71	5	5056.09	2.39	5	6068.415	1.75	5	7108.715	1.13	5	8175.59	0.76	5
2145.405	1.83	5	2333.305	6.88	5	3179.39	5.15	5	4098.208	3.71	5	5070.714	2.39	5	6085.967	1.75	5	7129.275	1.13	5	8199.236	0.76	5
2151.61	1.83	5	2340.053	6.88	5	3188.586	5.15	5	4110.061	3.71	5	5085.38	2.39	5	6103.569	1.75	5	7149.895	1.13	5	8222.951	0.76	5
2157.833	1.83	5	2346.822	6.88	5	3197.808	5.15	5	4121.948	3.71	5	5100.088	2.39	5	6121.222	1.75	5	7170.575	1.13	5	8246.734	0.76	5
2164.074	1.83	5	2353.609	6.88	5	3207.057	5.15	5	4133.87	3.71	5	5114.839	2.39	5	6138.927	1.75	5	7191.314	1.13	5	8270.586	0.76	5
2170.333	1.83	5	2360.417	6.88	5	3216.333	5.15	5	4145.827	3.71	5	5129.633	2.39	5	6156.682	1.75	5	7212.113	1.13	5	8294.507	0.76	5
2176.611	1.83	5	2367.244	6.88	5	3225.635	5.15	5	4157.818	3.71	5	5144.469	2.39	5	6174.489	1.75	5	7232.973	1.13	5	8318.497	0.76	5
2182.906	1.83	5	2374.09	6.88	5	3234.965	5.15	5	4169.843	3.71	5	5159.348	2.39	5	6192.347	1.75	5	7253.893	1.13	5	8342.556	0.76	5
2189.22	1.83	5	2380.957	6.88	5	3244.321	5.15	5	4181.904	3.71	5	5174.271	2.39	5	6210.257	1.75	5	7274.873	1.13	5	8366.685	0.76	5
2195.551	1.83	5	2387.843	6.88	5	3253.705	5.15	5	4193.999	3.71	5	5189.236	2.39	5	6228.219	1.75	5	7295.914	1.13	5	8390.884	0.76	5
2201.902	1.83	5	2394.75	6.88	5	3263.115	5.15	5	4206.129	3.71	5	5204.245	2.39	5	6246.233	1.75	5	7317.016	1.13	5	8415.153	0.76	5

Test file
20t

Duration	Hum_freq	Hum_int	Hum_dec	Prm_freq	Prm_int	Prm_dec	Trc_freq	Trc_int	Trc_dec	Qnt_freq	Qnt_int	Qnt_dec	Nom_freq	Nom_int	Nom_dec	Mp1_freq	Mp1_int	Mp1_dec	Mp2_freq	Mp2_int	Mp2_dec
0.25	500	3.14	1	890.8987	4.02	5	1189.207	6.78	5	1542.211	0.82	5	2000	10	5	2244.924	1.25	5	2378.414	2.43	5
0.25	500	3.14	1	890.8987	4.02	5	1189.207	6.78	5	1542.211	0.82	5	2000	10	5	2519.842	1.25	5	2747.907	2.43	5
0.25	500	3.14	1	1000	4.02	5	1189.207	6.78	5	1542.211	0.82	5	2000	10	5	2244.924	1.25	5	2378.414	2.43	5
0.25	500	3.14	1	1000	4.02	5	1189.207	6.78	5	1542.211	0.82	5	2000	10	5	2519.842	1.25	5	2747.907	2.43	5
0.25	500	3.14	1	890.8987	4.02	5	1259.921	6.78	5	1542.211	0.82	5	2000	10	5	2244.924	1.25	5	2378.414	2.43	5
0.25	500	3.14	1	890.8987	4.02	5	1259.921	6.78	5	1542.211	0.82	5	2000	10	5	2519.842	1.25	5	2747.907	2.43	5
0.25	500	3.14	1	1000	4.02	5	1259.921	6.78	5	1542.211	0.82	5	2000	10	5	2244.924	1.25	5	2378.414	2.43	5
0.25	500	3.14	1	1000	4.02	5	1259.921	6.78	5	1542.211	0.82	5	2000	10	5	2519.842	1.25	5	2747.907	2.43	5
0.25	500	3.14	1	890.8987	4.02	5	1189.207	6.78	5	1542.211	0.82	5	2000	10	5	2244.924	1.25	5	2378.414	2.43	5
0.25	500	3.14	1	890.8987	4.02	5	1189.207	6.78	5	1542.211	0.82	5	2000	10	5	2519.842	1.25	5	2747.907	2.43	5
0.25	500	3.14	1	1000	4.02	5	1189.207	6.78	5	1542.211	0.82	5	2000	10	5	2244.924	1.25	5	2378.414	2.43	5
0.25	500	3.14	1	1000	4.02	5	1189.207	6.78	5	1542.211	0.82	5	2000	10	5	2519.842	1.25	5	2747.907	2.43	5
0.25	500	3.14	1	890.8987	4.02	5	1259.921	6.78	5	1542.211	0.82	5	2000	10	5	2244.924	1.25	5	2378.414	2.43	5
0.25	500	3.14	1	890.8987	4.02	5	1259.921	6.78	5	1542.211	0.82	5	2000	10	5	2519.842	1.25	5	2747.907	2.43	5
0.25	500	3.14	1	1000	4.02	5	1259.921	6.78	5	1542.211	0.82	5	2000	10	5	2244.924	1.25	5	2378.414	2.43	5
0.25	500	3.14	1	1000	4.02	5	1259.921	6.78	5	1542.211	0.82	5	2000	10	5	2519.842	1.25	5	2747.907	2.43	5

Test file continued
20t

Mp3_freq	Mp3_int	Mp3_dec	SQ_freq	SQ_int	SQ_dec	ON_freq	ON_int	ON_dec	I-7_freq	I-7_int	I-7_dec	I-8_freq	I-8_int	I-8_dec	I-9_freq	I-9_int	I-9_dec	I-10_freq	I-10_int	I-10_dec	I-11_freq	I-11_int	I-11_dec
2568.335	1.83	5	2886.752	6.88	5	3863.745	5.15	5	4907.794	3.71	5	5998.082	2.39	5	7123.317	1.75	5	8267.85	1.13	5	9429.496	0	5
2869.568	1.83	5	2886.752	6.88	5	3863.745	5.15	5	4907.794	3.71	5	5998.082	2.39	5	7123.317	1.75	5	8267.85	1.13	5	9429.496	0	5
2568.335	1.83	5	2886.752	6.88	5	3863.745	5.15	5	4907.794	3.71	5	5998.082	2.39	5	7123.317	1.75	5	8267.85	1.13	5	9429.496	0	5
2869.568	1.83	5	2886.752	6.88	5	3863.745	5.15	5	4907.794	3.71	5	5998.082	2.39	5	7123.317	1.75	5	8267.85	1.13	5	9429.496	0	5
2568.335	1.83	5	2886.752	6.88	5	3863.745	5.15	5	4907.794	3.71	5	5998.082	2.39	5	7123.317	1.75	5	8267.85	1.13	5	9429.496	0	5
2869.568	1.83	5	2886.752	6.88	5	3863.745	5.15	5	4907.794	3.71	5	5998.082	2.39	5	7123.317	1.75	5	8267.85	1.13	5	9429.496	0	5
2568.335	1.83	5	2886.752	6.88	5	3863.745	5.15	5	4907.794	3.71	5	5998.082	2.39	5	7123.317	1.75	5	8267.85	1.13	5	9429.496	0	5
2869.568	1.83	5	2886.752	6.88	5	3863.745	5.15	5	4907.794	3.71	5	5998.082	2.39	5	7123.317	1.75	5	8267.85	1.13	5	9429.496	0	5
2568.335	1.83	5	3019.834	6.88	5	4189.176	5.15	5	5479.629	3.71	5	6863.97	2.39	5	8325.808	1.75	5	9843.458	1.13	5	11415.84	0	5
2869.568	1.83	5	3019.834	6.88	5	4189.176	5.15	5	5479.629	3.71	5	6863.97	2.39	5	8325.808	1.75	5	9843.458	1.13	5	11415.84	0	5
2568.335	1.83	5	3019.834	6.88	5	4189.176	5.15	5	5479.629	3.71	5	6863.97	2.39	5	8325.808	1.75	5	9843.458	1.13	5	11415.84	0	5
2869.568	1.83	5	3019.834	6.88	5	4189.176	5.15	5	5479.629	3.71	5	6863.97	2.39	5	8325.808	1.75	5	9843.458	1.13	5	11415.84	0	5
2568.335	1.83	5	3019.834	6.88	5	4189.176	5.15	5	5479.629	3.71	5	6863.97	2.39	5	8325.808	1.75	5	9843.458	1.13	5	11415.84	0	5
2869.568	1.83	5	3019.834	6.88	5	4189.176	5.15	5	5479.629	3.71	5	6863.97	2.39	5	8325.808	1.75	5	9843.458	1.13	5	11415.84	0	5
2568.335	1.83	5	3019.834	6.88	5	4189.176	5.15	5	5479.629	3.71	5	6863.97	2.39	5	8325.808	1.75	5	9843.458	1.13	5	11415.84	0	5
2869.568	1.83	5	3019.834	6.88	5	4189.176	5.15	5	5479.629	3.71	5	6863.97	2.39	5	8325.808	1.75	5	9843.458	1.13	5	11415.84	0	5

Reference file
20r

Duration	Hum_freq	Hum_int	Hum_dec	Prm_freq	Prm_int	Prm_dec	Trc_freq	Trc_int	Trc_dec	Qnt_freq	Qnt_int	Qnt_dec	Nom_freq	Nom_int	Nom_dec	Mp1_freq	Mp1_int	Mp1_dec	Mp2_freq	Mp2_int	Mp2_dec
0.25	489.286	3.14	1	978.5721	4.02	5	1177.247	6.78	5	1509.164	0.82	5	1957.144	10	5	2327.45	1.25	5	2501.714	2.43	5
0.25	490.7012	3.14	1	981.4024	4.02	5	1180.652	6.78	5	1513.529	0.82	5	1962.805	10	5	2334.181	1.25	5	2508.949	2.43	5
0.25	492.1204	3.14	1	984.2409	4.02	5	1184.066	6.78	5	1517.907	0.82	5	1968.482	10	5	2340.932	1.25	5	2516.206	2.43	5
0.25	493.5438	3.14	1	987.0876	4.02	5	1187.491	6.78	5	1522.297	0.82	5	1974.175	10	5	2347.703	1.25	5	2523.484	2.43	5
0.25	494.9713	3.14	1	989.9425	4.02	5	1190.926	6.78	5	1526.7	0.82	5	1979.885	10	5	2354.493	1.25	5	2530.782	2.43	5
0.25	496.4029	3.14	1	992.8057	4.02	5	1194.37	6.78	5	1531.116	0.82	5	1985.611	10	5	2361.303	1.25	5	2538.102	2.43	5
0.25	497.8386	3.14	1	995.6772	4.02	5	1197.825	6.78	5	1535.544	0.82	5	1991.354	10	5	2368.133	1.25	5	2545.443	2.43	5
0.25	499.2785	3.14	1	998.557	4.02	5	1201.289	6.78	5	1539.985	0.82	5	1997.114	10	5	2374.982	1.25	5	2552.805	2.43	5
0.25	500.7225	3.14	1	1001.445	4.02	5	1204.764	6.78	5	1544.439	0.82	5	2002.89	10	5	2381.851	1.25	5	2560.188	2.43	5
0.25	502.1708	3.14	1	1004.342	4.02	5	1208.248	6.78	5	1548.906	0.82	5	2008.683	10	5	2388.74	1.25	5	2567.593	2.43	5
0.25	503.6232	3.14	1	1007.246	4.02	5	1211.743	6.78	5	1553.386	0.82	5	2014.493	10	5	2395.649	1.25	5	2575.019	2.43	5
0.25	505.0798	3.14	1	1010.16	4.02	5	1215.247	6.78	5	1557.879	0.82	5	2020.319	10	5	2402.578	1.25	5	2582.467	2.43	5
0.25	506.5407	3.14	1	1013.081	4.02	5	1218.762	6.78	5	1562.385	0.82	5	2026.163	10	5	2409.527	1.25	5	2589.936	2.43	5
0.25	508.0057	3.14	1	1016.011	4.02	5	1222.287	6.78	5	1566.904	0.82	5	2032.023	10	5	2416.496	1.25	5	2597.427	2.43	5
0.25	509.475	3.14	1	1018.95	4.02	5	1225.822	6.78	5	1571.436	0.82	5	2037.9	10	5	2423.485	1.25	5	2604.94	2.43	5
0.25	510.9486	3.14	1	1021.897	4.02	5	1229.368	6.78	5	1575.981	0.82	5	2043.794	10	5	2430.495	1.25	5	2612.474	2.43	5

Reference file continued
20r

Mp3_freq	Mp3_int	Mp3_dec	SQ_freq	SQ_int	SQ_dec	ON_freq	ON_int	ON_dec	I-7_freq	I-7_int	I-7_dec	I-8_freq	I-8_int	I-8_dec	I-9_freq	I-9_int	I-9_dec	I-10_freq	I-10_int	I-10_dec	I-11_freq	I-11_int	I-11_dec
2656.604	1.83	5	2889.276	6.88	5	3936.963	5.15	5	5074.714	3.71	5	6278.945	2.39	5	7536.109	1.75	5	8828.013	1.13	5	10152.92	0	5
2664.288	1.83	5	2897.633	6.88	5	3948.35	5.15	5	5089.391	3.71	5	6297.105	2.39	5	7557.905	1.75	5	8853.546	1.13	5	10182.29	0	5
2671.994	1.83	5	2906.014	6.88	5	3959.77	5.15	5	5104.111	3.71	5	6315.319	2.39	5	7579.765	1.75	5	8879.153	1.13	5	10211.74	0	5
2679.722	1.83	5	2914.419	6.88	5	3971.223	5.15	5	5118.874	3.71	5	6333.584	2.39	5	7601.688	1.75	5	8904.834	1.13	5	10241.27	0	5
2687.473	1.83	5	2922.848	6.88	5	3982.709	5.15	5	5133.679	3.71	5	6351.903	2.39	5	7623.674	1.75	5	8930.59	1.13	5	10270.89	0	5
2695.245	1.83	5	2931.302	6.88	5	3994.228	5.15	5	5148.527	3.71	5	6370.274	2.39	5	7645.724	1.75	5	8956.419	1.13	5	10300.6	0	5
2703.041	1.83	5	2939.78	6.88	5	4005.78	5.15	5	5163.418	3.71	5	6388.699	2.39	5	7667.837	1.75	5	8982.324	1.13	5	10330.39	0	5
2710.859	1.83	5	2948.283	6.88	5	4017.366	5.15	5	5178.352	3.71	5	6407.177	2.39	5	7690.015	1.75	5	9008.303	1.13	5	10360.27	0	5
2718.699	1.83	5	2956.81	6.88	5	4028.986	5.15	5	5193.33	3.71	5	6425.708	2.39	5	7712.257	1.75	5	9034.358	1.13	5	10390.23	0	5
2726.563	1.83	5	2965.362	6.88	5	4040.639	5.15	5	5208.35	3.71	5	6444.293	2.39	5	7734.563	1.75	5	9060.488	1.13	5	10420.29	0	5
2734.449	1.83	5	2973.938	6.88	5	4052.325	5.15	5	5223.414	3.71	5	6462.932	2.39	5	7756.933	1.75	5	9086.694	1.13	5	10450.42	0	5
2742.358	1.83	5	2982.54	6.88	5	4064.046	5.15	5	5238.522	3.71	5	6481.625	2.39	5	7779.369	1.75	5	9112.975	1.13	5	10480.65	0	5
2750.289	1.83	5	2991.166	6.88	5	4075.8	5.15	5	5253.673	3.71	5	6500.371	2.39	5	7801.869	1.75	5	9139.332	1.13	5	10510.96	0	5
2758.244	1.83	5	2999.818	6.88	5	4087.589	5.15	5	5268.868	3.71	5	6519.172	2.39	5	7824.434	1.75	5	9165.766	1.13	5	10541.36	0	5
2766.221	1.83	5	3008.494	6.88	5	4099.411	5.15	5	5284.107	3.71	5	6538.028	2.39	5	7847.065	1.75	5	9192.276	1.13	5	10571.85	0	5
2774.222	1.83	5	3017.195	6.88	5	4111.268	5.15	5	5299.391	3.71	5	6556.938	2.39	5	7869.761	1.75	5	9218.863	1.13	5	10602.43	0	5

Parameter files for regression tests (section 8.2)

Test file

Regression test

Duration	Hum_freq	Hum_int	Hum_dec	Prm_freq	Prm_int	Prm_dec	Trc_freq	Trc_int	Trc_dec	Qnt_freq	Qnt_int	Qnt_dec	Nom_freq	Nom_int	Nom_dec	Mp1_freq	Mp1_int	Mp1_dec	Mp2_freq	Mp2_int	Mp2_dec
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5

Test file continued

Regression test

Mp3_freq	Mp3_int	Mp3_dec	SQ_freq	SQ_int	SQ_dec	ON_freq	ON_int	ON_dec	I-7_freq	I-7_int	I-7_dec	I-8_freq	I-8_int	I-8_dec	I-9_freq	I-9_int	I-9_dec	I-10_freq	I-10_int	I-10_dec	I-11_freq	I-11_int	I-11_dec
1357.388	1.83	5	1443.061	6.88	5	1931.873	5.15	5	2454.08	3.71	5	3039.794	2.39	5	3561.839	1.75	5	4133.854	1.13	5	4715.16	0.76	5
1357.388	1.83	5	1447.401	6.88	5	1942.316	5.15	5	2472.18	3.71	5	3067.583	2.39	5	3599.074	1.75	5	4182.206	1.13	5	4775.638	0.76	5
1357.388	1.83	5	1451.754	6.88	5	1952.815	5.15	5	2490.412	3.71	5	3095.626	2.39	5	3636.698	1.75	5	4231.124	1.13	5	4836.891	0.76	5
1357.388	1.83	5	1456.12	6.88	5	1963.372	5.15	5	2508.78	3.71	5	3123.925	2.39	5	3674.715	1.75	5	4280.613	1.13	5	4898.93	0.76	5
1357.388	1.83	5	1460.499	6.88	5	1973.985	5.15	5	2527.283	3.71	5	3152.484	2.39	5	3713.13	1.75	5	4330.682	1.13	5	4961.765	0.76	5
1357.388	1.83	5	1464.891	6.88	5	1984.656	5.15	5	2545.922	3.71	5	3181.303	2.39	5	3751.946	1.75	5	4381.336	1.13	5	5025.405	0.76	5
1357.388	1.83	5	1469.297	6.88	5	1995.384	5.15	5	2564.698	3.71	5	3210.386	2.39	5	3791.168	1.75	5	4432.583	1.13	5	5089.862	0.76	5
1357.388	1.83	5	1473.716	6.88	5	2006.171	5.15	5	2583.614	3.71	5	3239.734	2.39	5	3830.8	1.75	5	4484.429	1.13	5	5155.146	0.76	5
1357.388	1.83	5	1478.148	6.88	5	2017.016	5.15	5	2602.668	3.71	5	3269.351	2.39	5	3870.847	1.75	5	4536.881	1.13	5	5221.267	0.76	5
1357.388	1.83	5	1482.593	6.88	5	2027.919	5.15	5	2621.863	3.71	5	3299.239	2.39	5	3911.312	1.75	5	4589.947	1.13	5	5288.236	0.76	5
1357.388	1.83	5	1487.052	6.88	5	2038.881	5.15	5	2641.2	3.71	5	3329.4	2.39	5	3952.2	1.75	5	4643.634	1.13	5	5356.064	0.76	5
1357.388	1.83	5	1491.524	6.88	5	2049.903	5.15	5	2660.679	3.71	5	3359.837	2.39	5	3993.515	1.75	5	4697.948	1.13	5	5424.763	0.76	5
1357.388	1.83	5	1496.009	6.88	5	2060.984	5.15	5	2680.303	3.71	5	3390.552	2.39	5	4035.262	1.75	5	4752.898	1.13	5	5494.342	0.76	5
1357.388	1.83	5	1500.508	6.88	5	2072.125	5.15	5	2700.07	3.71	5	3421.547	2.39	5	4077.446	1.75	5	4808.491	1.13	5	5564.813	0.76	5
1357.388	1.83	5	1505.021	6.88	5	2083.326	5.15	5	2719.984	3.71	5	3452.826	2.39	5	4120.071	1.75	5	4864.734	1.13	5	5636.189	0.76	5
1357.388	1.83	5	1509.547	6.88	5	2094.588	5.15	5	2740.044	3.71	5	3484.391	2.39	5	4163.141	1.75	5	4921.634	1.13	5	5708.48	0.76	5

Reference file
Regression test

Duration	Hum_freq	Hum_int	Hum_dec	Prm_freq	Prm_int	Prm_dec	Trc_freq	Trc_int	Trc_dec	Qnt_freq	Qnt_int	Qnt_dec	Nom_freq	Nom_int	Nom_dec	Mp1_freq	Mp1_int	Mp1_dec	Mp2_freq	Mp2_int	Mp2_dec
0.25	246.7719	3.14	1	493.5438	4.02	5	593.7455	6.78	5	761.1486	0.82	5	987.0876	10	5	1173.852	1.25	5	1261.742	2.43	5
0.25	247.1999	3.14	1	494.3998	4.02	5	594.7753	6.78	5	762.4687	0.82	5	988.7996	10	5	1175.887	1.25	5	1263.93	2.43	5
0.25	247.6286	3.14	1	495.2573	4.02	5	595.8069	6.78	5	763.7911	0.82	5	990.5145	10	5	1177.927	1.25	5	1266.122	2.43	5
0.25	248.0581	3.14	1	496.1162	4.02	5	596.8402	6.78	5	765.1158	0.82	5	992.2324	10	5	1179.97	1.25	5	1268.318	2.43	5
0.25	248.4883	3.14	1	496.9767	4.02	5	597.8754	6.78	5	766.4428	0.82	5	993.9533	10	5	1182.016	1.25	5	1270.518	2.43	5
0.25	248.9193	3.14	1	497.8386	4.02	5	598.9123	6.78	5	767.7721	0.82	5	995.6772	10	5	1184.066	1.25	5	1272.721	2.43	5
0.25	249.351	3.14	1	498.702	4.02	5	599.951	6.78	5	769.1037	0.82	5	997.4041	10	5	1186.12	1.25	5	1274.929	2.43	5
0.25	249.7835	3.14	1	499.567	4.02	5	600.9916	6.78	5	770.4376	0.82	5	999.1339	10	5	1188.177	1.25	5	1277.14	2.43	5
0.25	250.2167	3.14	1	500.4334	4.02	5	602.0339	6.78	5	771.7738	0.82	5	1000.867	10	5	1190.238	1.25	5	1279.355	2.43	5
0.25	250.6507	3.14	1	501.3013	4.02	5	603.0781	6.78	5	773.1124	0.82	5	1002.603	10	5	1192.302	1.25	5	1281.574	2.43	5
0.25	251.0854	3.14	1	502.1708	4.02	5	604.124	6.78	5	774.4532	0.82	5	1004.342	10	5	1194.37	1.25	5	1283.797	2.43	5
0.25	251.5209	3.14	1	503.0417	4.02	5	605.1718	6.78	5	775.7964	0.82	5	1006.083	10	5	1196.442	1.25	5	1286.023	2.43	5
0.25	251.9571	3.14	1	503.9142	4.02	5	606.2214	6.78	5	777.1419	0.82	5	1007.828	10	5	1198.517	1.25	5	1288.254	2.43	5
0.25	252.3941	3.14	1	504.7882	4.02	5	607.2728	6.78	5	778.4898	0.82	5	1009.576	10	5	1200.595	1.25	5	1290.488	2.43	5
0.25	252.8318	3.14	1	505.6637	4.02	5	608.326	6.78	5	779.84	0.82	5	1011.327	10	5	1202.678	1.25	5	1292.726	2.43	5
0.25	253.2703	3.14	1	506.5407	4.02	5	609.3811	6.78	5	781.1925	0.82	5	1013.081	10	5	1204.764	1.25	5	1294.968	2.43	5

Reference file continued
Regression test

Mp3_freq	Mp3_int	Mp3_dec	SQ_freq	SQ_int	SQ_dec	ON_freq	ON_int	ON_dec	I-7_freq	I-7_int	I-7_dec	I-8_freq	I-8_int	I-8_dec	I-9_freq	I-9_int	I-9_dec	I-10_freq	I-10_int	I-10_dec	I-11_freq	I-11_int	I-11_dec
1339.861	1.83	5	1456.872	6.88	5	1985.611	5.15	5	2559.64	3.71	5	3212.486	2.39	5	3801.049	1.75	5	4452.336	1.13	5	5121.11	0.76	5
1342.185	1.83	5	1459.399	6.88	5	1989.055	5.15	5	2564.079	3.71	5	3218.057	2.39	5	3807.641	1.75	5	4460.058	1.13	5	5129.992	0.76	5
1344.513	1.83	5	1461.93	6.88	5	1992.505	5.15	5	2568.526	3.71	5	3223.639	2.39	5	3814.245	1.75	5	4467.793	1.13	5	5138.89	0.76	5
1346.845	1.83	5	1464.465	6.88	5	1995.961	5.15	5	2572.981	3.71	5	3229.23	2.39	5	3820.86	1.75	5	4475.542	1.13	5	5147.802	0.76	5
1349.18	1.83	5	1467.005	6.88	5	1999.422	5.15	5	2577.444	3.71	5	3234.83	2.39	5	3827.487	1.75	5	4483.305	1.13	5	5156.731	0.76	5
1351.52	1.83	5	1469.55	6.88	5	2002.89	5.15	5	2581.914	3.71	5	3240.441	2.39	5	3834.125	1.75	5	4491.08	1.13	5	5165.674	0.76	5
1353.864	1.83	5	1472.098	6.88	5	2006.364	5.15	5	2586.392	3.71	5	3246.061	2.39	5	3840.775	1.75	5	4498.869	1.13	5	5174.633	0.76	5
1356.213	1.83	5	1474.652	6.88	5	2009.844	5.15	5	2590.878	3.71	5	3251.691	2.39	5	3847.436	1.75	5	4506.672	1.13	5	5183.608	0.76	5
1358.565	1.83	5	1477.209	6.88	5	2013.33	5.15	5	2595.371	3.71	5	3257.33	2.39	5	3854.109	1.75	5	4514.488	1.13	5	5192.598	0.76	5
1360.921	1.83	5	1479.771	6.88	5	2016.821	5.15	5	2599.872	3.71	5	3262.98	2.39	5	3860.794	1.75	5	4522.318	1.13	5	5201.604	0.76	5
1363.281	1.83	5	1482.338	6.88	5	2020.319	5.15	5	2604.382	3.71	5	3268.639	2.39	5	3867.49	1.75	5	4530.162	1.13	5	5210.626	0.76	5
1365.646	1.83	5	1484.909	6.88	5	2023.823	5.15	5	2608.899	3.71	5	3274.308	2.39	5	3874.197	1.75	5	4538.019	1.13	5	5219.663	0.76	5
1368.014	1.83	5	1487.484	6.88	5	2027.333	5.15	5	2613.423	3.71	5	3279.987	2.39	5	3880.917	1.75	5	4545.889	1.13	5	5228.716	0.76	5
1370.387	1.83	5	1490.064	6.88	5	2030.85	5.15	5	2617.956	3.71	5	3285.675	2.39	5	3887.648	1.75	5	4553.773	1.13	5	5237.784	0.76	5
1372.764	1.83	5	1492.648	6.88	5	2034.372	5.15	5	2622.496	3.71	5	3291.374	2.39	5	3894.39	1.75	5	4561.671	1.13	5	5246.869	0.76	5
1375.145	1.83	5	1495.237	6.88	5	2037.9	5.15	5	2627.045	3.71	5	3297.083	2.39	5	3901.145	1.75	5	4569.583	1.13	5	5255.969	0.76	5

Parameter files for amplitude test (section 8.3)

Test file
Amplitude test

Duration	Hum_freq	Hum_int	Hum_dec	Prm_freq	Prm_int	Prm_dec	Trc_freq	Trc_int	Trc_dec	Qnt_freq	Qnt_int	Qnt_dec	Nom_freq	Nom_int	Nom_dec	Mp1_freq	Mp1_int	Mp1_dec	Mp2_freq	Mp2_int	Mp2_dec
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5
0.25	250	3.14	1	500	4.02	5	601.5125	6.78	5	771.1054	0.82	5	1000	10	5	1189.207	1.25	5	1278.247	2.43	5

Test file continued
Amplitude test

Mp3_freq	Mp3_int	Mp3_dec	SQ_freq	SQ_int	SQ_dec	ON_freq	ON_int	ON_dec	I-7_freq	I-7_int	I-7_dec	I-8_freq	I-8_int	I-8_dec	I-9_freq	I-9_int	I-9_dec	I-10_freq	I-10_int	I-10_dec	I-11_freq	I-11_int	I-11_dec
1357.388	1.83	5	1509.413	0	5	2094.588	0	5	2739.707	3.71	5	3432.568	2.39	5	4162.444	1.75	5	4919.952	1.13	5	5703.961	0.76	5
1357.388	1.83	5	1442.933	0	5	1931.873	1.716667	5	2453.778	3.71	5	2999.43	2.39	5	3561.243	1.75	5	4132.775	1.13	5	4712.189	0.76	5
1357.388	1.83	5	1442.933	0	5	1931.873	3.433333	5	2453.778	3.71	5	2999.43	2.39	5	3561.243	1.75	5	4132.775	1.13	5	4712.189	0.76	5
1357.388	1.83	5	1509.413	0	5	2094.588	5.15	5	2739.707	3.71	5	3432.568	2.39	5	4162.444	1.75	5	4919.952	1.13	5	5703.961	0.76	5
1357.388	1.83	5	1442.933	2.293333	5	1931.873	0	5	2453.778	3.71	5	2999.43	2.39	5	3561.243	1.75	5	4132.775	1.13	5	4712.189	0.76	5
1357.388	1.83	5	1509.413	2.293333	5	2094.588	1.716667	5	2739.707	3.71	5	3432.568	2.39	5	4162.444	1.75	5	4919.952	1.13	5	5703.961	0.76	5
1357.388	1.83	5	1509.413	2.293333	5	2094.588	3.433333	5	2739.707	3.71	5	3432.568	2.39	5	4162.444	1.75	5	4919.952	1.13	5	5703.961	0.76	5
1357.388	1.83	5	1442.933	2.293333	5	1931.873	5.15	5	2453.778	3.71	5	2999.43	2.39	5	3561.243	1.75	5	4132.775	1.13	5	4712.189	0.76	5
1357.388	1.83	5	1442.933	4.586667	5	1931.873	0	5	2453.778	3.71	5	2999.43	2.39	5	3561.243	1.75	5	4132.775	1.13	5	4712.189	0.76	5
1357.388	1.83	5	1509.413	4.586667	5	2094.588	1.716667	5	2739.707	3.71	5	3432.568	2.39	5	4162.444	1.75	5	4919.952	1.13	5	5703.961	0.76	5
1357.388	1.83	5	1509.413	4.586667	5	2094.588	3.433333	5	2739.707	3.71	5	3432.568	2.39	5	4162.444	1.75	5	4919.952	1.13	5	5703.961	0.76	5
1357.388	1.83	5	1442.933	4.586667	5	1931.873	5.15	5	2453.778	3.71	5	2999.43	2.39	5	3561.243	1.75	5	4132.775	1.13	5	4712.189	0.76	5
1357.388	1.83	5	1509.413	6.88	5	2094.588	0	5	2739.707	3.71	5	3432.568	2.39	5	4162.444	1.75	5	4919.952	1.13	5	5703.961	0.76	5
1357.388	1.83	5	1442.933	6.88	5	1931.873	1.716667	5	2453.778	3.71	5	2999.43	2.39	5	3561.243	1.75	5	4132.775	1.13	5	4712.189	0.76	5
1357.388	1.83	5	1442.933	6.88	5	1931.873	3.433333	5	2453.778	3.71	5	2999.43	2.39	5	3561.243	1.75	5	4132.775	1.13	5	4712.189	0.76	5
1357.388	1.83	5	1509.413	6.88	5	2094.588	5.15	5	2739.707	3.71	5	3432.568	2.39	5	4162.444	1.75	5	4919.952	1.13	5	5703.961	0.76	5

Reference file

Amplitude test

Duration	Hum_freq	Hum_int	Hum_dec	Prm_freq	Prm_int	Prm_dec	Trc_freq	Trc_int	Trc_dec	Qnt_freq	Qnt_int	Qnt_dec	Nom_freq	Nom_int	Nom_dec	Mp1_freq	Mp1_int	Mp1_dec	Mp2_freq	Mp2_int	Mp2_dec
0.25	246.7719	3.14	1	493.5438	4.02	5	593.7455	6.78	5	761.1486	0.82	5	987.0876	10	5	1173.852	1.25	5	1261.742	2.43	5
0.25	247.1999	3.14	1	494.3998	4.02	5	594.7753	6.78	5	762.4687	0.82	5	988.7996	10	5	1175.887	1.25	5	1263.93	2.43	5
0.25	247.6286	3.14	1	495.2573	4.02	5	595.8069	6.78	5	763.7911	0.82	5	990.5145	10	5	1177.927	1.25	5	1266.122	2.43	5
0.25	248.0581	3.14	1	496.1162	4.02	5	596.8402	6.78	5	765.1158	0.82	5	992.2324	10	5	1179.97	1.25	5	1268.318	2.43	5
0.25	248.4883	3.14	1	496.9767	4.02	5	597.8754	6.78	5	766.4428	0.82	5	993.9533	10	5	1182.016	1.25	5	1270.518	2.43	5
0.25	248.9193	3.14	1	497.8386	4.02	5	598.9123	6.78	5	767.7721	0.82	5	995.6772	10	5	1184.066	1.25	5	1272.721	2.43	5
0.25	249.351	3.14	1	498.702	4.02	5	599.951	6.78	5	769.1037	0.82	5	997.4041	10	5	1186.12	1.25	5	1274.929	2.43	5
0.25	249.7835	3.14	1	499.567	4.02	5	600.9916	6.78	5	770.4376	0.82	5	999.1339	10	5	1188.177	1.25	5	1277.14	2.43	5
0.25	250.2167	3.14	1	500.4334	4.02	5	602.0339	6.78	5	771.7738	0.82	5	1000.867	10	5	1190.238	1.25	5	1279.355	2.43	5
0.25	250.6507	3.14	1	501.3013	4.02	5	603.0781	6.78	5	773.1124	0.82	5	1002.603	10	5	1192.302	1.25	5	1281.574	2.43	5
0.25	251.0854	3.14	1	502.1708	4.02	5	604.124	6.78	5	774.4532	0.82	5	1004.342	10	5	1194.37	1.25	5	1283.797	2.43	5
0.25	251.5209	3.14	1	503.0417	4.02	5	605.1718	6.78	5	775.7964	0.82	5	1006.083	10	5	1196.442	1.25	5	1286.023	2.43	5
0.25	251.9571	3.14	1	503.9142	4.02	5	606.2214	6.78	5	777.1419	0.82	5	1007.828	10	5	1198.517	1.25	5	1288.254	2.43	5
0.25	252.3941	3.14	1	504.7882	4.02	5	607.2728	6.78	5	778.4898	0.82	5	1009.576	10	5	1200.595	1.25	5	1290.488	2.43	5
0.25	252.8318	3.14	1	505.6637	4.02	5	608.326	6.78	5	779.84	0.82	5	1011.327	10	5	1202.678	1.25	5	1292.726	2.43	5
0.25	253.2703	3.14	1	506.5407	4.02	5	609.3811	6.78	5	781.1925	0.82	5	1013.081	10	5	1204.764	1.25	5	1294.968	2.43	5

Reference file continued

Amplitude test

Mp3_freq	Mp3_int	Mp3_dec	SQ_freq	SQ_int	SQ_dec	ON_freq	ON_int	ON_dec	I-7_freq	I-7_int	I-7_dec	I-8_freq	I-8_int	I-8_dec	I-9_freq	I-9_int	I-9_dec	I-10_freq	I-10_int	I-10_dec	I-11_freq	I-11_int	I-11_dec
1339.861	1.83	5	1456.742	6.88	5	1985.611	5.15	5	2559.325	3.71	5	3167.267	2.39	5	3800.412	1.75	5	4450.994	1.13	5	5117.47	0.76	5
1342.185	1.83	5	1459.269	6.88	5	1989.055	5.15	5	2563.764	3.71	5	3172.76	2.39	5	3807.003	1.75	5	4458.713	1.13	5	5126.346	0.76	5
1344.513	1.83	5	1461.8	6.88	5	1992.505	5.15	5	2568.21	3.71	5	3178.263	2.39	5	3813.606	1.75	5	4466.447	1.13	5	5135.237	0.76	5
1346.845	1.83	5	1464.335	6.88	5	1995.961	5.15	5	2572.665	3.71	5	3183.775	2.39	5	3820.22	1.75	5	4474.193	1.13	5	5144.143	0.76	5
1349.18	1.83	5	1466.875	6.88	5	1999.422	5.15	5	2577.126	3.71	5	3189.297	2.39	5	3826.846	1.75	5	4481.953	1.13	5	5153.065	0.76	5
1351.52	1.83	5	1469.419	6.88	5	2002.89	5.15	5	2581.596	3.71	5	3194.828	2.39	5	3833.483	1.75	5	4489.726	1.13	5	5162.002	0.76	5
1353.864	1.83	5	1471.968	6.88	5	2006.364	5.15	5	2586.074	3.71	5	3200.369	2.39	5	3840.132	1.75	5	4497.513	1.13	5	5170.955	0.76	5
1356.213	1.83	5	1474.52	6.88	5	2009.844	5.15	5	2590.559	3.71	5	3205.92	2.39	5	3846.792	1.75	5	4505.314	1.13	5	5179.924	0.76	5
1358.565	1.83	5	1477.078	6.88	5	2013.33	5.15	5	2595.052	3.71	5	3211.48	2.39	5	3853.464	1.75	5	4513.127	1.13	5	5188.908	0.76	5
1360.921	1.83	5	1479.64	6.88	5	2016.821	5.15	5	2599.553	3.71	5	3217.05	2.39	5	3860.147	1.75	5	4520.955	1.13	5	5197.907	0.76	5
1363.281	1.83	5	1482.206	6.88	5	2020.319	5.15	5	2604.061	3.71	5	3222.629	2.39	5	3866.842	1.75	5	4528.796	1.13	5	5206.922	0.76	5
1365.646	1.83	5	1484.777	6.88	5	2023.823	5.15	5	2608.578	3.71	5	3228.219	2.39	5	3873.549	1.75	5	4536.65	1.13	5	5215.953	0.76	5
1368.014	1.83	5	1487.352	6.88	5	2027.333	5.15	5	2613.102	3.71	5	3233.818	2.39	5	3880.267	1.75	5	4544.519	1.13	5	5224.999	0.76	5
1370.387	1.83	5	1489.931	6.88	5	2030.85	5.15	5	2617.634	3.71	5	3239.426	2.39	5	3886.997	1.75	5	4552.401	1.13	5	5234.061	0.76	5
1372.764	1.83	5	1492.515	6.88	5	2034.372	5.15	5	2622.174	3.71	5	3245.045	2.39	5	3893.738	1.75	5	4560.296	1.13	5	5243.139	0.76	5
1375.145	1.83	5	1495.104	6.88	5	2037.9	5.15	5	2626.722	3.71	5	3250.673	2.39	5	3900.491	1.75	5	4568.205	1.13	5	5252.233	0.76	5